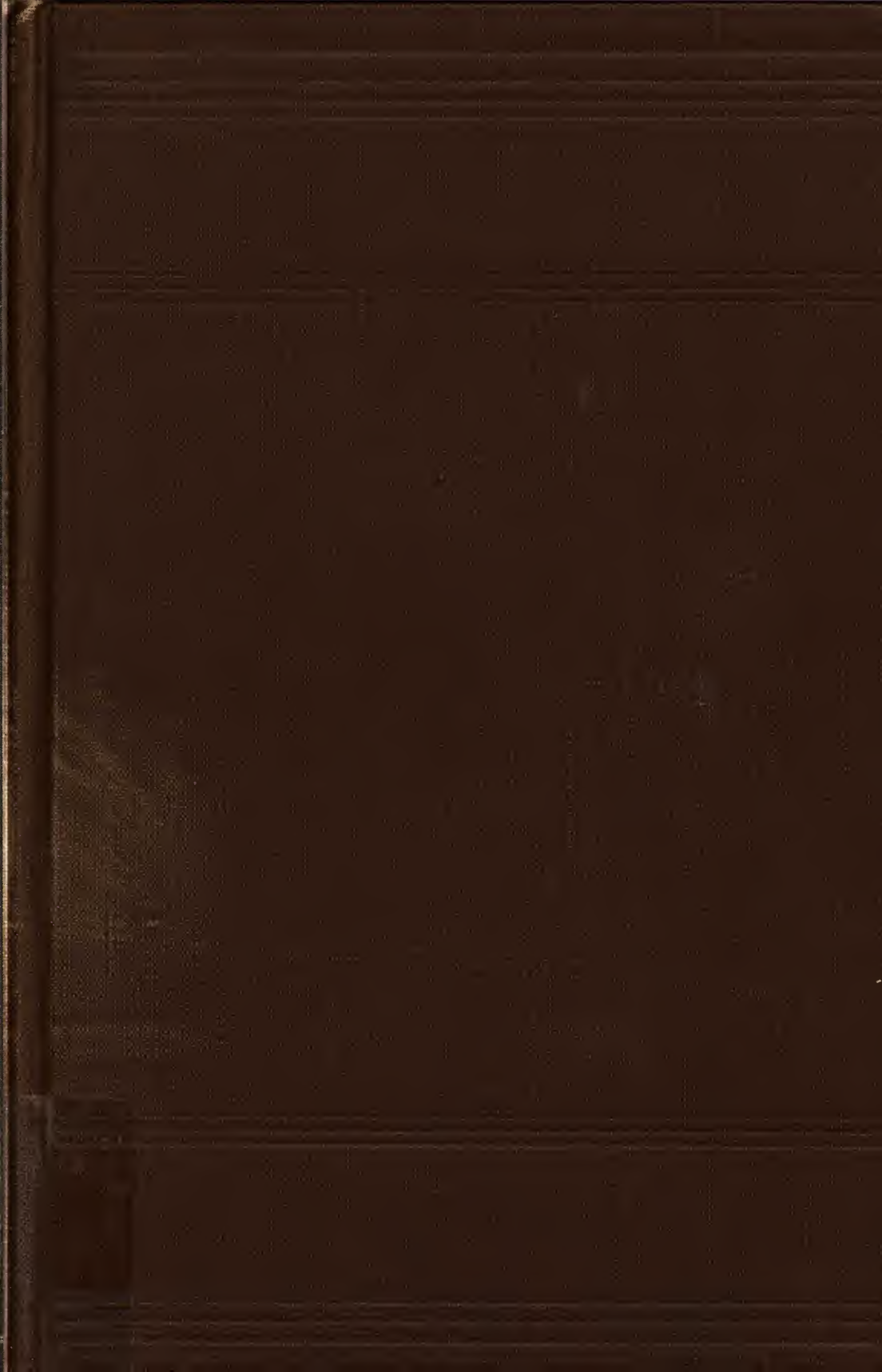
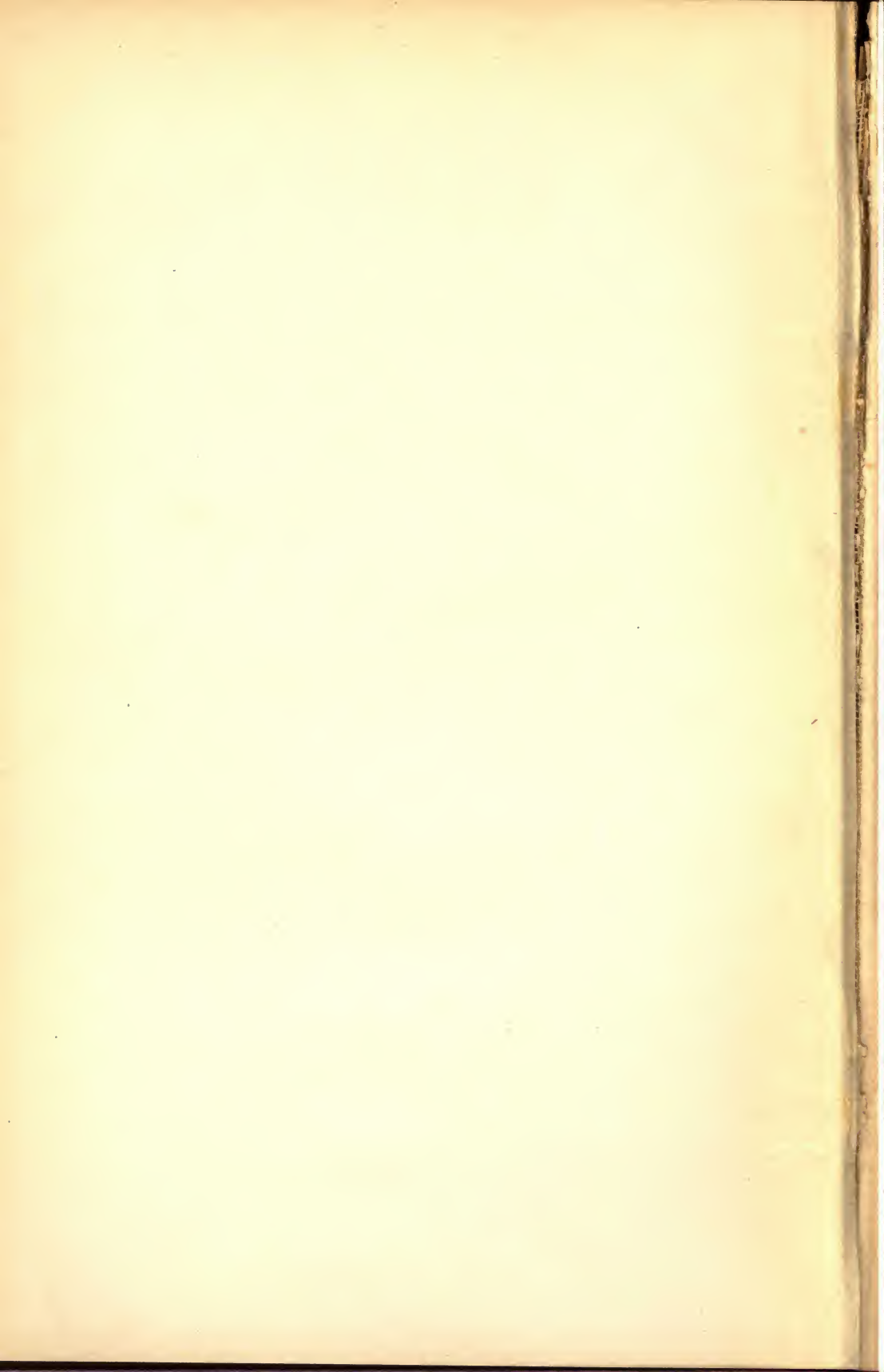
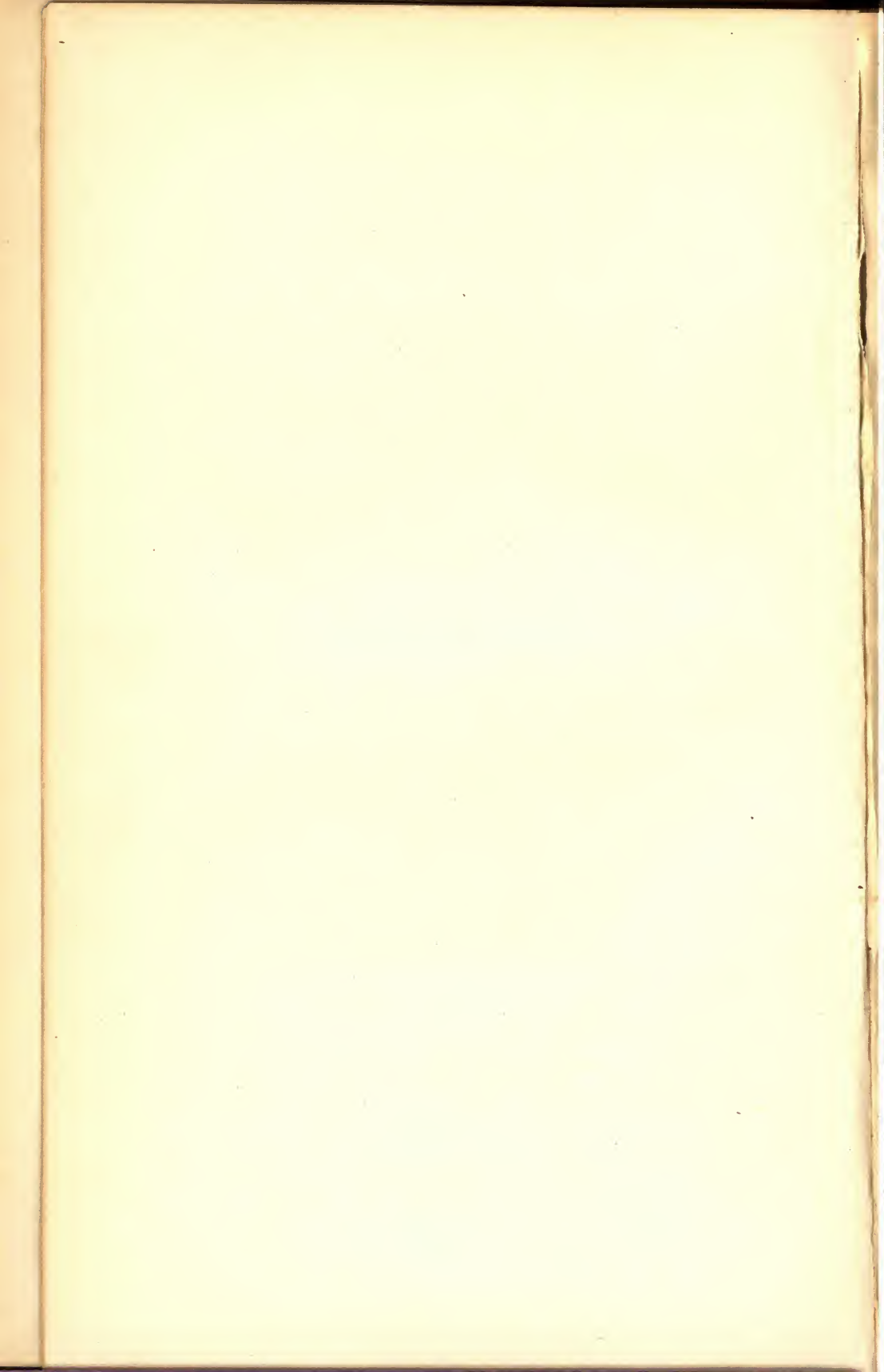






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TO THE ELASTIC THEORY

BY

MALVERD A. HOWE, M. AM. SOC. C. E.

*Professor of Civil Engineering, Rose Polytechnic Institute*

SECOND EDITION, REVISED AND ENLARGED

FIRST THOUSAND

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## PREFACE TO THE SECOND EDITION.

---

AN attempt has been made to correct the errors found in the first edition of this book and to simplify the demonstrations of the formulas. A large percentage of the text has been rewritten and considerable new matter added.

In general the unit load method has been employed. This is the only satisfactory method to use if maximum stresses are desired. In Article 89 is given a method by which the effects of certain fields of loading can be found directly.

The second example in Chapter IV has been worked out in detail and the computations arranged in tabular form so that they can be easily followed. By using these tabular forms the numerical work can be subdivided and assigned to two or more computers.

In Appendix B is given a series of arch coefficients and an example illustrating their use in transforming a given arch to one having different dimensions.

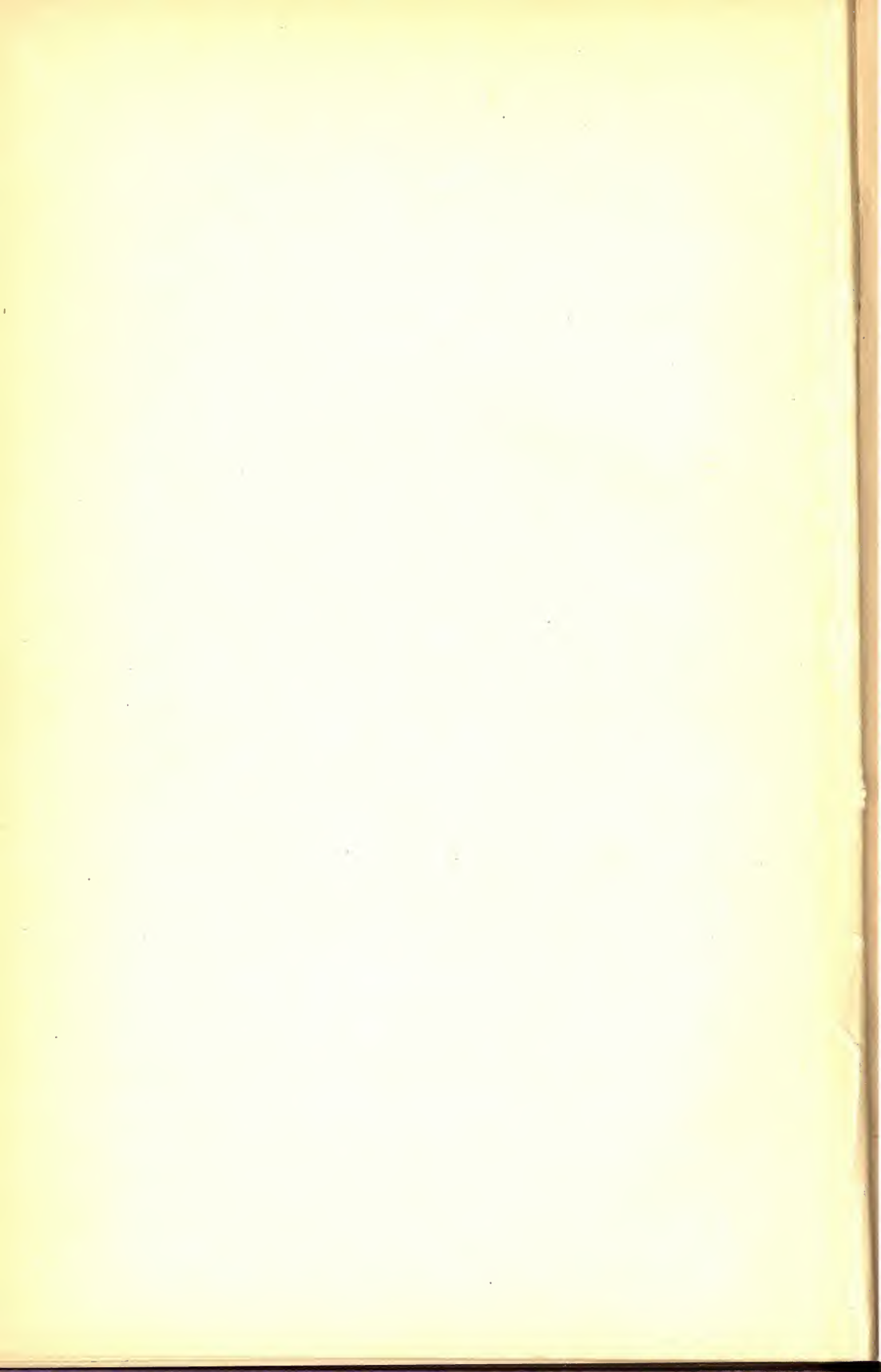
The latest knowledge concerning the maximum range of temperature in concrete arch rings is given in Appendix E.

The author is indebted to Prof. H. A. Thomas for suggestions and a careful reading of the proof.

M. A. H.

OCTOBER, 1913.





## PREFACE TO THE FIRST EDITION.

---

THE object of this book is to present in a simple form the method to be employed in the designing of masonry arches according to the *elastic theory*.

The entire subject of arches has been fully treated in the author's Treatise on Arches, in which formulas for special cases and conditions are given. Considering the fact that masonry arches are constructed of materials and under conditions which are more or less uncertain in character, the use of comprehensive or rigid formulas is not necessary or warranted. Consequently the formulas and methods here presented are somewhat approximate, but quite accurate enough for the purpose for which they are intended.

The greater portion of the book is taken up with the solution of examples, giving each step in detail so as to be easily followed by the undergraduate or the engineer who has not the time to review the theory of arches in a comprehensive manner.

The first and second examples have been solved by a somewhat longer method than necessary. This method was used in order to show clearly the several processes and checks.



In the third example will be found the simplest solution of the formulas for the horizontal thrusts and bending moments at the supports presented up to this time.

The numerical and graphical work has been given with such discrepancies as may be expected unless extraordinary care is exercised and many decimal places used. The discrepancies are of no practical importance, as the results are much nearer being exact than any masonry structure can be built, so as to fulfil the conditions upon which the calculations are based.

For the benefit of those who desire to follow precedents and as an aid in making preliminary calculations and estimates, the general data for over five hundred arch bridges have been given in tabular form with references to periodicals, etc., where more extended descriptions can be found. Without any doubt many errors exist in this table, which is quite incomplete in some particulars. The data have been derived from many sources and in some cases supplied from drawings by scaling and in others by calculations.

M. A. H.

TERRE HAUTE, July 1906.

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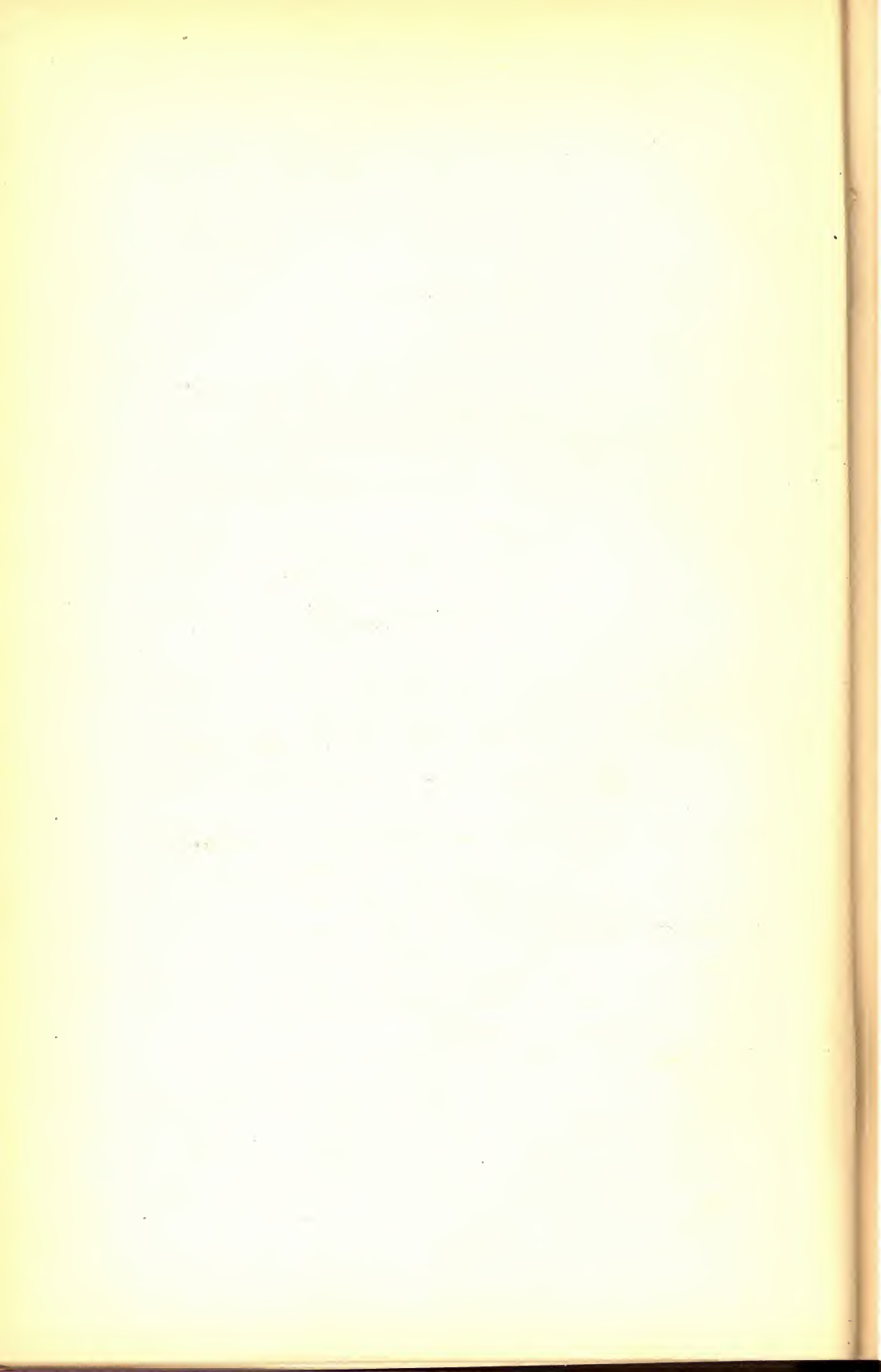
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## NOMENCLATURE.

---

THE following nomenclature refers in general to the formulas in Chapters I, II, and III:

- $A$  = the area of the concrete in a radial section of a reinforced concrete arch.
- $a$  = the area of the steel in a radial section of a reinforced concrete arch.
- $a$  = the horizontal distance from the left support to the point of application of a concentrated load.
- $a = k\delta x \div 2$  when the span is divided into  $n$  parts, each equal to  $\delta x$ .
- $b$  = the breadth of any radial section.
- $b$  = the ordinate of the point of application of a concentrated load.
- $D$  = the change in the function denoted by its subscript.
- $d$  = the distance of the reinforcing steel from the gravity axis in reinforced concrete arches.
- $E$  = Young's modulus of elasticity.
- $E_c$  = Young's modulus of elasticity for concrete.
- $E_s$  = Young's modulus of elasticity for steel.
- $e$  = the coefficient of expansion.
- $F$  = the area of any radial section.
- $F_x$  = the area of a radial section distant  $x$  from the left support.

$f$  = the rise of the arch axis in arch formulas.

$f$  = the clear rise of the intrados in descriptions and in formulas, for depth of ring.

$H$  = the value of  $H_1$  with the effect of the axial stress neglected.

$H_a$  = the horizontal thrust produced by the effect of the axial stress.

$H_1$  = the horizontal thrust at the left support produced by vertical loading.

$H_2$  = the horizontal thrust at the right support produced by vertical loading.

$H_t$  = the horizontal thrust produced by change of temperature.

$H_x$  = the horizontal component of the resultant force acting on a radial section at a distance  $x$  from the left support.

$h$  = the depth of any radial section.

[  $h_1$  = the horizontal thrust at the left support produced by horizontal loading.

$h_2$  = the horizontal thrust at the right support produced by horizontal loading.

$h_1'$  = the value of  $h_1$  with the effect of the axial stress neglected.

$h_2'$  = the value of  $h_2$  with the effect of the axial stress neglected.

$I$  = the moment of inertia.

$I_x$  = the moment of inertia of a radial section distant  $x$  from the left support.

$k = 2a \div \delta x$  when the span is divided into  $n$  equal parts of  $\delta x$  each.

$l$  = the span of an arch axis in theoretical formulas.

$l$  = the clear span in descriptions and formulas for ring thickness.

$M_1$  = the bending moment at the left support.

$M_2$  = the bending moment at the right support.

$M_x$  = the bending moment at a point in the arch axis distant  $x$  from the left support.

$m_x$  = the bending moment at a distance  $x$  from the left support considering the arch as a simple beam on two supports (vertical loading).

$m_x'$  = the bending moment at a distance  $x$  from the left support considering the arch as hinged at the right support and on rollers at the left support (horizontal loading).

$N_x$  = the normal component of the resultant force acting on a radial section distant  $x$  from the left support.

$n$  = the number of equal parts into which the span is assumed divided.

$n = E_s \div E_c$  for reinforced concrete fiber stress formulas.

$n'$  = the number of divisions between the load and the crown (Art. 30).

$P$  = any vertical concentrated load.

$p$  = the intensity of stress in the extreme fibers.

$p_u$  = the intensity of stress in the fibers distant  $u$  from the gravity axis.

$p_0$  = the average intensity of the fiber stresses for any radial section.

$Q$  = any horizontal concentrated load.

$R$  = the radius of the intrados (Appendix A, Table II).

$R_1$  = the vertical reaction at the left support considering the arch as hinged at the right support and on rollers at the left support.



$R_2$  = the corresponding reaction at the right support.

$R_x$  = the resultant force acting on a radial section distant  $x$  from the left support.

$s$  = the length of the arch axis.

$t^\circ$  = the number of degrees change of temperature.

$t_0$  = the depth of the arch ring at the crown.

$t_s$  = the depth of the arch ring at the support.

$T_x$  = the radial component of the resultant force acting on a radial section distant  $x$  from the left support.

$u$  = any distance above or below the gravity axis. Art. 1.

$V_1$  = the vertical reaction at the left support.

$V_2$  = the vertical reaction at the right support.

$V_x$  = the vertical component of the resultant force acting on a radial section distant  $x$  from the left support.

$x$  and  $y$  = coordinates of any point on the axis of the arch.

$x = z\delta x \div 2$  when the span is divided into  $n$  equal parts of  $\delta x$  each.

$x' = l - x$ .

$x_1$  = the horizontal distance of the equilibrium polygon from the left support.

$x_2$  = the horizontal distance of the equilibrium polygon from the right support.

$x_0$  = the distance of the resultant force on any section from the gravity axis of the section.

$x_0'$  = the abscissa of the apex of the equilibrium polygon for a single horizontal load.

$y$  = the ordinate to any point on the axis of the arch.

$y_a$  = the average value of  $y$ .

$y_1$  = the ordinate to the equilibrium polygon at the left support.



$y_2$  = the ordinate to the equilibrium polygon at the right support.

$y_0$  = the ordinate to the apex of the equilibrium polygon for a single vertical load.

$z$  = for radial sections, the distance to the outer fibers.

$z = 2x \div \delta x$  when the span is divided into  $n$  equal parts of  $\delta x$  each.

$z_1$  = the distance of the extreme fiber above the gravity axis of a section.

$-z_2$  = the distance of the extreme fiber below the gravity axis of a section.

$-\beta$  = the angle which the radial section at the right support makes with the vertical.

$\Delta = \delta s \div I$ .

$\Sigma$  = the sign of summation and when no limits are shown the sum is to be taken over the entire span.

$\phi_0$  = the angle which the radial section at the left support makes with the vertical.

$\phi$  = the angle which any radial section makes with the vertical.

$\left. \begin{array}{l} \delta s \\ \delta x \\ \delta y \end{array} \right\} \text{See Fig. 5, Art. 15.}$

$\delta u$  See Art. 1.

$$B = \Delta \left( y - \frac{\Sigma y \Delta}{\Sigma \Delta} \right).$$

$$C = H_1 \frac{\Sigma y \Delta}{\Sigma \Delta}.$$

$$D = \frac{\Sigma m_x \Delta}{\Sigma \Delta} = \left\{ \frac{a}{\Sigma z \Delta} + k \frac{1/2}{\Sigma \Delta} \right\} \frac{\delta x}{2 \Sigma \Delta} P.$$

$$\mathbf{E} = \left\{ (n-k) \sum_0^a (z-n)z\Delta - k \sum_a^{l/2} (z-n)^2\Delta \right\} \frac{-\delta x}{4 \sum_0^{l/2} (z-n)^2\Delta} P.$$

$$\mathbf{F} = h_1 \frac{\Sigma y \Delta}{\Sigma \Delta}.$$

$$\mathbf{G} = \left\{ \Sigma y \Delta - b \sum_a^{l/2} \Delta - \sum_0^a y \Delta \right\} \frac{Q}{\Sigma \Delta}.$$

$$\mathbf{H} = \left\{ b \sum_0^{l/2} (n-z)^2\Delta - n \sum_0^a y(z-n)\Delta + nb \sum_0^a (z-n)\Delta \right\} \frac{-Q}{2 \sum_0^{l/2} (z-n)^2\Delta}.$$

$$\mathbf{C}' = H_1 y_a.$$

$$\mathbf{D}' = \frac{1}{n} \left\{ \sum_0^a x + n'a \right\} P = \frac{\Sigma m_x}{n}.$$

$$\mathbf{E}' = (R_1 - R_2) \left\{ \sum_0^{l/2} \left( \frac{l}{2} - x \right) x + 2R_2 \sum_a^{l/2} \left( \frac{l}{2} - x \right)^2 \right\} \frac{P}{\frac{4}{l} \sum_0^{l/2} \left( \frac{l}{2} - x \right)^2}.$$

$$\mathbf{F}' = h_1 y_a.$$

$$\mathbf{G}' = \left\{ \Sigma y - \sum_0^a y - n'b \right\} \frac{Q}{n}.$$

$$\mathbf{H}' = \left\{ b \sum_0^{l/2} \left( \frac{l}{2} - x \right)^2 - \frac{l}{2} \sum_0^a \left( x - \frac{l}{2} \right) y + \frac{bl}{2} \sum_0^a \left( x - \frac{l}{2} \right) \right\} \frac{-Q}{2 \sum_0^{l/2} \left( \frac{l}{2} - x \right)^2}.$$

## DEFINITIONS AND GENERAL RELATIONS.

---

If loads acting vertically downward are placed on any structure having two supports and the resulting reactions at these supports are inclined upward, such a structure is an arch.

If the supports of an arch are immovable under all conditions of loading and the structure is either a beam or a truss continuous from one support to the other and without hinges or joints of articulation, the structure is called a fixed arch. When such a structure has one or more hinges it becomes a hinged arch. One-, two-, or three-hinged arch, according to the number of hinges.

Arches with one hinge are not common but there are many examples of two- and three-hinged arches.

The fixed arch alone will be considered, as nearly all masonry arches belong to this type. From the above definition it is evident that the fixed arch may be considered as a very general case of a beam fixed at the ends. In the case of the arch the beam is curved instead of straight. As the stresses in a straight beam or truss, fixed at the ends, cannot be found by static methods, it is clear that the stresses in a fixed arch are statically indeterminate and that the determination of the stresses must be based upon the theory of elasticity which is the foundation of the analysis of straight beams fixed at the ends.

As shown in Fig. 1, which represents a fixed arch supporting a vertical load, there are reactions and bending moments at the supports, which, if known, permit of the determination of the stresses in the arch by static methods.

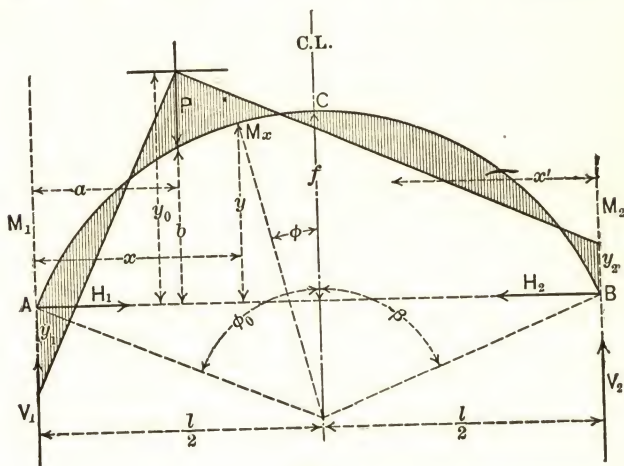


FIG. I.

Let  $H_1$  = the horizontal component of the reaction at the left support, usually designated as horizontal thrust.

$H_2$  = the horizontal component of the reaction at the right support.

$V_1$  = the vertical component of the reaction at the left support.

$V_2$  = the vertical component of the reaction at the right support.

$M_1$  = the bending moment at the left support.

$M_2$  = the bending moment at the right support.

$P$  = any vertical load acting downward.

$a$  = the horizontal distance of  $P$  from the left support.



$x$  and  $y$  = the coordinates of any point in the axis of the arch, the line which passes through the centers of gravity of the radial sections of the arch.

and  $M_x$  = the bending moment at any point having the coordinates  $x$  and  $y$ .

The arch is assumed to be symmetrical about the center of the span. Since the algebraic sum of the horizontal components of the forces acting upon the arch must equal zero for equilibrium,

$$H_1 = H_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The bending moment at any point having the coordinates  $x$  and  $y$ , for a single vertical load acting downward, is

$$M_x = M_1 + V_1x - H_1y - P(x-a)^{x>a}. \quad . \quad . \quad . \quad (2)$$

The expression  $(x-a)^{x>a}$  means that the term is used in (2) only when the load  $P$  is upon the left of the center of moments, or when  $x$  is greater than  $a$ .

If in (2),  $x=l$ , then  $y=0$ ,  $M_x=M_2$  and

$$M_2 = M_1 + V_1l - P(l-a). \quad . \quad . \quad . \quad . \quad . \quad (3)$$

From (3)

$$V_1 = \frac{M_2 - M_1}{l} + P \frac{(l-a)}{l}. \quad . \quad . \quad . \quad . \quad (4)$$

The term  $P \frac{l-a}{l}$  represents the vertical reaction at the left support of the load  $P$  when placed upon a straight beam supported at the ends. Let this be called the common reaction and be designated by  $R_1$ , then

$$V_1 = \frac{M_2 - M_1}{l} + R_1. \quad . \quad . \quad . \quad . \quad (5)$$



Substituting (5) in (2) it becomes

$$M_x = M_1 - H_1 y + \frac{M_2 - M_1}{l} x + R_1 x - P(x-a)^{x>a}. \quad (6)$$

The terms  $R_1 x - P(x-a)^{x>a}$  represent the bending moment produced by  $P$  when placed on a straight beam supported

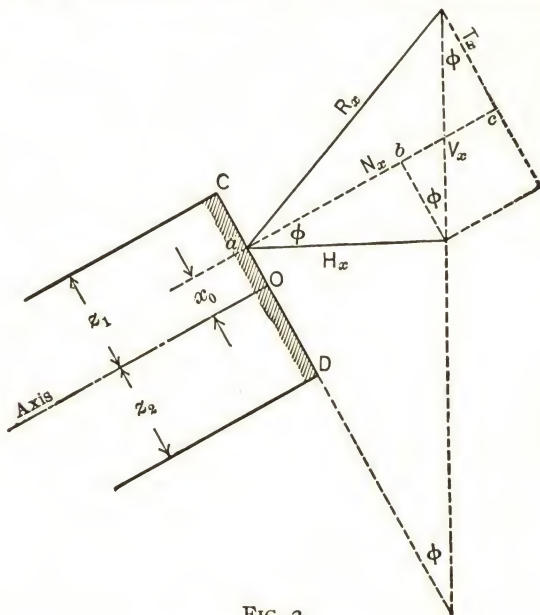


FIG. 2.

at the ends, the span of the beam being that of the arch and the distances  $a$  and  $x$  remaining unchanged in value. Let this moment be called the common moment and be represented by  $m_x$ , then

$$M_x = M_1 + \frac{M_2 - M_1}{l} x - H_1 y + m_x. \quad (7)$$

This equation is true for a straight beam fixed at the ends by making  $y$  equal zero.

In Fig. 2 let  $R_x$  be the direction and magnitude of the resultant of all forces acting upon the radial section  $CD$  taken through the point  $O$ , on the axis, having the coordinates  $x$  and  $y$ , then

$N_x$  = the normal component of  $R_x$ ;

$T_x$  = the radial component of  $R_x$ ;

$V_x$  = the vertical component of  $R_x$ ;

$H_x$  = the horizontal component of  $R_x$ .

Considering vertical loads only and assuming that the section makes the angle  $\phi$  with the vertical,

$$N_x = cb + ba = V_x \sin \phi + H_x \cos \phi. \quad \dots \quad (8)$$

But  $V_x = V_1 - (P \text{ when } x > a) \quad \dots \quad (9)$

and  $H_x = H_1 \quad \dots \quad (10)$

hence

$$N_x = V_1 \sin \phi + H_1 \cos \phi - P \sin \phi \quad \dots \quad (11)$$

In like manner

$$T_x = V_x \cos \phi - H_x \sin \phi, \quad \dots \quad (12)$$

or

$$T_x = V_1 \cos \phi - H_1 \sin \phi - P \cos \phi \quad \dots \quad (13)$$

Since the moment of the external forces equals the moment of the internal forces at any section  $M_x = N_x x_0$ . If  $I_x$  represents the moment of inertia of the section  $CD$

about an axis through  $O$  normal to the page, and  $F_x$  represents the area of the section, the fiber stress at  $C$  is

$$p = \frac{N_x}{F_x} + \frac{M_x z_1}{I_x} = \frac{N_x}{F_x} + \frac{N_x x_0 z_1}{I_x}, \quad . \quad . \quad . \quad . \quad (14)$$

and the fiber stress at  $D$  is

$$p = \frac{N_x}{F_x} - \frac{M_x z_2}{I_x} = \frac{N_x}{F_x} - \frac{N_x x_0 z_2}{I_x}. \quad . \quad . \quad . \quad . \quad (15)$$

# SYMMETRICAL MASONRY ARCHES.

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## CHAPTER I.

### FUNDAMENTAL FORMULAS.

1. **Angular Distortion Produced by Pure Bending.** Let Fig. 3 represent a curved beam fixed at  $B$  and free at  $A$ . The cross-section of the beam is assumed to be rectangular at all points as shown in Fig. 3a. The line  $AOO'UB$  which passes through the centers of gravity of the cross-sections is called the axis of the beam. The nomenclature used is indicated in Figs. 3 and 3a.

Assume that all parts of the beam are rigid except the short piece between the two radial sections  $CD$  and  $KL$ , and let the angle made between the  $CD$  and  $KL$  be assumed so small that  $CK$  equals  $DL$  equals  $\delta s$  for practical purposes. Of course, this is not the case unless the beam is straight, but when the depth  $CD$  is small in comparison with the radius of curvature, the assumption leads to very small error.

When this short length of the beam is subjected to pure bending the fibers on one side of the axis are lengthened and those upon the other side are shortened. In this case the upper fibers are lengthened and the lower fibers shortened, so that  $C$  moves to  $C'$  and  $D$  to  $D'$ . According to



the common theory of flexure the plane section  $CD$  before bending remains a plane throughout the distortion, so

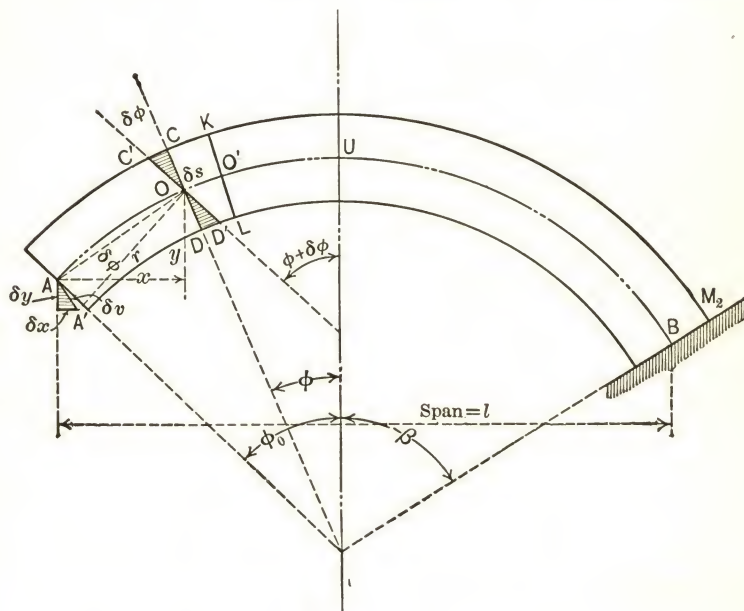


FIG. 3.

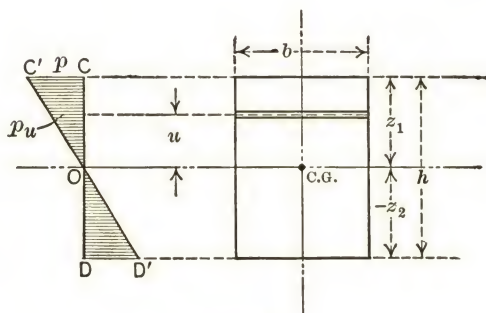


FIG. 3a.

that  $C'D'$  in Fig. 3 represents the new position of the section  $CD$ .

If  $p$  represents the intensity of the fiber stress at  $C$  and  $p_u$  that at a distance  $u$  above  $O$  (Fig. 3a), then

$$p_u = \frac{pu}{z_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Let the area of each fiber be  $\delta u$  and the breadth of the section  $b$ , then the total stress on the fibers at a distance  $u$  above  $O$ , is

$$p_u(b\delta u) = \frac{pu}{z_1}(b\delta u), \quad . \quad . \quad . \quad . \quad . \quad (17)$$

and the moment of this stress about the axis through  $O$ , is

$$p_u u(b\delta u) = \frac{p}{z_1}(b\delta u)u^2. \quad . \quad . \quad . \quad . \quad . \quad (18)$$

The sum of the moments of all of the fiber stresses above and below the axis through  $O$ , is

$$\sum_{-z_2}^{z_1} p_u u(b\delta u) = \frac{p}{z_1} \sum_{z_1 - z_2}^{z_1} (b\delta u)u^2 = \frac{p}{z_1} I_x, \quad . \quad . \quad (19)$$

where  $I_x$  is the moment of inertia of the section with reference to the horizontal axis shown in Fig. 3a.

Since the external and internal moments are equal

$$M_x = \frac{p}{z_1} I_x, \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

which is the usual expression for straight beams subjected to pure bending.

Let  $E_x$  represent the modulus of elasticity (Young's modulus) for this section of the beam, then, since

$$E_x = \frac{\text{Stress per unit area}}{\text{Strain per unit length}}, \quad \dots \quad (20a)$$

$$E_x = \frac{p}{\frac{CC'}{\delta s}} = \frac{p}{z_1 \delta \phi}.$$

$$\therefore p = E_x z_1 \frac{\delta \phi}{\delta s}. \quad \dots \quad (21)$$

Substituting this value of  $p$  in (20) and solving for  $\delta \phi$ ,

$$\delta \phi = \frac{M_x \delta s}{E_x I_x}. \quad \dots \quad (22)$$

This represents the change in the angle  $\phi$  produced by pure bending distorting only the small portion of the beam  $CKLD$ . If all of the beam between  $A$  and  $U$  is distorted by pure bending, the change in the angle  $\phi_0$  is

$$D_{\phi_0} = \sum_A^U \delta \phi = \sum_A^U \frac{M_x \delta s}{E_x I_x}. \quad \dots \quad (23)$$

If the beam is elastic for its entire length between  $A$  and  $B$ , the effect of pure bending upon the total angle  $\phi_0 + (-\beta)$  is

$$D_{\phi_0} + D_{-\beta} = \sum_A^U \frac{M_x \delta s}{E_x I_x} + \sum_U^B \frac{M_x \delta s}{E_x I_x} = \sum_A^B \frac{M_x \delta s}{E_x I_x} = \sum_0^l \frac{M_x \delta s}{E_x I_x}. \quad (A')$$

If the top fibers are shortened instead of lengthened, then

$$D_{\phi_0} + D_{-\beta} = - \sum_0^l \frac{M_x \delta s}{E_x I_x}. \quad \dots \quad (A)$$

**2. Changes in the Coordinates  $x$  and  $y$  Produced by Pure Bending.**—Referring to Fig. 3 it appears that, when the short portion of the beam is distorted, the point  $A$  moves to  $A'$ , revolving about  $O$  as a center, and that  $x$  is shortened an amount  $\delta x$  and  $y$  lengthened an amount  $\delta y$ . From Fig. 3,

$$\left. \begin{aligned} \delta y : \delta v :: x : AO & \text{ or } (AO) \delta y = x \delta v = x(AO) \delta \phi \\ \delta x : \delta v :: y : AO & \text{ or } (AO) \delta x = y \delta v = y(AO) \delta \phi \end{aligned} \right\} \quad (25)$$

Substituting the value of  $\delta \phi$  from (22),

$$\delta y = \frac{M_x}{E_x} \cdot \frac{\delta s}{I_x} x \quad \text{and} \quad \delta x = \frac{M_x}{E_x} \frac{\delta s}{I_x} y. \quad (26)$$

If the entire beam is elastic the change in the horizontal distance between  $A$  and  $B$  is, when the upper fibers are shortened,

$$D_l = + \sum_A^B \frac{M_x}{E_x} \cdot \frac{\delta s}{I_x} x = \sum_0^l \frac{M_x}{E_x} \frac{\delta s}{I_x} y, \quad (B)$$

and the total change in  $C$ , the vertical distance between  $A$  and  $B$  is

$$D_c = \sum_A^B \frac{M_x}{E_x} \frac{\delta s}{I_x} x = + \sum_0^l \frac{M_x}{E_x} \frac{\delta s}{I_x} x. \quad (C)$$

Pure bending produces no change in  $s$ , the length of the axis of the beam, as the axis is assumed to lie in the neutral plane, in which the fibers remain unchanged in length.

**3. Changes in  $s$ ,  $x$ , and  $y$  Produced by a Direct or Axial Stress.**—A direct or axial stress is one producing a uniform normal intensity over the entire area of the section of the beam considered. In Fig. 4, let  $R_x$  be the



resultant force acting upon the section  $CD$  and let  $N_x$  be the normal component of  $R_x$ . If two equal and opposite forces  $N_x$  are applied at  $O$  no change in equilibrium

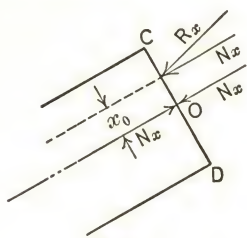


FIG. 4.

will follow. One of these forces and the normal component of  $R_x$  form a couple with an arm  $x_0$ . The moment of this couple is  $N_x x_0$  and produces pure bending, the effect of which has been considered above. The other force  $N_x$  being applied at the center of gravity of the section produces a

uniform intensity of stress over the entire section and is a direct or axial stress. Let  $F_x$  equal the area of the section and  $p_0$  the uniform intensity or  $p_0 = N_x \div F_x$ .

Referring to Fig. 3, it is evident if each fiber of the portion of the beam represented by  $CKLD$  is  $\delta s$  in length, that each one will be changed in length the same amount by the force having the intensity  $p_0$ , assuming Young's modulus to be constant. If  $N_x$  is a compressive stress each fiber will be shortened an amount represented by  $p_0 \delta s \div E_x$ . Assuming the entire beam to be elastic and each section subjected to an axial stress, the resulting decrease in the length of  $s$ , the length of the axis between  $A$  and  $B$  is

$$D_s = - \sum_A^B \frac{p_0 \delta s}{E_x} = - \sum_0^l \frac{p_0 \delta s}{E_x} \dots \dots \dots (27)$$

Corresponding changes in  $x$  and  $y$  occur, and for the entire beam between  $A$  and  $B$ ,

$$D_i = - \sum_A^B \frac{p_0 \delta x}{E_x} = - \sum_0^l \frac{p_0 \delta x}{E_x}, \dots \dots \dots (B_1)$$

and

$$D_c = + \sum_A^B \frac{p_0 \delta y}{E_x} = + \sum_0^l \frac{p_0 \delta y}{E_x}. \quad \dots \quad (C_1)$$

**4. Changes in  $s$ ,  $x$ , and  $y$  Produced by a Rise in Temperature.**

Let  $e$  = the coefficient of expansion for a change of one degree in temperature and  $t^\circ$  = the number of degrees change in temperature. Then

$$D_s = et^\circ \sum_A^B \delta s = et^\circ \sum_0^l \delta s = et^\circ s; \quad \dots \quad (28)$$

$$D_l = et^\circ \sum_A^B \delta x = et^\circ \sum_0^l \delta x = et^\circ l; \quad \dots \quad (B_2)$$

$$-D_c = et^\circ \sum_A^B \delta y = et^\circ \sum_0^l \delta y. \quad \dots \quad (C_2)$$

**5. The Combination of the Effects of Pure Bending, Axial Stress and Changes in Temperature.**—Combining the formulas deduced in the previous articles,

$$D\phi_0 + D_{-\beta} = - \sum_0^l \frac{M_x \delta s}{E_x I_x}; \quad \dots \quad (29)$$

$$D_l = + \sum_0^l \frac{M_x \delta s}{E_x I_x} y - \sum_0^l \frac{p_0 \delta x}{E_x} + et^\circ l; \quad \dots \quad (30)$$

$$-D_c = - \sum_0^l \frac{M_x \delta s}{E_x I_x} x - \sum_0^l \frac{p_0 \delta y}{E_x} + et^\circ \sum_0^l \delta y. \quad \dots \quad (31)$$

These three equations are general and apply to any curved beam, with long radii of curvature, fixed at one end and free at the other, or to a portion of the beam if the proper limits of summation are used. The beam may have any form of cross-section to which the common theory of flexure applies.

Referring to Fig. 3, if the beam is unsupported at  $A$ , there is a bending moment at  $B$  which will be called  $M_2$ . This condition is that of a curved cantilever beam. The internal stresses are statically determinate and the distortions or changes in the central angle, the span, etc., are found from the three general equations (29), (30), and (31).

If there is a support at  $A$  which prevents vertical movement but permits of horizontal movement and there is no restraint against turning about  $A$ , the structure is a curved beam fixed at one end and supported at the other. Since all vertical movement is prevented  $D_v = 0$  or

$$e t^0 \sum_0^l \delta y - \sum_0^l \frac{M_x \delta s}{E_x I_x} x - \sum_0^l \frac{p_0 \delta y}{E_x} = 0. \quad (32)$$

From (7)

$$M_x = M_2 \frac{x}{l} + m_x, \quad \text{since } M_1 \text{ and } H_1 \text{ are zero.}$$

From (11) and (5) for a single vertical load, .

$$\begin{aligned} N_x &= V_1 \sin \phi + H_1 \cos \phi - (P \sin \phi)^{x>a} = V_1 \sin \phi - (P \sin \phi)^{x>a} \\ &= \left\{ \frac{M_2}{l} + R_1 \right\} \sin \phi - (P \sin \phi)^{x>a}. \quad (33) \end{aligned}$$

By definition  $p_0 = N_x \div F_x$ , hence

$$p_0 = \left\{ \frac{M_2}{l} + R_1 \right\} \frac{\sin \phi}{F_x} - \frac{P \sin \phi}{F_x}^{x>a}. \quad (34)$$

If the values of  $M_x$  and  $p_0$  just obtained are substituted in the general equation (32),  $M_2$  is the only unknown and hence its value can be found.

When both horizontal and vertical movements are prevented at  $A$ , but rotation about  $A$  is not restrained, the end  $B$ , Fig. 1, remaining fixed, the structure becomes an arch, according to the definition of an arch, fixed at one end and hinged at the other. This is a special case of the one-hinged arch.

Since no vertical displacement can take place,  $D_v=0$ , and since no horizontal movement is permitted,  $D_h=0$ , hence, from (30) and (31),

$$+\sum_0^l \frac{M_x \delta s}{E_x I_x} y - \sum_0^l \frac{p_0 \delta x}{F_x} + e t^o l = 0 \quad . \quad . \quad . \quad (35)$$

$$-\sum_0^l \frac{M_x \delta s}{E_x I_x} x - \sum_0^l \frac{p_0 \delta y}{F_x} + e t^o \sum_0^l \delta y = 0 \quad . \quad . \quad . \quad (36)$$

From (7)

$$M_x = M_2 \frac{x}{l} - H_1 y + m_x, \quad . \quad . \quad . \quad (37)$$

and from (11) and (5)

$$N_x = \left\{ \frac{M_2}{l} + R_1 \right\} \sin \phi + H_1 \cos \phi - P \sin^{\frac{x>a}{}} \phi, \quad . \quad (38)$$

or

$$p_0 = \left\{ \frac{M_2}{l} + R_1 \right\} \frac{\sin \phi}{F_x} + H_1 \frac{\cos \phi}{F_x} - \frac{P \sin^{\frac{x>a}{}} \phi}{F_x}. \quad . \quad (39)$$

After (37) and (39) are substituted in (35) and (36) but two unknowns appear,  $M_2$  and  $H_1$ , and hence their values can be obtained by the usual methods of elimination. In case there are hinges at  $A$  and  $B$ , but all vertical and horizontal movements prevented, the equations just given hold true when  $M_2$  is made zero. The value of the remaining unknown  $H_1$  can be found



from either of the two summation equations (35) and (36). This type of structure is a two-hinged arch.

In case horizontal and vertical movements at  $A$  and  $B$ , Fig. 1, are prevented and there are no hinges, the central angle, the span and the relative elevations of the supports remain unchanged, or

$$D_{\phi} + D_{-\beta} = 0, \quad D_t = 0, \quad \text{and} \quad D_c = 0.$$

When these conditions are imposed the structure is called a fixed arch. Before the general equations are written in full, the effect of  $N_x$ , the axial stress, will be considered.

**6. Effect of the Axial Stress and Assuming the Modulus of Elasticity as Constant.\*** (Fixed Arch.)—The effect of the axial stress  $N_x$  upon the value of  $H_1$ , the horizontal thrust, is small, excepting in arches, which are quite flat. For symmetrical fixed arches having a ratio of rise to span of one-tenth the effect of the axial stress is to reduce the magnitude of the horizontal thrust,  $H_1$ , about 30%, while for a ratio of one-fifth this percentage drops to about 10%. While the effect of the axial stress upon  $H_1$  may be small, the effect upon the fiber stresses may be quite large. In important structures its effect should be considered. In the formulas to be deduced the axial stress terms will be given. These terms are dropped when the effect of the axial stress is neglected.

From (11) and (5),

$$N_x = \frac{M_2 - M_1}{l} \sin \phi + R_1 \sin \phi + H_1 \cos \phi - (P \sin \phi)^{x>a}. \quad (40)$$

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\* See "A Treatise on Arches," by Malverd A. Howe, John Wiley & Sons, New York.

Investigation has shown that the terms containing  $\sin \phi$  can be neglected without serious error when the effect of the axial stress is considered, so that

$$N_x = H_1 \cos \phi, \text{ approximately.} \quad . \quad . \quad . \quad (41)$$

The terms, in the general summation equations which contain  $p_0 = N_x \div F_x$ , now become

$$\sum_0^l \frac{p_0 \delta x}{E_x} = H_1 \sum_0^l \frac{\delta x \cos \phi}{E_x F_x} = H_1 \sum_0^l \frac{(\delta x)^2}{E_x F_x \delta s}, \quad . \quad . \quad (42)$$

and

$$\sum_0^l \frac{p_0 \delta y}{E_x} = H_1 \sum_0^l \frac{\delta y \cos \phi}{E_x F_x} = H_1 \sum_0^l \frac{\delta x \delta y}{E_x F_x \delta s}. \quad . \quad . \quad (43)$$

Assuming the modulus of elasticity  $E_x$  as constant, the three general equations (29), (30), and (31) become for the symmetrical fixed arch,

$$D_{\phi_0} + D_{-\beta} = \frac{1}{E} \left\{ - \sum_0^l M_x \Delta \right\} = 0; \quad . \quad . \quad . \quad . \quad (44)$$

$$D_l = \frac{1}{E} \left\{ + \sum_0^l M_x y \Delta - H_1 \sum_0^l \frac{(\delta x)^2}{F_x \delta s} \right\} + et^0 l = 0; \quad (45)$$

$$D_c = \frac{1}{E} \left\{ - \sum_0^l M_x x \Delta \right\} = 0, \quad . \quad . \quad . \quad . \quad (46)$$

where  $\Delta = \delta s \div I_x$ . The terms containing  $\delta y$  reduce to zero since for each  $+\delta y$  there is a  $-\delta y$ .

These equations are perfectly general for any *symmetrical* fixed arch subjected to any loading and are sufficiently exact for all practical purposes. While vertical loads alone have been considered the expressions for  $M_x$  and  $N_x$  are readily modified to include horizontal loads.

Equations (44), (45), and (46) may be written as follows:

$$-\sum_0^l M_x \Delta = 0; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

$$+\sum_0^l M_x y \Delta - H_1 \sum_0^l \frac{(\delta x)^2}{F_x \delta s} + e t^\circ l E = 0; \quad . \quad . \quad (48)$$

$$-\sum_0^l M_x x \Delta = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

## CHAPTER II.

### SYMMETRICAL ARCHES FIXED AT THE ENDS

(VERTICAL LOADS.)

**7. Conditions which Must be Satisfied.** The last three equations in Art. 6 express the conditions which must obtain for a fixed arch. From these equations the values of  $H_1$ ,  $M_1$ , and  $M_2$  are found for any vertical loading and the intermediate bending moments then found by static methods.

For convenience the three equations of condition are repeated with the limits of summation omitted wherever they are  $l$  and  $o$ . The sign  $\Sigma$  indicates that the sum is to be taken over the entire span of the arch.

$$-\Sigma M_x \Delta = 0; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

$$+\Sigma M_{xy}\Delta-H_1\Sigma\frac{(\delta x)^2}{F_x\delta S}+et^0lE=0; \quad . \quad . \quad (5I)$$

$$-\Sigma M_x x \Delta = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (52)$$

8. Determination of the Horizontal Thrust,  $H_1$ , Produced by a Single Vertical Load and a Change of Temperature. From (7),

$$M_x = M_1 + \frac{M_2 - M_1}{l} x - H_1 y + m_x. \quad . \quad . \quad (7)$$



Substituting (7) in (50), (51), and (52) they become

$$-M_1 \Sigma \Delta - \frac{M_2 - M_1}{l} \Sigma x \Delta + H_1 \Sigma y \Delta - \Sigma m_x \Delta = 0 \quad . \quad . \quad . \quad (53)$$

$$+ M_1 \Sigma y \Delta + \frac{M_2 - M_1}{l} \Sigma xy \Delta - H_1 \Sigma y^2 \Delta + \Sigma m_x y \Delta \\ - H_1 \Sigma \frac{(\delta x)^2}{F_x \delta s} + et^0 l E = 0. \quad . \quad . \quad . \quad . \quad . \quad (54)$$

$$-M_1 \Sigma x \Delta - \frac{M_2 - M_1}{l} \Sigma x^2 \Delta + H_1 \Sigma xy \Delta - \Sigma m_x x \Delta = 0, \quad . \quad (55)$$

in which there are but three unknowns,  $H_1$ ,  $M_1$ , and  $M_2$ .

These equations can be simplified by replacing  $\Sigma x \Delta$  by  $\frac{1}{2} l \Sigma \Delta$  and  $\Sigma xy \Delta$  by  $\frac{1}{2} l \Sigma y \Delta$ . That the expressions are equivalent is shown as follows: For any two symmetrical points of the arch the values of  $\Delta$  are the same and the sum of the values of  $x$  for the points is  $l$ . If the sum  $x \Delta$  for each pair of symmetrical points be taken and then the sum of the results  $\Sigma x \Delta = \frac{1}{2} \Sigma l \Delta = \frac{1}{2} l \Sigma \Delta$ . In like manner  $\Sigma xy \Delta = \frac{1}{2} l \Sigma y \Delta$ .

Replacing  $\Sigma x \Delta$  and  $\Sigma xy \Delta$  by their equivalents (53), (54), and (55) become

$$-M_1 \Sigma \Delta - \frac{1}{2} (M_2 - M_1) \Sigma \Delta + H_1 \Sigma y \Delta - \Sigma m_x \Delta = 0 \quad . \quad . \quad (56)$$

$$+ M_1 \Sigma y \Delta + \frac{1}{2} (M_2 - M_1) \Sigma y \Delta - H_1 \Sigma y^2 \Delta + \Sigma m_x y \Delta$$

$$- H_1 \Sigma \frac{(\delta x)^2}{F_x \delta s} + et^0 l E = 0 \quad . \quad . \quad . \quad . \quad . \quad (57)$$

$$- \frac{1}{2} M_1 l \Sigma \Delta - \frac{1}{l} (M_2 - M_1) \Sigma x^2 \Delta + \frac{1}{2} H_1 l \Sigma y \Delta - \Sigma m_x x \Delta = 0. \quad (58)$$

If (56) is multiplied by  $\Sigma y \Delta$  and (57) by  $\Sigma \Delta$  and the

results added, the terms containing  $M_1$  and  $M_2$  cancel and the following expression for  $H_1$  is obtained:

$$H_1 = \frac{\Sigma m_x y \Delta - \Sigma m_x \Delta \frac{\Sigma y \Delta}{\Sigma \Delta} + et^{\circ} l E}{\Sigma y^2 \Delta - \Sigma y \Delta \frac{\Sigma y \Delta}{\Sigma \Delta} + \Sigma \frac{(\delta x)^2}{F_x \delta s}} \quad (59)$$

This reduces to

$$H_1 = \frac{\Sigma m_x \Delta \left\{ y - \frac{\Sigma y \Delta}{\Sigma \Delta} \right\} + et^{\circ} l E}{\Sigma y \Delta \left\{ y - \frac{\Sigma y \Delta}{\Sigma \Delta} \right\} + \Sigma \frac{(\delta x)^2}{F_x \delta s}} \quad (60)$$

where

$$m_x = P \frac{l-a}{l} x - P(x-a)^{x>a}, \quad (6a)$$

and

$$\Delta = \delta s \div I_x.$$

**9. The Horizontal Thrust Produced by a Single Vertical Load, Changes of Temperature and the Axial Stress, Separated into Parts.**

Let  $H$  = the horizontal thrust produced by a vertical load with the effect of the axial stress neglected;

$H_t$  = the horizontal thrust produced by a change of temperature with the effect of the axial stress neglected;

and  $H_a$  = the horizontal thrust produced by the axial stress alone.

Then, from (60),

$$H = \frac{\Sigma m_x \Delta \left( y - \frac{\Sigma y \Delta}{\Sigma \Delta} \right)}{\Sigma y \Delta \left( y - \frac{\Sigma y \Delta}{\Sigma \Delta} \right)} = \frac{\Sigma m_x \mathbf{B}}{\Sigma y \mathbf{B}}; \quad (61)$$

$$H_t = \frac{et^0 l E}{\Sigma y \mathbf{B}}; \quad . . . . . (62)$$

$$H_a = \left\{ \frac{-\Sigma \frac{(\delta x)^2}{F_x \delta s}}{\Sigma y \mathbf{B} + \Sigma \frac{(\delta x)^2}{F_x \delta s}} \right\} \left\{ \begin{array}{l} H \text{ for vertical loads} \\ H_t \text{ for changes of tempt.} \end{array} \right\}. \quad (63)$$

The negative sign signifies that  $H_a$  is opposite in character to  $H$  or  $H_t$ .

The value of  $m_x$  in (61) can be found by computation or from an equilibrium polygon. Although  $m_x$  has been considered as the common moment for a single vertical load, yet any number of vertical loads can be considered by determining  $m_x$  for each value of  $x$  for all of the loads. A single equilibrium polygon is sufficient.

Usually it is better to consider a single unit load moving across the span and draw an influence line for  $H$ , then the value of  $H$  for any vertical load can be found by simple multiplication.

**10. Determination of the Bending Moment  $M_1$ , at the Left Support Produced by a Single Vertical Load.**—Multiplying (56)

by  $\Sigma x^2 \Delta$  and (58) by  $\frac{l}{2} \Sigma \Delta$  they become

$$\begin{aligned} -M_1 \Sigma \Delta \Sigma x^2 \Delta - \frac{1}{2} (M_2 - M_1) \Sigma \Delta \Sigma x^2 \Delta + H_1 \Sigma y \Delta \Sigma x^2 \Delta \\ - \Sigma m_x \Delta \Sigma x^2 \Delta = 0, \quad . . . . . (64) \end{aligned}$$

$$\begin{aligned} -\frac{1}{4} l^2 M_1 \Sigma \Delta \Sigma \Delta - \frac{1}{2} (M_2 - M_1) \Sigma \Delta \Sigma x^2 \Delta + \frac{1}{4} l^2 H_1 \Sigma y \Delta \Sigma \Delta \\ - \frac{1}{2} l \Sigma m_x \Delta \Sigma \Delta = 0. \quad . . . . . (65) \end{aligned}$$

subtracting (65) from (64) and solving for  $M_1$ ,

$$M_1 = H_1 \frac{\Sigma y \Delta \left( \frac{1}{4} l^2 \Sigma \Delta - \Sigma x^2 \Delta \right)}{\Sigma \Delta \left( \frac{1}{4} l^2 \Sigma \Delta - \Sigma x^2 \Delta \right)} - \frac{\Sigma m_x \Delta \left( \frac{1}{2} l x \Sigma \Delta - \Sigma x^2 \Delta \right)}{\Sigma \Delta \left( \frac{1}{4} l^2 \Sigma \Delta - \Sigma x^2 \Delta \right)} \quad (66)$$

$$= H_1 \frac{\Sigma y \Delta}{\Sigma \Delta} - \frac{\Sigma m_x \Delta \left( \frac{1}{2} l x - \frac{\Sigma x^2 \Delta}{\Sigma \Delta} \right)}{\Sigma \Delta \left( \frac{1}{4} l^2 - \frac{\Sigma x^2 \Delta}{\Sigma \Delta} \right)} \quad \dots \quad (67)$$

$$= H_1 \frac{\Sigma y \Delta}{\Sigma \Delta} - \frac{\Sigma m_x \Delta}{\Sigma \Delta} - \frac{\Sigma m_x \Delta \left( x - \frac{1}{2} l \right)}{\Sigma \Delta \left( \frac{1}{4} l^2 - \frac{\Sigma x^2 \Delta}{\Sigma \Delta} \right)} \frac{l}{2} \quad \dots \quad (68)$$

If the effect of the axial stress is neglected,  $H_1$  is replaced by  $H$ , as given by (61).

$$m_x = P \frac{l-a}{l} x - P(x-a)^{x>a} \quad \dots \quad (6a)$$

**II. Determination of the Bending Moment  $M_2$  at the Right Support Produced by a Single Vertical Load.**—From (56),

$$M_2 = -M_1 + 2H_1 \frac{\Sigma y \Delta}{\Sigma \Delta} - 2 \frac{\Sigma m_x \Delta}{\Sigma \Delta} \quad \dots \quad (69)$$

Substituting (68) in (69),

$$M_2 = H_1 \frac{\Sigma y \Delta}{\Sigma \Delta} - \frac{\Sigma m_x \Delta}{\Sigma \Delta} + \frac{\Sigma m_x \Delta \left( x - \frac{1}{2} l \right)}{\Sigma \Delta \left( \frac{1}{4} l^2 - \frac{\Sigma x^2 \Delta}{\Sigma \Delta} \right)} \frac{l}{2} \quad \dots \quad (70)$$

If the effect of the axial stress is neglected,  $H_1$  is replaced by  $H$ , as given by (61).

$$m_x = P \frac{l-a}{l} x - P(x-a)^{x>a} \quad \dots \quad (6a)$$



A comparison of (68) and (70) shows that all the terms are identical in form, the only difference in the equations being the sign of the last term.

**12. Determination of the Bending Moments  $M_1$  and  $M_2$  for Two Symmetrical and Equal Vertical Loads.** If there are two equal symmetrical loads the values of  $m_x$  are symmetrical, and there are equal values at equal distances on each side of the center of the span. Then

$$\Sigma m_x \Delta (x - \frac{1}{2}l) = \Sigma m_x x \Delta - \frac{1}{2}l \Sigma m_x \Delta = \frac{1}{2}l \Sigma m_x \Delta - \frac{1}{2}l \Sigma m_x \Delta = 0,$$

or the last terms in (68) and (70) are zero.

For any symmetrical and equal vertical loading  $M_1 = M_2$  and

$$M_1 = M_2 = H_1 \frac{\Sigma y \Delta}{\Sigma \Delta} - \frac{\Sigma m_x \Delta}{\Sigma \Delta}, \quad . . . . . (71)$$

where

$$m_x = R_1 x - \sum_0^x P(x-a)^{x>a}. \quad . . . . . (72)$$

The values of  $m_x$  can be found from a single equilibrium polygon.

**13. Determination of the Bending Moments  $M_1$  and  $M_2$  Produced by Changes of Temperature.** Since the arch is symmetrical about the center of the span, a change of temperature has like effects at each support, or  $M_1 = M_2$ . From (56),

$$M_1 = H_t \frac{\Sigma y \Delta}{\Sigma \Delta} = M_2. \quad . . . . . (73)$$

The moment at any point in the arch axis is

$$M_x = M_1 - H_t y = -H_t \left( y - \frac{\Sigma y \Delta}{\Sigma \Delta} \right). \quad . . . . . (74)$$

The signs in (73) and (74) are for an assumed increase of temperature when  $H_t$  is positive.

**14. Determination of the Bending Moments  $M_1$  and  $M_2$  Produced by the Axial Stress Alone.** The horizontal thrust  $H_a$  produced by the axial stress alone is a fixed percentage of  $H$  or  $H_t$ , as shown by (63). This percentage is dependent upon the shape of the arch axis and the properties of the radial sections of the arch and is not necessarily the same for any two arches. As in the case of change of temperature the moments at the two supports are equal, or  $M_1 = M_2$ . From (71),

$$M_1 = H_a \frac{\Sigma y \Delta}{\Sigma \Delta} = M_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (75)$$

The moment at any point in the arch axis is

$$M_x = M_1 - H_a y = -H_a \left( y - \frac{\Sigma y \Delta}{\Sigma \Delta} \right). \quad . \quad . \quad (76)$$

**15. Working Formula for  $H$ , the Horizontal Thrust Produced by a Single Vertical Load, Neglecting the Effect of the Axial Stress.**—From Art. 9,

$$H = \frac{\Sigma m_x \Delta \left( y - \frac{\Sigma y \Delta}{\Sigma \Delta} \right)}{\Sigma y \Delta \left( y - \frac{\Sigma y \Delta}{\Sigma \Delta} \right)} = \frac{\Sigma m_x \mathbf{B}}{\Sigma y \mathbf{B}}, \quad . \quad . \quad . \quad (61)$$

where

$$m_x = P \frac{l-a}{l} x - P(x-a)^{x>a}. \quad . \quad . \quad . \quad . \quad (6a)$$

Substituting (6a) in the expression  $\Sigma m_x \mathbf{B}$ ,

$$\Sigma m_x \mathbf{B} = \left\{ \frac{l-a}{l} \Sigma x \mathbf{B} - \Sigma_a^l (x-a) \mathbf{B} \right\} P. \quad . \quad . \quad (77)$$

Since the values of  $\mathbf{B}$  are symmetrical about the center of the span,

$$\Sigma x\mathbf{B} = \frac{1}{2}l \Sigma \mathbf{B} = \frac{1}{2}l \Sigma \Delta \left( y - \frac{\Sigma y \Delta}{\Sigma \Delta} \right) = \frac{1}{2}l (\Sigma y \Delta - \Sigma y \Delta) = 0,$$

and the first term in the second member of (77) is zero. The second term may be written

$$- \sum_0^l x\mathbf{B} + a \sum_0^l \mathbf{B} + \sum_0^a x\mathbf{B} - a \sum_0^a \mathbf{B} = \sum_0^a x\mathbf{B} - a \sum_0^a \mathbf{B};$$

hence,

$$\Sigma m_x \mathbf{B} = \left( \sum_0^a x\mathbf{B} - a \sum_0^a \mathbf{B} \right) P$$

and

$$H = \frac{\sum_0^a x\mathbf{B} - a \sum_0^a \mathbf{B}}{\Sigma y \mathbf{B}}. \quad \dots \quad (78)$$

In applying (78) the span is divided into  $n$  equal parts, usually twenty, and verticals drawn from the centers of these divisions until they cut the axis of the arch in points numbered 1, 2, 3, . . . . 3', 2', 1', as shown in Fig. 5. The values of  $x$ ,  $y$ ,  $\delta s$ ,  $\delta x$ ,  $\delta y$ ,  $F$ , and  $I$  are determined for each point. A unit load is assumed to occupy each point 1, 2, 3, etc., in turn and the corresponding values of  $H$  computed. These values plotted give the influence line for  $H$ . (See Plate III.)

If  $a = k \frac{\delta x}{2}$  and  $x = z \frac{\delta x}{2}$ , (78) becomes,

$$H = \frac{\sum_0^a z\mathbf{B} - k \sum_0^a \mathbf{B}}{\Sigma y \mathbf{B}} \left( P \frac{\delta x}{2} \right), \quad \dots \quad (79)$$

$$\mathbf{B} = \Delta \left( y - \frac{\Sigma y \Delta}{\Sigma \Delta} \right).$$

Since the loads are vertical and the arch symmetrical a unit load at point 1, Fig. 5, has the same  $H$  as a unit load at point 1', and, therefore, it is only necessary to compute the values of  $H$  for points 1 to 10 inclusive. The factor  $\mathbf{B}$  is a constant for a given arch and is obtained

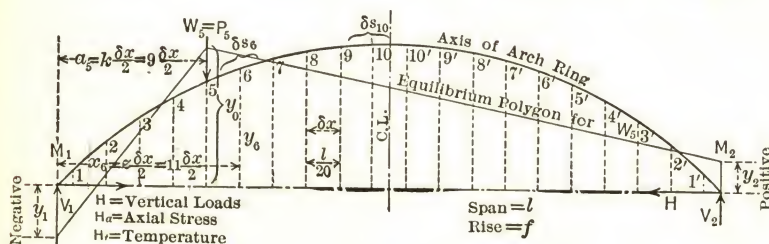


FIG. 5.

as shown in Table I, Chapter IV. The values of  $H$  are quickly found, as shown in Table II, Chapter IV.

16. Working Formulas for  $M_1$  and  $M_2$ , the Bending Moments at the Supports Produced by a Single Vertical Load.—From Arts. 10 and 11,

$$\left. \begin{matrix} M_1 \\ M_2 \end{matrix} \right\} = H_1 \frac{\Sigma y \Delta}{\Sigma \Delta} - \frac{\Sigma m_x \Delta}{\Sigma \Delta} \mp \frac{\Sigma m_x \Delta (x - \frac{1}{2}l)}{\Sigma \Delta \left( \frac{1}{4}l^2 - \frac{\Sigma x^2 \Delta}{\Sigma \Delta} \right)} \frac{l}{2} \quad (80)$$

For convenience let

$$M_1 = \mathbf{C} - \mathbf{D} - \mathbf{E} \quad \text{and} \quad M_2 = \mathbf{C} - \mathbf{D} + \mathbf{E}.$$

In (80)  $H_1$  is the horizontal thrust, and  $m_x$  the common moment as found from (6a). In case the effect of the axial stress is neglected,  $H_1$  becomes  $H$  from (60).



*Term D.*

Substituting the value of  $m_x$  in  $\Sigma m_x \Delta$ ,

$$\begin{aligned}\Sigma m_x \Delta &= \left\{ \frac{l-a}{l} \Sigma x \Delta - \Sigma_a^l (x-a) \Delta \right\} P \\ &= \left\{ \frac{l}{2} \Sigma_0^l \Delta - \frac{a}{2} \Sigma_0^l \Delta - \frac{l}{2} \Sigma_0^l \Delta + a \Sigma_0^l \Delta + \frac{a}{0} \Sigma_0^l x \Delta - a \Sigma_0^l \Delta \right\} P \\ &= \left\{ \frac{l/2}{0} \Sigma_0^l \Delta - a \Sigma_0^l \Delta + \frac{a}{0} \Sigma_0^l x \Delta \right\} P = \left\{ \frac{a}{0} \Sigma_0^l x \Delta + a \Sigma_a^{l/2} \Delta \right\} P.\end{aligned}$$

If  $x$  is replaced by  $z \frac{\delta x}{2}$  and  $a$  by  $k \frac{\delta x}{2}$ ,

$$\frac{\Sigma m_x \Delta}{\Sigma \Delta} = \left\{ \Sigma_{x=0}^{x=a} z \Delta + k \Sigma_a^{l/2} \Delta \right\} \frac{\delta x}{2 \Sigma \Delta} P = \mathbf{D}. \quad (81)$$

This expression is very quickly worked out in any particular case, as shown in Table III, Chapter IV. Since the value of  $M_1$  for a load at point 1 is equal to the value of  $M_2$  for a load at point 1', etc. (see Fig. 5), it is not necessary to carry the computations for  $\mathbf{D}$  beyond the first ten points when  $n = 20$ .

*Term E.*

Replacing  $x$  by  $z \frac{\delta x}{2}$  and  $l$  by  $n \delta x$  the expression for  $\mathbf{E}$  becomes

$$\mathbf{E} = \frac{\Sigma m_x \Delta (z-n)n}{n^2 \Sigma \Delta - \Sigma z^2 \Delta},$$

where

$$m_x = \left\{ \frac{2n-k}{2n} z - (z-k) \right\} P \frac{\delta x}{2}.$$

Substituting the value of  $m_x$  the numerator becomes  $\Sigma m_x \Delta(z-n)n$

$$= \left\{ \frac{2n-k}{2} \sum_0^l (z-n)z\Delta - n \sum_a^l (z-n)z\Delta + k \sum_a^l (z-n)n\Delta \right\} P \frac{\delta x}{2}.$$

(a)
(b)
(c)

$$(a) = n \sum_0^l (z-n)z\Delta - \frac{1}{2}k \sum_0^l (z-n)z\Delta;$$

$$(b) = -n \sum_0^l (z-n)z\Delta + n \sum_0^a (z-n)z\Delta;$$

$$(c) = k \sum_0^l (z-n)n\Delta - k \sum_0^a (z-n)n\Delta.$$

The first terms of (a) and (b) cancel and the first term of (c) is zero, hence

$$\Sigma m_x \Delta(z-n)n$$

$$= \left\{ -\frac{1}{2}k \sum_0^l (z-n)z\Delta + n \sum_0^a (z-n)z\Delta - k \sum_0^a (z-n)n\Delta \right\} P \frac{\delta x}{2}. \quad (82)$$

$$\sum_0^l (z-n)z\Delta = 2 \sum_0^{l/2} (z-n)^2 \Delta,$$

since the values of  $\Delta$  are symmetrical. For example, take any two symmetrical points,

$$(z-n)z\Delta + (z'-n)z'\Delta = \{(z-n)z + (z'-n)z'\}\Delta,$$

but  $z' = 2n - z$ , hence the expression becomes

$$\{(z-n)z + (2n-z)(2n-z-n)\}\Delta = 2(z-n)^2 \Delta.$$

As this is the sum for a pair of symmetrical points, the total sum will be the sum of this expression for points 1-10 inclusive, when  $n = 20$ .

Increasing and decreasing (82) by  $k \sum_0^a (z-n)^2 \Delta$ , it becomes

$$\begin{aligned} & \Sigma m_x \Delta (z-n)n \\ &= \left\{ -k \sum_0^{1/2} (z-n)^2 \Delta + k \sum_0^a (z-n)^2 \Delta - k \sum_0^a (z-n)z \Delta + k \sum_0^a (z-n)n \Delta \right. \\ & \quad \left. + n \sum_0^a (z-n)z \Delta - k \sum_0^a (z-n)n \Delta \right\} P \frac{\delta x}{2}, \end{aligned}$$

which reduces to

$$\Sigma m_x \Delta (z-n)n = \left\{ (n-k) \sum_0^a (z-n)z \Delta - k \sum_0^{1/2} (z-n)^2 \Delta \right\} P \frac{\delta x}{2}. \quad (83)$$

The denominator of **E** can be simplified as follows:

Since

$$\Sigma z^2 \Delta = 2 \sum_0^{1/2} (n-z)^2 \Delta + 2n^2 \sum_0^{1/2} \Delta \quad \text{and} \quad \Sigma \Delta = 2 \sum_0^{1/2} \Delta;$$

$$n^2 \Sigma \Delta - \Sigma z^2 \Delta = \left( 2n^2 \sum_0^{1/2} \Delta - 2 \sum_0^{1/2} (n-z)^2 \Delta - 2n^2 \sum_0^{1/2} \Delta \right) = -2 \sum_0^{1/2} (n-z)^2 \Delta$$

Finally

$$\begin{aligned} & \frac{\Sigma m_x \Delta (x - \frac{1}{2}l)}{\Sigma \Delta \left( \frac{1}{4}l^2 - \frac{\Sigma x^2 \Delta}{\Sigma \Delta} \right)} = \mathbf{E} \\ &= \left\{ (n-k) \sum_0^a (z-n)z \Delta - k \sum_0^{1/2} (z-n)^2 \Delta \right\} \frac{\frac{\delta x}{1/2}}{-4 \sum_0^{1/2} (z-n)^2 \Delta} P. \quad (84) \end{aligned}$$

This expression is long but is very quickly solved as shown in Table IV, Chapter IV.

**17. Determination of Horizontal Thrust Produced by any Number of Vertical Loads when the Effect of the Axial Stress is Neglected.**—For each vertical load the horizontal thrust  $H$  is, from (61), Art. 15,

$$H = \frac{\Sigma m_x \mathbf{B}}{\Sigma y \mathbf{B}} \dots \dots \dots (61)$$

Since  $y$  and  $B$  do not depend upon the loading in any manner, and  $m_x$  is the only quantity which does depend upon the loading, the value of  $H$  can be found from (61) for any number of vertical loads by simply substituting for  $m_x$  the common moment for all of the loads on the arch. The values of  $m_x$  may be found by means of an equilibrium polygon or computed by the formula,

$$m_x = R_1 x - \sum_0^x P(x-a). \quad \dots \quad (61a)$$

This method is convenient when the dead load is not symmetrical about the center of the span, as is the case when the roadway of an arch bridge is not level. If the arch is divided, as shown in Fig. 5, Art. 15, and a unit load placed at each point, the value of  $H$  for the twenty unit loads is found, as shown in Table VII, Chapter IV.

**18. Determination of the Bending Moments  $M_1$  and  $M_2$  Produced by any Number of Vertical Loads.** For each load, from (80), and Art. 16,

$$\left. \begin{matrix} M_1 \\ M_2 \end{matrix} \right\} = \left\{ H_1 \frac{\sum y \Delta}{\sum \Delta} - \frac{\sum m_x \Delta}{\sum \Delta} \mp \frac{n \sum m_x \Delta (z-n)}{-2 \sum_0^{l/2} (z-n)^2 \Delta} \right\}. \quad \dots \quad (85)$$

For a number of vertical loads  $H_1$  is the thrust produced by these loads, including the effect of the axial stress, or  $H_1$  may be replaced by  $H$  when the effect of the axial stress is neglected.  $m_x$  is the common bending moment as found from (61a) or from an equilibrium polygon.

For equal and symmetrical loading the last term of (85) is zero and

$$M_1 = M_2 = H_1 \frac{\sum y \Delta}{\sum \Delta} - \frac{\sum m_x \Delta}{\sum \Delta}. \quad \dots \quad (86)$$



19. A Graphical Representation of the Bending Moments  $M_x$  Produced by a Vertical Load.—From (7),

$$M_x = M_1 \frac{l-x}{l} + \frac{M_2 x}{l} - H_1 y + m_x. \quad . \quad . \quad (7)$$

Dividing (7) by  $H_1$ ,

$$\frac{M_x}{H_1} = \frac{M_1}{H_1} \frac{l-x}{l} + \frac{M_2}{H_1} \frac{x}{l} - y + \frac{m_x}{H_1}. \quad . \quad . \quad (7a)$$

In Fig. 6 let  $ABC$  represent the axis of the arch and  $A'B'C'$  the equilibrium polygon, for a single vertical load  $P$ , drawn with  $H_1$  as a pole distance. The pole is taken opposite the point in the force polygon shown in Fig. 6a. The point  $A'$  is located by making the ordinate  $AA'$  equal  $M_1 \div H_1$ . Placing  $A'$  below  $A$  is equivalent to assuming  $M_1$  negative.

From Fig. 6,

$$-df = -\frac{M_1}{H_1} \frac{l-x}{l}, \quad ef = \frac{M_2}{H_1} \frac{x}{l}, \quad ad = y, \quad be = \frac{m_x}{H_1},$$

and

$$-df + ef - ad + be = -ab.$$

$$\therefore \frac{M_x}{H_1} = -ab \quad \text{and} \quad M_x = H_1(-ab).$$

From this it appears that the bending moment at any point in the arch axis equals the vertical ordinate between the axis and the true equilibrium polygon. The true equilibrium polygon is constructed, as shown in Figs. 6 and 6a, after the values of  $V_1$ ,  $H_1$ , and  $M_1$  have been determined. Any number of loads may be considered

by using the values of  $V_1$ ,  $H_1$ , and  $M_1$  for these loads. Ordinates below the arch axis are considered negative and correspond to a bending moment producing tension in the upper fibers of the arch.

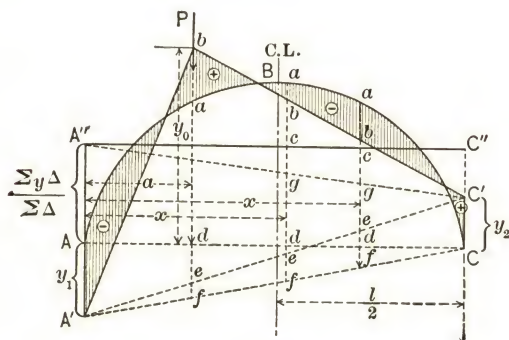


FIG. 6.

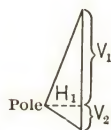


FIG. 6a.

The equilibrium polygon for a single vertical load can be at once located by means of the three ordinates  $y_1$ ,  $y_0$ , and  $y_2$ ,

$$y_1 = \frac{M_1}{H_1}, \quad y_0 = \frac{M_1}{H_1} + \frac{V_1}{H_1} a, \quad y_2 = \frac{M_2}{H_1}.$$

**20. Maximum Bending Moments Produced by Vertical Loads.**—Let  $K$  in Fig. 7 be any point on the arch axis where the maximum bending moment is wanted. Since the bending moment at any point is proportional to the ordinate between the axis and the equilibrium polygon at the point, Art. 19, any single load having its equilibrium polygon passing through the point will produce no moment at the point. In Fig. 7 the loads  $P$  and  $P'$  have equilibrium polygons passing through  $K$  and therefore these loads produce no bending at  $K$ . Loads in the shaded portions

of the span produce negative moments at  $K$  and those in the unshaded portion produce positive moments.

If equilibrium polygons for unit loads at each of the twenty points shown in Fig. 5 are constructed by means of the ordinates  $y_1$ ,  $y_0$ , and  $y_2$ , Art. 19, the loads producing like moments at any point can be determined

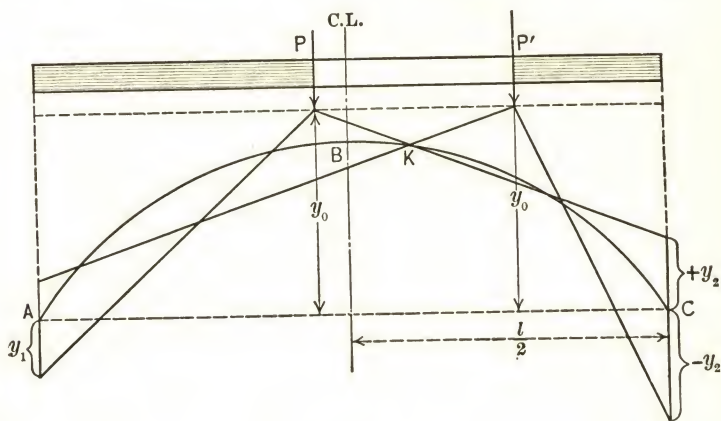


FIG. 7.

almost at a glance. Plate III shows a number of such polygons.

The loading which produces the maximum moment at any point does not produce maximum fiber stresses in the radial section passing through the point. For shallow arch rings this loading is nearly correct and will give fiber stresses sufficiently correct for practical purposes. The true maximum fiber stresses are best determined by computing the fiber stresses at any given section for a unit load at each of the twenty points and then plotting the results, thus obtaining an influence diagram from which the maximum stresses can be at once obtained. Plate

VI shows six influence diagrams for the stresses in the extreme fibers.

21. The Maximum Radial Shear  $T_z$  Produced by Vertical Loads.—From (13),

$$T_z = V_1 \cos \phi - H_1 \sin \phi - \sum_0^x P \cos \phi.$$

For loads upon the right of  $B$ , Fig. 8,

$$T_z = V_1 \cos \phi - H_1 \sin \phi.$$

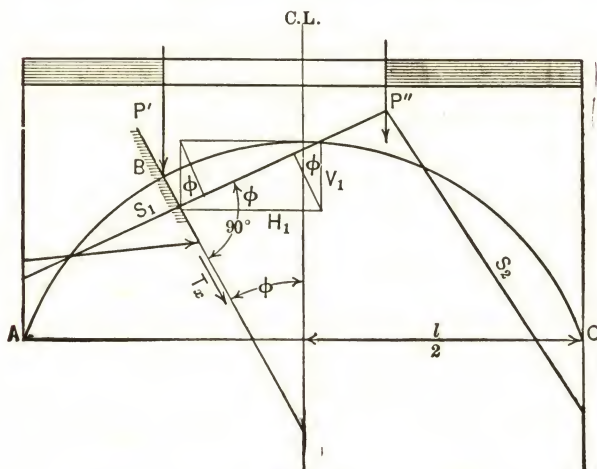


FIG. 8.

If  $S_1$  is normal to the radius passing through  $B$ , Fig. 8, it is evident from the figure that  $T_z$  is zero, since

$$V_1 \cos \phi = H_1 \sin \phi.$$

Therefore, all loads upon the right of  $P''$  produce the same kind of radial shear, and those upon the left produce the opposite kind until  $P'$  is reached. Since  $V_1 - \sum_0^x P$  results



in a downward force, the loads upon the left of  $B$  produce the same kind of radial shear as those upon the right of  $P''$ . The fields of loading producing like radial shears at  $B$  are shown in Fig. 8. This method assumes that the arch rib is shallow. The absolute maximum value of  $T_x$  is best found from an influence diagram. Usually  $T_x$  is so small that it is not necessary to consider it in masonry arches.

**22. Formulas for Vertical Loads, Changes of Temperature, etc.**

*(a) General Formulas with Axial Stress Neglected.*

$$H = \frac{\Sigma m_x \mathbf{B}}{\Sigma y \mathbf{B}} \quad \text{(Art. 15).} \quad (61)$$

$$\mathbf{B} = \Delta \left( y - \frac{\Sigma y \Delta}{\Sigma \Delta} \right). \quad \Delta = \delta s \div I_x.$$

$$\left. \begin{matrix} M_1 \\ M_2 \end{matrix} \right\} = H \frac{\Sigma y \Delta}{\Sigma \Delta} - \frac{\Sigma m_x \Delta}{\Sigma \Delta} \mp \frac{n \Sigma m_x \Delta (z - n)}{\frac{1}{2} \Sigma (z - n)^2 \Delta} \quad \text{(Art. 18).} \quad (85)$$

$$m_x = R_1 x - \sum_0^x P(x - a) = \text{the common moment.}$$

*(b) Formulas for a Single Load with Axial Stress Neglected.*

$$H = \frac{\sum_0^a z \mathbf{B} - k \sum_0^a \mathbf{B}}{\Sigma y \mathbf{B}} P \frac{\delta x}{2} \quad \text{(Art. 15).} \quad (79)$$

$$\mathbf{B} = \Delta \left( y - \frac{\Sigma y \Delta}{\Sigma \Delta} \right).$$

$$\left. \begin{matrix} M_1 \\ M_2 \end{matrix} \right\} = \mathbf{C} - \mathbf{D} \mp \mathbf{E} \quad \text{(Art. 18).} \quad (85)$$

$$C = H \frac{\Sigma y \Delta}{\Sigma \Delta}.$$

$$D = \left\{ \sum_{x=0}^{x=a} z \Delta + k \sum_a^{1/2} \right\} \frac{\delta x}{2 \Sigma \Delta} P. \quad \dots \quad (\text{Art. 16}). \quad (81)$$

$$E = \left\{ (n-k) \sum_0^a (z-n) z \Delta - k \sum_a^{1/2} (z-n)^2 \Delta \right\} \frac{\delta x}{-4 \sum_0^{1/2} (z-n)^2 \Delta} P. \\ (\text{Art. 16}). \quad (84)$$

(c) *Formula for  $M_1$  and  $M_2$  when the Loading is Symmetrical and the Axial Stress is Neglected.*

$$M_1 = M_2 = H \frac{\Sigma y \Delta}{\Sigma \Delta} - \frac{\Sigma m_x \Delta}{\Sigma \Delta}. \quad \dots \quad (\text{Art. 18}). \quad (86)$$

$$m_x = R_1 x - \sum_0^x P(x-a) = \text{the common moment.}$$

(d) *Change of Temperature with Axial Stress Neglected.*

$$H_t = \frac{e t^\circ l E}{\Sigma y \mathbf{B}}. \quad \dots \quad (\text{Art. 9}). \quad (62)$$

$$M_1 = M_2 = H_t \frac{\Sigma y \Delta}{\Sigma \Delta}. \quad \dots \quad (\text{Art. 13}). \quad (73)$$

(e) *Formulas Showing Effect of Axial Stress.*

$$H_a = - \frac{\Sigma \frac{(\delta x)^2}{F_x \delta s}}{\Sigma y \mathbf{B} + \Sigma \frac{(\delta x)^2}{F_x \delta s}} \left\{ \begin{array}{l} H \text{ for vertical loads} \\ H_t \text{ for changes in temperature} \end{array} \right\} \\ \dots \quad (\text{Art. 9}). \quad (63)$$

$$M_1 = M_2 = H_a \frac{\Sigma y \Delta}{\Sigma \Delta}. \quad \dots \quad (\text{Art. 14}). \quad (75)$$

(f) *Formulas for Intermediate Bending Moments.*

Vertical Loads:

$$M_x = M_1 \frac{l-x}{l} + M_2 \frac{x}{l} - H_1 y + m_x, \quad . \quad . \quad . \quad . \quad . \quad (7)$$

or

$$M_x = M_1 + V_1 x - H_1 y - \sum_a^x P(x-a),$$

$$m_x = R_1 x - \sum_0^x P(x-a).$$

Changes of Temperature:

$$M_x = -H_1 \left( y - \frac{\sum y \Delta}{\sum \Delta} \right) = M_1 - H_1 y. \quad . \quad (\text{Art. 13}). \quad (74)$$

Effect of the Axial Stress:

$$M_x = -H_a \left( y - \frac{\sum y \Delta}{\sum \Delta} \right) = M_1 - H_a y. \quad . \quad (\text{Art. 14}). \quad (76)$$

(g) *Formula for the Left Reaction  $V_1$ .*

$$V_1 = \frac{M_2 - M_1}{l} + R_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$V_2 = \sum P - V_1.]$$

# CHAPTER III.

## SYMMETRICAL ARCHES FIXED AT THE ENDS.

(HORIZONTAL LOADS.)

23. **Conditions which Must be Satisfied.**—The conditions are the same for horizontal loads as for vertical loads

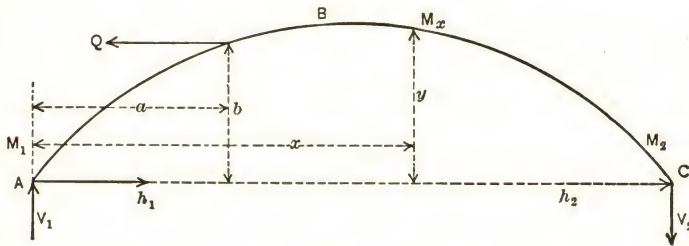


FIG. 9.

(see Art. 7), but the general equations are slightly different owing to the effect of the axial stress.

From (29), (30), and (31),

$$-\Sigma M_x \Delta = 0; \quad . . . . . (87)$$

$$+\Sigma M_x y \Delta - \Sigma p_0 \delta x + et^o l E = 0; \quad . . . (88)$$

$$-\Sigma M_x x \Delta - \Sigma p_0 \delta y = 0. \quad . . . . . (89)$$



From Fig. 9 for a single load acting from right to left,

$$M_x = M_1 + V_1 x - h_1 y + Q(y - b)^{x > a}, \quad \dots \quad (90)$$

when  $x = l$  in (90).

$$M_2 = M_1 + V_1 l + Q(-b) \quad \text{and} \quad V_1 = \frac{M_2 - M_1}{l} + Q \frac{b}{l}. \quad (91)$$

Substituting (91) in (90),

$$M_x = M_1 \frac{l-x}{l} + M_2 \frac{x}{l} - h_1 y + Q \frac{b}{l} x + Q(y - b)^{x > a}. \quad (92)$$

But  $Q \frac{b}{l} x + Q(y - b)^{x > a}$  is the common moment for  $Q$

upon a curved beam hinged at  $C$  and on rollers at  $A$ . Let this moment be represented by  $m_x'$ , then,

$$M_x = M_1 \frac{l-x}{l} + M_2 \frac{x}{l} - h_1 y + m_x', \quad \dots \quad (93)$$

where

$$m_x' = Q \frac{b}{l} x + Q(y - b)^{x > a} \quad \text{for a single load.} \quad \dots \quad (94)$$

Comparing (93) and (7) it is seen that the two equations are identical in form.

Referring to Fig. 4 and equation (8) it follows for horizontal loads, that

$$N_x = V_x \sin \phi + h_x \cos \phi. \quad \dots \quad (95)$$

But  $V_x = V_1$  and  $h_x = h_1 - (Q \text{ when } x > a)$ , therefore,

$$N_x = V_1 \sin \phi + h_1 \cos \phi - Q \cos \phi^{x > a}. \quad \dots \quad (96)$$

Neglecting  $V_1 \sin \phi$  and substituting (96) in  $N_x = F_x p_0$  and solving for  $p_0$ ,

$$p_0 = h_1 \frac{\cos \phi}{F_x} - Q \frac{\cos \phi}{F_x}; \quad . . . . . (97)$$

hence

$$\Sigma p_0 \delta x = h_1 \Sigma \frac{(\delta x)^2}{F_x \delta s} - \Sigma Q \frac{(\delta x)^2}{F_x \delta s}, \quad . . . . . (98)$$

and

$$\Sigma p_0 \delta y = h_1 \Sigma \frac{\delta x \delta y}{F_x \delta s} - \Sigma Q \frac{\delta x \delta y}{F_x \delta s}. \quad . . . . . (99)$$

The first term in the second member of (99) is zero and the second term always comparatively small, so that  $\Sigma p_0 \delta y$  in (89) can be neglected without serious error. The condition equations now become (see Art. 8), neglecting the temperature term,

$$-M_1 \Sigma \Delta - \frac{1}{2}(M_2 - M_1) \Sigma \Delta + h_1 \Sigma y \Delta - \Sigma m_x' \Delta = 0 \quad . . . (100)$$

$$+ M_1 \Sigma y \Delta + \frac{1}{2}(M_2 - M_1) \Sigma y \Delta - h_1 \Sigma y^2 \Delta + \Sigma m_x' y \Delta$$

$$- h_1 \Sigma \frac{(\delta x)^2}{F_x \delta s} + \Sigma Q \frac{(\delta x)^2}{F_x \delta s} \quad . . . . . (101)$$

$$- \frac{1}{2} M_1 l \Sigma \Delta - \frac{1}{l} (M_2 - M_1) \Sigma x^2 \Delta + \frac{1}{2} h_1 l \Sigma y \Delta - \Sigma m_x' x \Delta = 0, \quad (102)$$

where

$$m_x' = Q \frac{b}{l} x + Q(y - b) \text{ for a single load, } \quad . . . (94)$$

and for any number of horizontal loads,

$$m_x' = \Sigma Q \frac{b}{l} x + \Sigma Q(y - b). \quad . . . . . (103)$$

Comparing (100), (101), and (102) with (56), (57), and (58) in Art. 8, it is seen that the equations are identical in form with the exception of one additional term. It is not necessary to demonstrate the following formulas, as the methods employed are the same as used when considering vertical loads.

**24. Horizontal Thrust Produced by a Single Horizontal Load Acting from Right to Left.**

$$h_1 = \frac{\sum y \mathbf{B} - \sum_0^a y \mathbf{B} + b \sum_0^a \mathbf{B} + \sum_a^l \frac{(\delta x)^2}{F_x \delta s}}{\sum y \mathbf{B} + \sum \frac{(\delta x)^2}{F_x \delta s}} Q; \quad \dots \quad (104)$$

$$\mathbf{B} = \Delta \left( y - \frac{\sum y \Delta}{\sum \Delta} \right).$$

The above formula includes the effect of the axial stress. If this is neglected, (104) becomes

$$h_1' = \left\{ 1 - \frac{\sum_0^a y \mathbf{B} - b \sum_0^a \mathbf{B}}{\sum y \mathbf{B}} \right\} Q. \quad \dots \quad (105)$$

Since the algebraic sum of the horizontal forces equals zero

$$h_2' = Q - h_1'. \quad \dots \quad (106)$$

If the values of  $h_1'$  and  $h_2'$  are computed for loads upon the left of the center of the span, it is not necessary to make the computations for symmetrically placed loads upon the right, as the values of  $h_1'$  and  $h_2'$  simply interchange.

**25. Horizontal Thrust  $h_1$  Produced by any Number of Horizontal Loads.**

$$h_1 = \frac{\sum m_x' \mathbf{B} + \left[ \sum_a^l \frac{(\delta x)^2}{F_x \delta s} Q \text{ for each load} \right]}{\sum y \mathbf{B} + \sum \frac{(\delta x)^2}{F_x \delta s}}. \quad (107)$$

$$m_x' = \sum Q \frac{b}{l} x + \sum_0^x Q (y - b); \quad (103)$$

$$\mathbf{B} = \Delta \left( y - \frac{\sum y \Delta}{\sum \Delta} \right).$$

If the effect of the axial stress is neglected,

$$h_1' = \frac{\sum m_x' \mathbf{B}}{\sum y \mathbf{B}}. \quad (108)$$

Comparing (108) and (61) it is seen that the two equations are identical, as  $m_x$  and  $m_x'$  are the common moments for the loading on a beam hinged at the right end and on rollers at the left end. Equations (61) and (108) can be used for loads of any inclination by using the common moment for the loading placed on a beam hinged at the right end and on rollers at the left end. The beam has the same shape as the arch.

**26. Bending Moments  $M_1$  and  $M_2$  at the Supports Produced by a Single Horizontal Load Acting from Right to Left.**

$$\left. \begin{matrix} M_1 \\ M_2 \end{matrix} \right\} = \mathbf{F} - \mathbf{G} \mp \mathbf{H}; \quad (109)$$

$$\mathbf{F} = h_1 \frac{\sum y \Delta}{\sum \Delta}; \quad (110)$$

$$\mathbf{G} = \left\{ \sum y \Delta - b \sum_a^{l/2} \Delta - \sum_0^a y \Delta \right\} \frac{Q}{\sum \Delta}; \quad (111)$$



$$\mathbf{H} = \left\{ b \sum_0^{l/2} (n-z)^2 \Delta - n \sum_0^a y(z-n) \Delta + nb \sum_0^a (z-n) \Delta \right\} \left\{ \frac{-Q}{2 \sum_0^{l/2} (z-n)^2 \Delta} \right\} \quad (112)$$

27. **Bending Moments  $M_1$  and  $M_2$  at the Supports Produced by any Number of Horizontal Loads.**

$$\left. \begin{matrix} M_1 \\ M_2 \end{matrix} \right\} = h_1 \frac{\sum y \Delta}{\sum \Delta} - \frac{\sum m_x' \Delta}{\sum \Delta} \mp \frac{\sum m_x' (z-n) \Delta}{2 \sum_0^{l/2} (z-n)^2 \Delta} n, \quad (113)$$

$$m_x' = \sum Q \frac{b}{l} x + \sum_0^x Q (y-b). \quad (114)$$

If equation (113) is compared with (85) it is seen that the two are identical in form.

28. **Symmetrical Horizontal Loads.**—If the horizontal loads are symmetrical the general formulas can be somewhat simplified. In arch bridges the condition of symmetrical horizontal loads occurs when the roadway is level and supported by an earth fill. The horizontal components of the earth thrust are symmetrical horizontal loads. From Art. 25, the general form of the formula for the horizontal thrust at the left support is

$$h_1' = \frac{\sum m_x' \mathbf{B}}{\sum y \mathbf{B}} \quad (\text{the effect of the axial stress neglected}). \quad (115)$$

$$m_x' = \sum_0^x Q (y-b); \quad (116)$$

$$\mathbf{B} = \Delta \left( y - \frac{\sum y \Delta}{\sum \Delta} \right);$$

$$M_1 = M_2 = h_1' \frac{\sum y \Delta}{\sum \Delta} - \frac{\sum m_x' \Delta}{\sum \Delta}, \quad (117)$$

where  $m_x'$  is found from (116).



From Fig. 10,

$$e'c' = -V_2 \frac{x_2 + x'}{h_2} \quad \text{and} \quad e'a' = y,$$

or

$$-e'c' + e'a' = -a'c',$$

and hence

$$M_x = h_2(-a'c').$$

In a similar manner

$$M_x = h_2(+a''c'').$$

The points marked *E* in Fig. 10 are points of no moment.

The following coordinates may be used in constructing Fig. 10:

$$y_1 = \frac{M_1}{h_1}, \quad y_2 = \frac{M_2}{h_2}, \quad x_1 = \frac{M_1}{V_1}, \quad x_2 = \frac{M_2}{V_2}$$

and

$$x_0' = (b - y_1) \frac{x_1}{y_1} = \left( b - \frac{M_1}{h_1} \right) \frac{h_1}{V_1} \quad . \quad . \quad . \quad . \quad (122)$$

**29. Intermediate Bending Moments  $M_x$ , Produced by Two Symmetrical Horizontal Loads.**—The values of  $M_x$  are symmetrical about the center of the span. For points between *A* and *B*, Fig. 11, assuming *Q* and *Q'* as unity,

$$M_x = M_1 - h_1 y \quad \text{or} \quad \frac{M_x}{h_1} = \frac{M_1}{h_1} - y = y_1 - y. \quad . \quad . \quad (123)$$

The shaded portion in Fig. 11 shows the values of  $y_1 - y$ .

$$M_x = h_1(af)Q.$$

For points between  $B$  and  $D$ ,

$$M_x = M_1 - h_1y + y - b \quad \text{or} \quad \frac{M_x}{1 - h_1} = \frac{M_1 - b}{1 - h_1} + y. \quad (124)$$

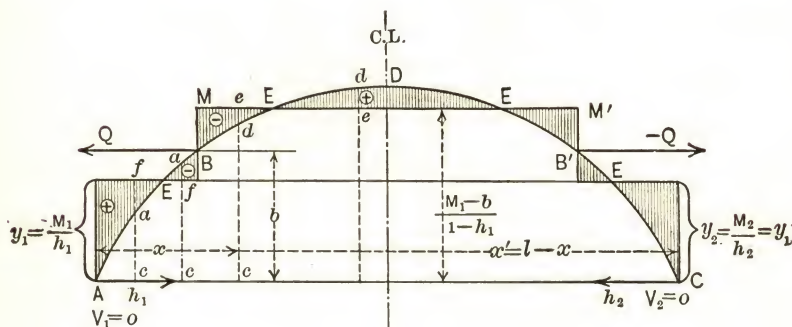


FIG. 11.

In Fig. 11 draw the line  $MM'$  parallel to  $AC$  and above  $AC$  a distance  $\frac{M_1 - b}{1 - h_1}$ , then,

$$\frac{M_x}{1 - h_1} = ed \quad \text{and} \quad M_x = (1 - h_1)(ed)Q.$$

The signs of the moments are indicated in the figure. If the directions of  $Q$  are reversed the signs will be reversed.

**30. Arch Ribs for which  $\Delta = \delta s \div I$  is Constant.**—Since  $\Delta = \delta s \div I$  if the arch axis is so divided into parts  $\delta s_1$ ,  $\delta s_2$ ,



$\delta s_3$ , etc., in length so that the quotient of each  $\delta s$  divided by the moment of inertia of the radial section of the rib for the portion  $\delta s$  long, is constant, it is evident that  $\Delta$  will be a constant wherever it appears in the formulas given in previous articles. In many instances it becomes a factor which can be eliminated.

Under the assumption that  $\Delta$  is constant the formulas given later can be used for

1°. Arch ribs of constant cross-section when the axis of the arch is divided into equal parts, each  $\delta s$  in length.

2°. Parabolic arch ribs for which  $EI \cos \phi$  is constant when the span is divided into equal parts, each  $\delta x$  in length.

3°. Any arch rib for which  $\delta s \div I$  is constant when the axis of the arch is divided into parts  $\delta s_1, \delta s_2, \delta s_3$ , etc., so that the moment of inertia, usually taken for the radial section at the center of  $\delta s$ , for each division bears a constant ratio to the length of the division.

The formulas for the above condition are readily deduced from the general equations, remembering that  $\Sigma \Delta \div \Delta = n$ , and  $\Sigma y \div n = y_a$ , where  $n$  is the number of divisions of the axis and  $y_a$  the average value of  $y$ .

$$H = \frac{\Sigma m_x(y - y_a)}{\Sigma y(y - y_a)}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (126)$$

$$H_t = \frac{et^2 l E}{\Delta \Sigma y(y - y_a)}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (127)$$

$$H_a = \frac{-\Sigma \frac{\delta x \cos \phi}{F_x \Delta}}{\Sigma y(y - y_a) + \Sigma \frac{\delta x \cos \phi}{F_x \Delta}} \left\{ \begin{array}{c} H \\ \text{or} \\ H_t \end{array} \right\} . \quad . \quad . \quad (128)$$

In (128) it is sometimes convenient to replace  $\Sigma \frac{\delta x \cos \phi}{F_x \Delta}$  by  $\frac{1}{\Delta} \Sigma \frac{(\delta x)^2}{F_x \delta s}$ .

$$\left. \begin{matrix} M_1 \\ M_2 \end{matrix} \right\} = H_1 y_a - \frac{\Sigma m_x}{n} \mp \frac{\Sigma m_x \left( x - \frac{l}{2} \right)}{\frac{1}{2} n l - \frac{2 \Sigma x^2}{l}}. \quad (129)$$

For vertical loads

$$m_x = R_1 x - \sum_0^x P(x-a). \quad (130)$$

For horizontal loads,

$$m_x = \Sigma Q \frac{b}{l} x + \sum_0^x Q(y-b). \quad (131)$$

Equation (128) is approximate for horizontal loads as one term in (107) has been neglected.

*For Symmetrical Loading*

$$H = \frac{\Sigma m_x (y - y_a)}{\Sigma y (y - y_a)}, \quad (126)$$

$$M_1 = M_2 = H_1 y_a - \frac{\Sigma m_x}{n}, \quad (132)$$

where

$$m_x = \left( \frac{1}{2} \Sigma P \right) x - \sum_0^x P(x-a) \text{ for symmetrical vertical loads, } (133)$$

and

$$m_x = + \sum_0^x Q(y-b) \text{ for symmetrical horizontal loads. } (134)$$

In the formulas given above all horizontal loads act from right to left except in case of symmetrical loading. In case they act from left to right the sign of  $Q$  is negative.

*For a Single Vertical Load* (effect of axial stress neglected):

$$H = \frac{\sum_0^a x(y - y_a) - a \sum_0^a (y - y_a)}{\sum y(y - y_a)} P. \quad . \quad . \quad . \quad . \quad . \quad . \quad (135)$$

$$\left. \begin{matrix} M_1 \\ M_2 \end{matrix} \right\} = \mathbf{C}' - \mathbf{D}' \mp \mathbf{E}'. \quad . \quad . \quad . \quad . \quad . \quad . \quad (136)$$

$\mathbf{C}' = Hy_a$  including the effect of the axial stress.

$\mathbf{C}' = Hy_a$  neglecting the effect of the axial stress.

$$\mathbf{D}' = \frac{\sum m_x}{n} = \frac{1}{n} \left\{ \sum_0^a x + n'a \right\} P,$$

where  $n'$  is the number of divisions between the load and the crown.

$$\mathbf{E}' = \frac{\sum m_x \left( x - \frac{l}{2} \right)}{\frac{1}{2}nl - \frac{2 \sum x^2}{l}} = \frac{(R_1 - R_2) \sum_0^a \left( \frac{l}{2} - x \right) x + 2R_2 \sum_a^{l/2} \left( \frac{l}{2} - x \right)^2}{\frac{4}{l} \sum_0^{l/2} \left( \frac{l}{2} - x \right)^2} P.$$

---

\* See page 79 for a demonstration of this expression.

For a Single Horizontal Load (effect of axial stress neglected):

$$h_1' = \left\{ 1 - \frac{\sum_0^a y(y - y_a) - b \sum_0^a (y - y_a)}{\sum y(y - y_a)} \right\} Q \dots \dots \dots (135a)$$

$$\left. \begin{matrix} M_1 \\ M_2 \end{matrix} \right\} = \mathbf{F}' - \mathbf{G}' \mp \mathbf{H}' \dots \dots \dots (109a)$$

$$\mathbf{F}' = h_1 y_a \quad \text{or} \quad h_1' y_a.$$

$$\mathbf{G}' = \left( \sum y - \sum_0^a y - n'b \right) \frac{Q}{n}.$$

$$\mathbf{H}' = \left\{ b \sum_0^{l/2} \left( \frac{l}{2} - x \right)^2 - \frac{l}{2} \sum_0^a \left( x - \frac{l}{2} \right) y + \frac{bl}{2} \sum_0^a \left( x - \frac{l}{2} \right) \right\} \frac{-Q}{2 \sum_0^{l/2} \left( \frac{l}{2} - x \right)^2}.$$

**31. Fiber Stresses for any Section.**—(a) In the case of a steel rib, to which the formulas given above probably more nearly apply than for ribs of any other material, the formula based upon the common theory of flexure may be used. This formula may be written

$$p = \frac{N_x}{F} \pm \frac{M_x z}{I} = \frac{N_x}{F} \pm M_x \frac{1}{S},$$

where  $p$  = the stress in the outer fiber;

$N_x$  = the axial stress or the normal component of the resultant stress upon the section being considered;

$F$  = the area of the section;



$M_x$  = the bending moment at the section;

$z$  = distance of outer fiber from the neutral axis;

$I$  = the moment of inertia of the section;

$S = \frac{I}{z}$  = the "section modulus."

The above formula considers that the modulus of elasticity  $E$  is constant throughout the section for all intensities which do not exceed the elastic limit of the steel.

(b) If the arch rib is composed of natural stone voussoirs, it will be incapable of resisting tension at the joints owing to the uncertainty of the adhesion between the mortar and the stone. Consequently the above formula applies only when the resultant pressure upon any joint lies within the middle third of the joint; that is, the entire joint or section will be in compression.

In case the resultant does not lie within the middle third but does lie within the section we may yet have a perfectly stable structure. Suppose that the resultant cuts the section outside the middle third but not outside the stone, as in Fig. 12.

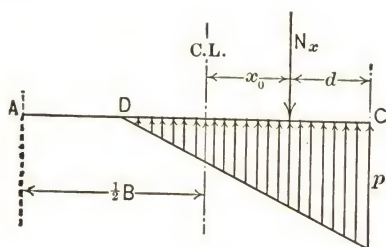


FIG. 12.

Let  $d$  = distance from edge C. The pressure may be assumed to be uniformly varying from C towards A, so that  $N_x$  will pass through the center of gravity of the intensities;

then

$$N_x = \frac{pCD}{2} = \frac{3pd}{2} = \frac{3}{2}p(\frac{1}{2}B - x_0),$$

or

$$p = \frac{2N_x}{3d}.$$

As long as  $p$  is so small that there is no danger of the stone being crushed the arch is stable. It is a recognized fact that this condition exists in a large number of arches now standing.

(c) Arch ribs constructed of plain concrete are capable of resisting a limited amount of tension, but it is better to treat them the same as if of natural stone. The ring may crack entirely through and yet be perfectly stable. Small rods of steel distributed laterally and circumferentially near the surfaces of the rib will prevent a considerable number of small cracks which might be produced by change of shape after removing the false work or changes of temperature.

(d) Reinforced-concrete ribs have circumferential steel rods or bars placed a short distance from the upper and lower surfaces of the rib to resist any tension which may occur. Even in this case the best designers limit the equilibrium polygons for dead and live load to nearly the middle third of the ring, so that there will be no tensile stresses.

The actual distribution of stress on a section of reinforced concrete is at present unknown. Many experiments have been made upon beams reinforced at the bottom, and various formulas advanced to aid in designing such beams, all giving fairly rational results. The elastic theory of the arch assumes that the linear arch is the neutral axis

of the material arch, and any great departure from the assumed form will affect the stresses; hence, since the experiments upon beams indicate that the neutral axis shifts for different loadings, it is evident that great refinement either in the calculation of stresses or the distribution of stress over a section is entirely out of place.

In the Melan system of reinforcement steel ribs are used spaced about 3 feet on centers. Here the steel may be assumed to resist the bending moments, and the concrete the direct compression. The concrete also prevents the steel ribs from buckling. It is questionable if the above assumption actually obtains. It is well on the side of safety, however.

One of the simplest methods in use merely replaces the steel reinforcement by an equivalent area of concrete and then employs the formula given above.

If the modulus of steel is  $E_s$  and that of concrete  $E_c$ , then the equivalent area of concrete will be  $\frac{E_s}{E_c} = n$  times the actual area of the steel. The fiber stress in the steel will actually be  $n$  times the fiber stress found for concrete in the position the steel occupies.

If  $a$  equals the area of the steel and  $A$  the area of the concrete, then

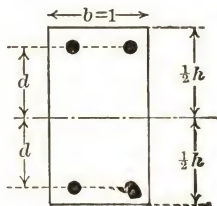


FIG. 13.

$$F = A + na,$$

$$I = \frac{h^3}{12} + nad^2,$$

and hence

$$p = \frac{N_x}{A + na} \pm M_x \frac{h}{\frac{h^3}{6} + 2nad^2}.$$



This formula assumes that the concrete resists tensile stresses which it is not capable of doing to any great extent, its tensile strength being somewhere near one tenth the compressive strength.

The above formula may be applied until the maximum safe tensile strength of the concrete or steel is reached, and then the method employed for stone arches when the resultant pressure lies within the ring until the safe compressive strength is reached.

All of the methods are quite approximate for reasons given above, and since the modulus of elasticity of concrete is not constant.

**32. Reliability of the Elastic Theory when Applied to Steel Ribs.**—There is but little doubt that the theory is correct for solid steel ribs having a depth which is comparatively small when compared with the radius of curvature, when the loading is applied at isolated points through vertical posts which are unbraced in the plane of the rib. The modulus of elasticity of steel is quite constant and it is capable of resisting both tension and compression. The deformation of steel either under direct stress or bending follows very closely that found by theory. In truth the theory is probably as exact for steel arch ribs as the common theory of flexure is for steel beams.

**33. Reliability of the Elastic Theory when Applied to Ribs Composed of Natural Stone Voussoirs.**—Here we have a material which cannot be trusted in tension; this is especially true of the joints between the voussoirs. In direct compression the modulus of elasticity is not constant but varies with the load, and then not according to any very definite law. However, within narrow limits it may be



considered as constant without serious error. Such being the case we may apply the elastic theory with confidence as long as the *equilibrium polygon lies within the middle third* of the ring, or when every section or joint is subjected to compressive stresses. We may not consider the theory as applicable when the polygon lies very much outside the middle third of the ring.

In case the equilibrium polygon passes without the ring at any joint, theoretically a free arch ring would fail. In practice this condition often obtains in stone bridges, yet they do not collapse or show serious signs of failure. It is true that some joints open slightly, but this appears to have little if any detrimental effect. This apparently proves that the elastic theory cannot be applied under such conditions. It is no fault in theory, but a failure to carry out in practice the assumptions made in applying the theory or basing the application of the theory upon wrong assumptions. For example, the elastic theory assumes a free rib capable of changing shape under various loads, while in practice the great majority of stone bridges have the ring securely clamped beneath the solid spandrel walls and by a mass of concrete backing of varying thickness. Such a structure may be said to become more and more stable under an increasing uniform loading, until the safe crushing strength of the arch stone is reached. This backing exerts a great passive force preventing any upward movement of the arch ring. It is evident, then, that if the ring is stable under the elastic theory assuming a free ring, it will be quite safe when clamped as explained above, and furthermore it does not necessarily follow, because the equilibrium polygon lies without the ring proper

at some joint, that the arch will fail, for the spandrel masonry will prevent a change in shape of the rib to any great extent. The question at once presents itself: What does happen? Probably the masonry readjusts itself until equilibrium exists, the arch joints are compressed unequally, and the friction of the spandrel masonry aids very materially in reducing the opening or compression of the joints at the extrados—in fact introducing an effective tension or compression, as the case may be.

Again, in bridges having a considerable depth of side wall above the crown a large portion, if not all, of the ring under the walls might be removed in many cases without complete failure, the wall masonry forming an arch in itself. In conclusion, for the dead and live loads the arch ring which is safe when assumed to be a free ring will be safe under the usual construction of the spandrels, or if the loads are transmitted to the ring through verticals as in steel structures. All arch rings should be so designed, using a factor of safety of *ten* for the crushing strength of the stone.

Provision for the stresses produced by changes in temperature was entirely neglected by the old builders, and for that matter by practically all modern builders. A temperature change of but  $\pm 40^{\circ}$  F., according to the elastic theory, produces a very wide range of stress both of tension and compression. These are a maximum at the supports. If any considerable change of temperature actually occurs and the elastic theory can be correctly applied, the arch ring, if free, should collapse. As stone arch bridges have stood for thousands of years without failure, we must conclude that either the stone does not

change in temperature through anything like the range of change in the air, or the arch ring adjusts itself with the aid of the spandrel masonry so as to resist the temperature stresses without excessive unit stresses, or the theory does not apply. Probably all three conclusions are more or less true. Even in the Northern States it is doubtful if any of the stonework, excepting possibly the more exposed surfaces, has a change of temperature of a great range,— $\pm 20^{\circ}$  F., say. The ring without any doubt adjusts itself to suit new conditions.

To show how small a change would be required in the mortar joints alone to provide for a change of  $40^{\circ}$  F., take a free rib of granite having a span of 60 ft. and a rise of 8 ft. (measurements taken for the axis). The length of the rib axis is 62.8 ft. The coefficient of expansion for  $1^{\circ}$  F. is 0.0000038. Then the total change in length of the rib is 0.0095 ft.; if there are 42 joints, the change in each joint would be 0.0002 ft. .0024 in., which is too small to be readily detected. Of course the joints do not all distort the same. Again, assume the rib under masonry spandrel walls, and let there be an increase of  $40^{\circ}$  F. in temperature, and also assume that the rib cannot rise; then the entire temperature effect must be used up in compressing the ring. The change in length per unit is  $40(0.0000038) = 0.00015$ . If the  $E = 6800000$ , the stress per square inch is a little over 1000 lbs. This might be increased to 10000 lbs. without the granite being crushed, even with the dead- and live-load stresses added.

Considering our ignorance of the actual temperature changes and the behavior of the stone under these changes, it is useless to attempt any theoretical treatment until our



knowledge of the subject has been very much increased. The temperature stresses appear to be able to take care of themselves as long as the rib is stable for the dead and live loads.

**34. Reliability of the Elastic Theory when Applied to Plain Concrete Ribs.**—Here we have a material which is fully as variable in its physical qualities as natural stone. Generally we have no joints to consider and no masonry spandrel backing, but we do have monolithic spandrel side walls clamping the rib, in many instances. As concrete resists tensile stresses but indifferently, it is not safe to permit more than *one tenth* its safe compressive strength in designing. As this amounts to about 50 lbs. per square inch, it may as well be neglected entirely, and the rib designed for the dead and live loads so that no tension can exist at any section.

The effects of changes of temperature are as uncertain as in stone arches. Having no joints, the ring cannot readily adjust itself, and hence probably resists some tension. As the modulus of elasticity is much less than for natural stone, although the coefficient of expansion is much greater, the theoretical stresses are very much smaller. For free rings no tension exceeding 50 lbs. per square inch should be allowed under any conditions, unless the concrete is reinforced with steel to prevent cracking. At present there appears to be no rational way of determining the amount of steel required so all that can be done is to experiment and follow previous builders where they have been successful. If the rib should crack through, it would not necessarily mean failure, as then the behavior would follow that of a voussoir ring.



**35. Reliability of the Elastic Theory when Applied to Reinforced Concrete Ribs.**—Concrete when reinforced with steel is very much more reliable than concrete without the steel. The principal difficulty experienced is the location of the neutral axis of any particular section. The location without any doubt shifts about under the action of different loads. As the elastic theory assumes the arch axis to pass through the neutral axis of each section of the rib, it is evident that we must assume the axis to lie at the center of gravity of the section and treat the material according to the common theory of flexure.

While a reinforced rib will safely resist tension by virtue of the steel, yet the best designers so proportion the arch rib that it is never subjected to tension under dead and live loads. For temperature stresses the compression in the concrete must not exceed about 800 pounds per square inch, including the effect of the dead and live loads. Under this assumption the concrete may crack on the tension side, and the steel resist all of the tension.

Even when considering the difficulties briefly mentioned above and our almost absolute ignorance of the actual distribution of stress over a reinforced section, we are compelled to accept the elastic theory as our best guide in designing reinforced-concrete ribs.

That the formulas based on the elastic theory are very nearly true has been shown experimentally. A comparison of the theoretical and experimental results is shown in Figs. 14 and 15.\*

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\* "Experimental Verification of Arch Formulas," Malverd A. Howe, *Railroad Age Gazette*, March 26, 1909.

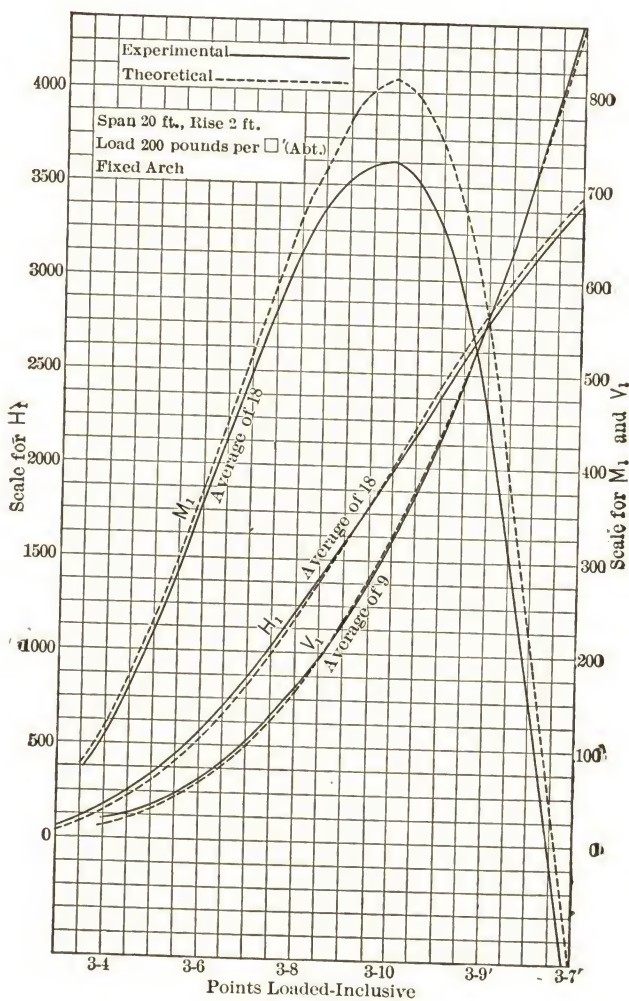


FIG. 14.

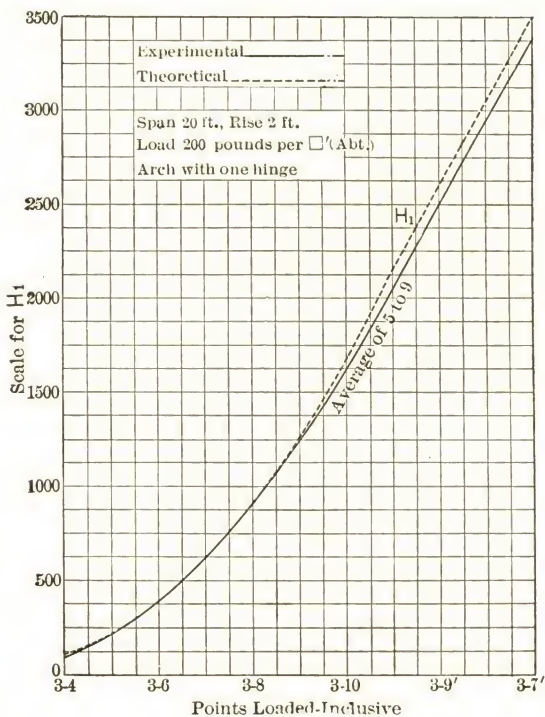


FIG. 15.

**36. Reliability of the Elastic Theory: Summary.**—For steel ribs it is without doubt quite reliable. For natural stone, concrete, and reinforced concrete the theory can be used with confidence as long as no tensile stresses occur in the rib. When tensile stresses obtain the theory applied under the usual assumptions is but an approximation.

**37. Depth of the Arch Rib.**—This must be assumed from the best data available, and then calculations made to see if it will answer under all conditions of loading, and

changes of temperature.\* If found necessary, the rib can be modified somewhat without making new calculations by changing the moments of inertia of all sections in the same ratio. The dead- and live-load stresses will remain sensibly unchanged, the change in weight of the rib being very small in comparison with the total dead load. The temperature and axial thrust stresses will be slightly modified. The question of the necessity of a new calculation must be decided by the designer according to his best judgment. In Table II are given the data for a large number of arch ribs to aid in assuming the dimensions of a proposed design.

The articles immediately following give the principal empirical formulas for the dimensions of arch rings, etc.

**38. Empirical Formulas for the Thickness of the Ring at the Crown in Stone Arches.**—Many formulas have been advanced for the depth of the arch ring at the crown. These are usually based upon the dimensions of arches constructed, and hence they merely indicate that an arch built like one which has been standing some time will probably stand also.

#### NOMENCLATURE.

$t_0$  = depth of arch ring at the crown, in feet;

$R$  = radius of curvature of intrados at the crown, in feet;

$l$  = clear span of arch, in feet;

$f$  = clear rise of arch, in feet.

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\* See Appendix B.



*Trautwine's Formulas.\**—The following formulas apply to circular and elliptical arches:

For first-class cut stone:

$$t_0 = 0.25\sqrt{R + 0.5l} + 0.2.$$

For second-class work:

$$t_0 = 0.281\sqrt{R + 0.5l} + 0.225.$$

For brickwork or fair rubble:

$$t_0 = 0.333\sqrt{R + 0.5l} + 0.267.$$

*Low's Formula:* †

$$t_0 = 0.125\sqrt{10(l-f) + 2H},$$

where  $H$  = the surcharge above the extrados at the crown.

*Rankine's Formulas:*

$$t_0 = \sqrt{0.12R} \text{ for a single arch;}$$

$$t_0 = \sqrt{0.17R} \text{ for an arch in a series.}$$

*Perronet's Formula for circular or elliptical arches:* ‡

$$t_0 = 1 + 0.035l.$$

*Dejardin's Formulas for circular arches:* ‡

$$\text{For } \frac{f}{l} = \frac{1}{2} \dots \dots \dots t_0 = 1 + 0.10R.$$

$$\text{For } \frac{f}{l} = \frac{1}{6} \dots \dots \dots t_0 = 1 + 0.05R.$$

\* Trautwine's "Engineer's Pocket-book."

† Engineering News, June 15, 1905.

‡ From paper by E. Sherman Gould, Van Nostrand's Mag., vol. xxix, p. 450.

For  $\frac{f}{l} = \frac{1}{8}$  . . . . .  $t_0 = 1 + 0.035R$ .

For  $\frac{f}{l} = \frac{1}{10}$  . . . . .  $t_0 = 1 + 0.02R$ .

*Dejardin's Formula* \* for elliptical and basket-handled arches:

For  $\frac{f}{l} = \frac{1}{3}$  . . . . .  $t_0 = 1 + 0.07R$ .

*Croizette-Desnoyer's Formulas*: \*

For  $\frac{f}{l} > \frac{1}{6}$  . . . . .  $t_0 = 0.50 + 0.28\sqrt{2R}$ .

For  $\frac{f}{l} = \frac{1}{6}$  . . . . .  $t_0 = 0.50 + 0.26\sqrt{2R}$ .

For  $\frac{f}{l} = \frac{1}{12}$  . . . . .  $t_0 = 0.50 + 0.20\sqrt{2R}$ .

For elliptical arches use  $R$  for circle having same rise and span.

*German and Russian Practice*: \*

$$t_0 = 1 + 0.035l + 0.02H,$$

where  $H$  = the surcharge over the extrados at the crown, including the moving load if any.

*Austrian Specifications for large arches of brick and stone*: †

$f/l$  between  $\frac{1}{2}$  and  $\frac{2}{3}$ .

For  $l = 30$  metres. . . .  $t_0 = 1.1$  m.

For  $l = 40$     "    . . . .  $t_0 = 1.4$  "

For  $l = 65$     "    . . .  $t_0 = 2.2$  "

\* From paper by E. Sherman Gould, Van Nostrand's Mag., vol. xxix, p. 450. 1883.

† "A Treatise on Arches," by Malverd A. Howe. Wiley.

For  $l = 80$  metres. . . .  $t_0 = 2.7$  m

For  $l = 100$  " . . . .  $t_0 = 3.4$  "

For  $l = 120$  " . . . .  $t_0 = 4.1$  "

**39. Thickness of Arch Ring of Stone at the Support.**—For semicircular stone arches it is generally assumed that the masonry for  $30^\circ$  from the spring line is self-supporting and consequently has no arch action. If this is so, then the maximum angle which a stone arch ring can be considered to subtend is  $60^\circ$  each way from the crown. If the loading is so arranged that the equilibrium polygon follows the axis of the ring, then the pressures will vary directly as the secant of the angle  $\phi$ ; consequently the ring thickness  $60^\circ$  from the crown should be  $t_s = t_0 \sec 60^\circ = 2t_0$ .

*\* Croizette-Desnoyer's Formulas for segmental arches:*

For  $\frac{f}{l} = \frac{1}{6}$ . . . . .  $t_s = 1.40t_0$ .

For  $\frac{f}{l} = \frac{1}{8}$ . . . . .  $t_s = 1.24t_0$ .

For  $\frac{f}{l} = \frac{1}{10}$ . . . . .  $t_s = 1.15t_0$ .

For  $\frac{f}{l} = \frac{1}{12}$ . . . . .  $t_s = 1.10t_0$ .

For basket-handled arches:

when  $\frac{f}{l} = \frac{1}{3}$ . . . . .  $t_s = 1.80t_0$ ;

$\frac{f}{l} = \frac{1}{4}$ . . . . .  $t_s = 1.60t_0$ ;

$\frac{f}{l} = \frac{1}{5}$ . . . . .  $t_s = 1.40t_0$ .

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\* Van Nostrand's Engineering Magazine, vol. xxix, p. 454.

**39a. Thickness of Arch Ring of Concrete.—***Weld's Formula for Reinforced Concrete Arches.\**

$$12t_0 = \sqrt{l} + 0.1l + 0.005W + 0.0025D;$$

 $W$  = moving load per square foot; $D$  = weight of fill above the crown per square foot.*Frye's Formulas for Concrete and Cut-stone Arches.†*

For highway bridges:

$$t_0 = 0.15 + \sqrt{\frac{l}{100} \left( 3 + \frac{l}{f} \right)}.$$

For high H.W. embankment or for railroad bridge:

$$t_0 = 0.20 + \sqrt{\frac{l}{100} \left( 4 + \frac{l}{f} \right)}.$$

For high railroad embankment:

$$t_0 = 0.25 + \sqrt{\frac{l}{100} \left( 5 + \frac{l}{f} \right)}.$$

$$t_s = t_0(1 + 0.002[l + 2f]).$$

*Formulas for Highway Bridges.‡*

| Material.                 | Span in Feet.          |                      |                                         |                        |
|---------------------------|------------------------|----------------------|-----------------------------------------|------------------------|
|                           | Under 20. <sup>1</sup> | 20-50. <sup>1</sup>  | 50-150. <sup>2</sup>                    | Over 150. <sup>3</sup> |
| 1st class, ashlar...      | $t_0$<br>0.04(6+l)     | $t_0$<br>0.020(30+l) | $t_0$<br>0.00012(11000+l <sup>2</sup> ) | $t_0$<br>0.018(75+l)   |
| 2d class, ashlar or brick | 0.06(6+l)              | 0.025(30+l)          | 0.00016(11000+l <sup>2</sup> )          | 0.025(76+l)            |
| Plain concrete...         | 0.04(6+l)              | 0.020(30+l)          | 0.00014(11000+l <sup>2</sup> )          | 0.020(75+l)            |
| Reinforced concrete       | 0.03(6+l)              | 0.015(30+l)          | 0.00010(11000+l <sup>2</sup> )          | 0.016(75+l)            |

<sup>1</sup> For culverts under a high fill, add 60%. For railroad arches, add 25%.<sup>2</sup> For railroad arches, add 20%.<sup>3</sup> For railroad arches, add 15%.

\* The Engineering Record, Nov. 4. 1905.

† "Civil Engineers' Pocket-book," A. I. Frye, page 766.

‡ "American Civil Engineers' Pocket-book," Mansfield Merriman, Editor-in-chief.



40. **Thickness of Abutment.**—Trautwine's rule for all kinds of stone arches is best explained by means of a diagram, Fig. 16. This form of abutment, according to

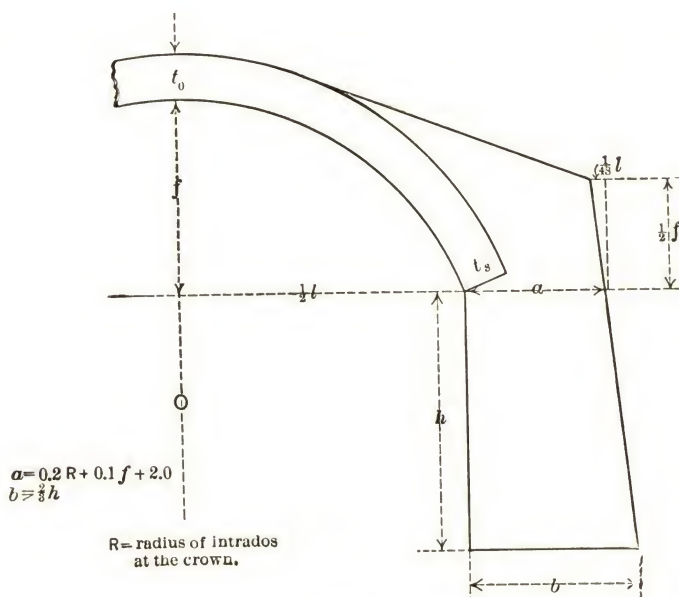


FIG. 16.

*Trautwine*, is sufficiently strong to take the thrust due to the dead load before the back filling of earth is in place.

*Rankine* states that in existing structures the thickness  $a$  varies from  $\frac{1}{3}$  to  $\frac{1}{2}$  the radius of the intrados at the crown.

*Baker*, in "A Treatise on Masonry Construction," gives a formula, said to represent *German* and *Russian* practice, which has the form

$$a = 1 + 0.04(5l + 4h),$$

where  $h$  is the distance from the spring line down to the top of the foundation.

**41. Thickness of Piers.**—In a series of arches it is customary to use several narrow piers and then introduce a much heavier pier, called an abutment pier. This should be of sufficient strength to resist the thrust from one side without any aid from the arches upon the other side. The thickness will then be the same as if it were an abutment in reality without earth backing. For the regular piers various rules have been used. Twice the thickness of the arch ring at the crown plus a fraction of a foot has been used in very important bridges. Usually piers are from  $2\frac{1}{2}$  to 3 times the thickness of the arch ring at the crown.

The vertical load upon piers is not very large when measured in tons per square foot, and as far as strength is concerned they could be made considerably smaller than outlined above. The only horizontal thrust to be resisted is the unbalanced thrust produced by the moving load, unless adjacent arches are of different dimensions. With the exception of high bridges the effect of the wind is of no moment.

**42. Remarks concerning Empirical Formulas.**—The formulas given in the previous articles are based, for the most part, upon actual structures and will without doubt lead to safe structures if the equivalents in materials and workmanship are held throughout. Apparently the formulas apply to all kinds of stone, as no mention is made of the quality of the materials (excepting Trautwine's formulas) used. Unquestionably the arch rings were constructed of average materials, probably no better if as good as those used now; hence the formulas will be of service in assuming dimensions which can be relied upon as

being safe for structures quite similar to those upon which the formulas are based.

**43. Albula Railroad Practice\*** (gage 1 m.).—The following dimensions were used in the construction of a great number of arches on the Albula Railroad.

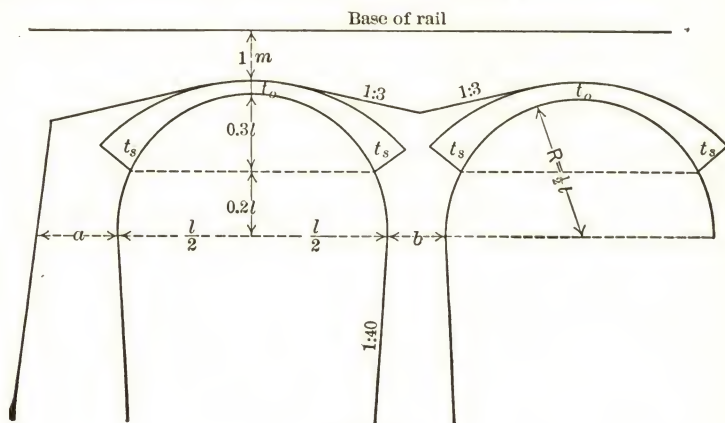


FIG. 17.

|                   |      |      |      |      |      |      |      |
|-------------------|------|------|------|------|------|------|------|
| Span..... $l$     | 6    | 8    | 10   | 12   | 15   | 20   | 25   |
| Key..... $t_0$    | 0.55 | 0.60 | 0.70 | 0.75 | 0.80 | 0.90 | 1.00 |
| Spring..... $t_s$ | 0.80 | 0.90 | 1.00 | 1.10 | 1.20 | 1.35 | 1.50 |
| Pier..... $b$     | 1.20 | 1.35 | 1.50 | 1.70 | 2.10 | 2.70 | 3.60 |
| Abutment..... $a$ | 1.70 | 1.90 | 2.10 | 2.80 | 3.50 | 4.20 | 5.30 |

Twenty-six viaducts were built of the spans given below:

|                                 |    |    |    |    |    |    |    |    |    |    |    |    |
|---------------------------------|----|----|----|----|----|----|----|----|----|----|----|----|
| Span..... $l$                   | 10 | 11 | 12 | 14 | 15 | 16 | 20 | 22 | 25 | 27 | 30 | 42 |
| Number of                       |    |    |    |    |    |    |    |    |    |    |    |    |
| spans.....                      | 33 | 3  | 7  | 1  | 16 | 14 | 15 | 1  | 1  | 1  | 1  | 1  |
| (All dimensions are in metres.) |    |    |    |    |    |    |    |    |    |    |    |    |

**43a. Stability of Piers and Abutments.**—The width of a pier at the spring line is assumed or derived according to Art. 41. The sides usually have a batter of one in

\* The Engineer, 1904.



twenty-four down to the top course of the foundation. The foundation courses are stepped down until sufficient bearing area is obtained. If the pier rests upon solid rock the width of the bottom course will be less than when the pier rests on gravel or is supported by piles.

When the spans adjacent to a pier are equal and there is no live load on the spans, the pier is subjected to a uniform compression at each horizontal section. The intensity of this compression equals the weight of the material above the section which is included between the centers of the two spans, divided by the area of the section. If both spans are fully loaded with a uniform live load, the load on any horizontal section of the pier is equal to the total weight above the section which is included between the centers of the spans. These loads produce uniform intensities which are usually considerably less than the allowable intensities for the materials composing the pier and the material upon which the pier rests. If only one span is loaded with a full live load each section of the pier is subjected to an uniform intensity produced by the total load above the section and in addition to a uniformly varying intensity produced by a bending moment which is due to the unequal horizontal thrusts and vertical reactions. This is equivalent to the bending moment produced by the horizontal thrust and vertical reaction of the live load alone, as the dead load forces balance. The effect of this moment is compression on the side of the pier adjacent to the unloaded span and tension on the side adjacent to the loaded span. The intensities of these forces combined algebraically with the uniform intensity must not exceed the allowable



intensities for the material composing or supporting the pier. While reinforced concrete piers can safely resist a certain tension, yet it is good practice to so proportion the pier that all sections are subjected to compression.

To illustrate the method to be followed in testing an assumed design, let Fig. 18 represent the section of a pier taken on the longitudinal axis of the bridge. The arch-rings are 20 ft. in width. The piers have round ends down to the level of  $DD'$ . Below  $DD'$  all sections are rectangular and 31 ft. long.

Considering the horizontal section at  $KK'$  the total weight of the material between the centers of the spans is  $1.351000 + 676000 + 600000 = 2.627000$  lbs. Since the area of the section  $KK'$  is  $15 \times 31 = 465$  sq.ft., the uniform intensity is equal to  $2627000 \div 465 = 5600$  lbs. per square foot.

The moment of the unsymmetrical forces about the center of  $KK'$  is obtained as follows:

$$\begin{array}{rcl}
 +1108000 \times 28.45 & = & +31522600 \\
 +600000 \times 3.50 & = & +2100000 \\
 \hline
 & & +33622600 \\
 -930000 \times 28.27 & = & -26291100 \\
 -676000 \times 3.50 & = & -2366090 \\
 \hline
 & & -28657100 \\
 \text{Total moment} & = & +4965500
 \end{array}$$

The intensity of the resulting stress at  $K'$  is

$$\frac{4965500(7.5)}{\frac{1}{12}(31)(15)^3} = 4300 \text{ lbs. per square foot compression,}$$

and that at  $K$  is 4300 lbs. per square foot tension.

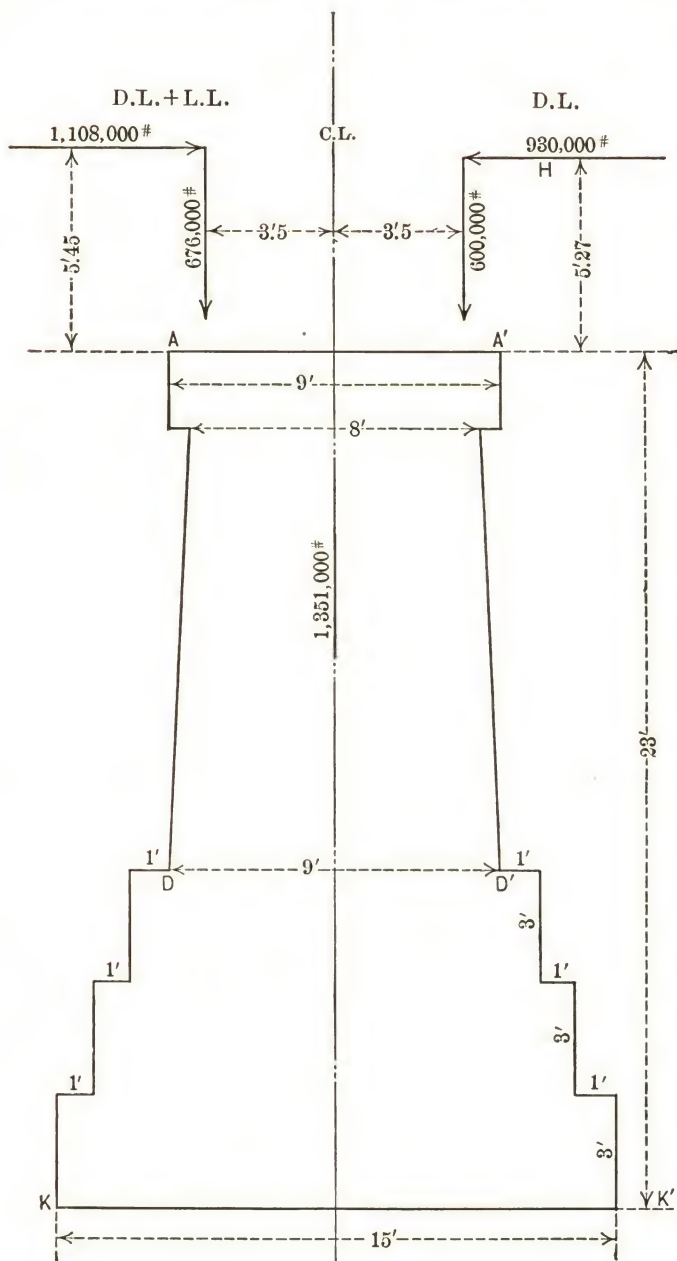


FIG. 18.

The total intensity at  $K' = 5600 + 4300 = 9900$  lbs. per sq. ft.

“ “  $K = 5600 - 4300 = 1300$  “ “ “

This shows that there is no tension at this section. If the pier rests on compact gravel or any harder material the maximum intensity, 9900, is within safe limits. If the material is not equal to hard gravel the base should be widened or the pier supported on piles.

As a matter of precaution the section at  $DD'$  will be considered to see if it is in compression throughout.

The section  $DD'$  is composed of a rectangle and two semicircular ends. The area is 244 sq.ft. and the moment of inertia is 1537 (foot-units).

The total vertical load at this section is  $807000 + 676000 + 600000 = 2083000$  lbs. and the resulting uniform intensity is equal to  $2083000 \div 244 = 8540$  lbs. per square foot.

The moment of all the unsymmetrical forces above  $DD'$  is 3363500 ft. lbs. The intensity at  $D'$  due to this moment is

$$\frac{3363500(4.5)}{1537} = 9850 \text{ lbs. per square foot compression}$$

and at  $D'$  the same amount, tension.

The total intensity at  $D' = 8540 + 9850 = 18390$  lbs. per sq.ft.

The total intensity at  $D = 8540 - 9850 = -310$  “ “

This indicates a very small tension at  $D$ , less than 3 lbs. per square inch. The intensity at  $D'$  is about 128 lbs. per square inch, which is very much less than the allowable intensity for any material which would be used in a pier.

If the adjacent spans are unequal, the projections of the footing courses may be increased on the side adjacent to the shorter span to counteract the effect of the unequal thrusts. This can be so adjusted that the pier will transmit a uniform pressure to the material upon which it rests. The same end may be obtained by raising the spring line of the shorter span above that of the longer span.

In the above treatment the pier has been considered non-elastic. While this assumption is not correct, it is doubtful if the data upon which the analysis is based, warrant the refinement of considering the elastic constants of the materials composing the pier.

The stability of the abutment is considered in a similar manner. The abutment should be stable under the thrust produced by the dead load alone without any assistance from the earth fill above or back of it. When the moving load is considered with the dead load the fill above the abutment and the thrust of the earth back of the abutment are included.

**44. The Dead Load.**—Very little is required in the way of discussion in reference to the dead load for steel ribs. The floor and all supports, and even the lateral systems, can all be designed and the actual weights computed. There remains, then, only the weight of the rib proper to be estimated. The weight of the assumed rib will be sufficiently close for all purposes, as a large error in the weight of the rib will be comparatively small for the entire load. The weight above the rib is usually transmitted to the rib through verticals extending up to the roadway.

In the case of masonry ribs with the spandrels com-



pletely filled with earth, sand, gravel, etc., the actual load supported by the rib is not very definite. If the filling is put in in horizontal layers well compacted, the load upon the ring will certainly not exceed the actual weight of the material, and it is very doubtful if such filling creates any considerable horizontal thrust against the rib. If perfectly dry and clean sand or gravel is employed, then there may be horizontal forces acting against the rib. These will be very small, however, for segmental arches. This thrust can be found according to the theory of earth pressure.\*

The consideration of the horizontal thrust of the spandrel filling is a refinement not warranted in works of this class. The weight of the spandrel filling with pavement, arch rib, etc., should be considered as divided into vertical loads, the horizontal projection of  $\delta s$  being the measure of each division. For computations the load may be assumed to act at the center of the projection of  $\delta s$ .

In case the spandrels are partially filled with concrete its weight may be taken as divided into vertical forces. This is probably not as near the truth as when the fill is made of sand or gravel, but the assumption is on the safe side. Overloading the haunches will cause an upward movement at the crown, and overloading the crown causes the haunches to rise; but when the spandrel filling is partially concrete the *passive* resistance to an upward movement is very much in excess of its weight; so also is that of sand or gravel. The arch rib, then, in this type of bridge is anything but a free member, and consequently

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\* "Retaining-walls for Earth," by Malverd A. Howe. John Wiley & Sons, Inc., New York.

any great refinement in its design is time wasted. If we can assure ourselves that the rib is safe by adding a few inches to the thickness of the ring, the very small percentage of extra cost need not be considered at all.

When the roadway is supported by longitudinal walls resting upon the rib, the problem is at least as complex as before, for there is no way of knowing how the weights transmitted by the walls are distributed. The only recourse is to treat the material as in the case of sand or gravel filling.

The use of lateral walls or columns to support the roadway places the problem in a shape to be carefully considered theoretically. The actual magnitudes of the loads can be computed and the points of application to the rib are definitely fixed. For long spans this is unquestionably the best and most economical type which can be built. There is an exception to this in very flat arches where the ring occupies the greater portion of the vertical projection of the bridge.

**45. Dead-load Equilibrium Polygon Following the Axis of the Arch Rib.**—It is assumed that the rib has been dimensioned and that the fill over the crown is known. Compute the weight of the shaded portion in Fig. 19 and call it  $P_1$ . Lay off the vertical line  $DE$ , and  $P_1$  from  $D$ . Draw  $DO$  horizontal and  $CO$  parallel to the tangent to the arch axis at  $b$ . Then from  $O$  draw lines parallel to the tangents at  $c, d, e$ , etc.; then these lines will cut off on  $DE$  the loads  $P_2, P_3$ , etc., for which an equilibrium polygon will pass through the points  $a, b, c, d$ , etc., and  $DO$  will be the horizontal thrust for this loading. A check calculation will show that this is the true horizontal thrust

according to the elastic theory, *neglecting the effect of the axial stress.*

By a similar construction the polygon may be made to pass through the points where the loads are applied to the

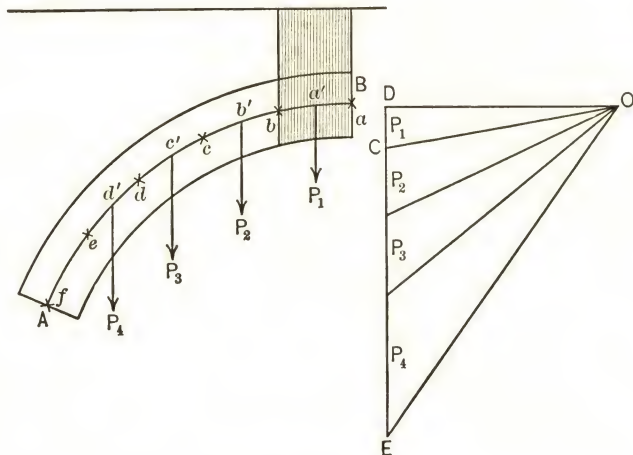


FIG. 19.

axis. In either case the bending moments due to the dead load are sensibly zero. This assumes that the loads are reasonably close together.

Filled spandrels can usually be made so that the above conditions are fulfilled by selecting proper filling materials.



## CHAPTER IV.

### EXAMPLES SHOWING THE APPLICATION OF THE FORMULAS, ETC.

**46. Preliminary.**— In the examples which follow, the computations will be given in detail, with suggestions as to methods and checks. In some cases it will be found that the algebraic work necessary to get the data into shape for applying the arch theory requires as much time as the computation of  $H_1$  for each load respectively. Some of this work will be found quite unnecessary by many. It is given in one case for the benefit of the few who may use the example as a guide for their first arch calculation.

**47. First Example: Data.**— Let us assume that the design shall be for a single-track railway bridge with an arch ring of Quincy, Mass., granite, and that the axis of the ring has a span of 60 ft. and a rise of 8 ft. Let the spandrel filling be cinders, sand, or gravel, in such proportions that the total dead load will have its equilibrium polygon following the axis of the ring. Since this is to be a railway bridge, there should be at least 3 ft. of fill between the base of the rail and the arch ring at the crown. This will distribute the moving load which may be assumed at 5000 lbs. per foot of span. If the ties are 8 ft. long, we may assume that the fill will distribute 5000 lbs. over at



least 13 ft. under the ties, or that the moving load will be about 400 lbs. per square foot. 30 lbs. per square foot will cover the weight of the track.

**48. Subdivision of the Arch Axis.**—This should not be decided upon until the shape of the arch ring is determined. In this case let the ring be of uniform depth throughout; then, in order that  $I$  may be constant, the *axis* should be divided into *equal parts*. In all summation formulas it is well known that the smaller the divisions are made the more accurate will be the results. (In this

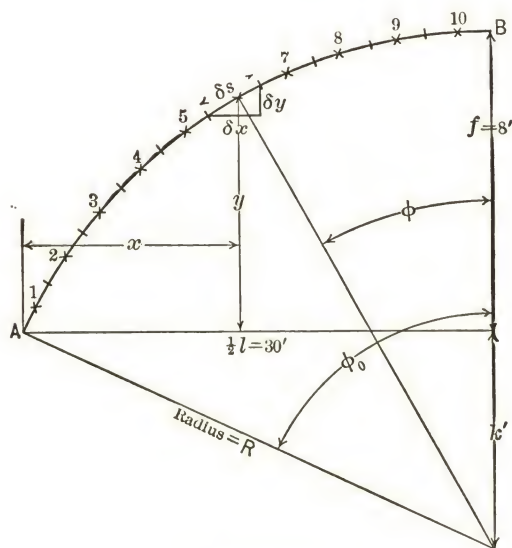


FIG. 20.

particular case  $\delta s$  might be replaced by  $ds$ , and the problem solved, as far as  $H_1$ ,  $M_1$ , etc., are concerned, by means of integration.)\*

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\* "A Treatise on Arches," by Malverd A. Howe, John Wiley & Sons, Inc., New York.

Ordinarily *twenty* divisions will give results sufficiently accurate for practical purposes. This number will be used.

From Fig. 20,

$$\frac{1}{2}l = 30 = \sqrt{R^2 - (R - 8)^2}. \quad \therefore R = 60.25 \text{ ft., and } k' = 52.25 \text{ ft.}$$

$$\sin \phi_0 = \frac{\frac{1}{2}l}{R} = \frac{30}{60.25} \quad \therefore \phi_0 = 29^\circ 51'.76.$$

$$\text{Arc } AB = \frac{2\pi R}{360} 29.8616 = 31.40 \text{ ft.}$$

Hence  $\delta s = 3.14$  ft., and the angle at the center for each division is  $2^\circ.98616$ .

**49. Computation of  $x$  and  $y$ .**—The values of  $x$  and  $y$  are computed for the center points of the divisions made above, as shown in detail in Table A.

TABLE A.

| Point. | $\phi$ .           | $\sin \phi$ . | $\cos \phi$ . | $R \sin \phi$ . | $R \cos \phi$ . | $x$                  | $y$                     |
|--------|--------------------|---------------|---------------|-----------------|-----------------|----------------------|-------------------------|
|        |                    |               |               |                 |                 | $30 - R \sin \phi$ . | $R \cos \phi - 52.25$ . |
| 1      | $28^\circ 22'.172$ | 0.47515       | 0.87991       | 28.628          | 53.015          | 1.372                | 0.765                   |
| 2      | $25 \ 22 \ .996$   | .42867        | .90346        | 25.827          | 54.433          | 4.173                | 2.183                   |
| 3      | $22 \ 23 \ .820$   | .38102        | .92457        | 22.956          | 55.705          | 7.044                | 3.455                   |
| 4      | $19 \ 24 \ .644$   | .33234        | .94316        | 20.023          | 56.625          | 9.977                | 4.575                   |
| 5      | $16 \ 25 \ .468$   | .28275        | .95919        | 17.036          | 57.791          | 12.964               | 5.541                   |
| 6      | $13 \ 26 \ .292$   | .23239        | .97262        | 14.001          | 58.600          | 15.999               | 6.350                   |
| 7      | $10 \ 27 \ .116$   | .18141        | .98335        | 10.930          | 59.247          | 19.070               | 7.000                   |
| 8      | $7 \ 27 \ .940$    | .12993        | .99152        | 7.828           | 59.739          | 22.172               | 7.489                   |
| 9      | $4 \ 28 \ .764$    | .07810        | .99694        | 4.705           | 60.065          | 25.295               | 7.815                   |
| 10     | $1 \ 29 \ .588$    | .02606        | .99966        | 1.570           | 60.229          | 28.430               | 7.979                   |
| C      | $0 \ 0$            | 0             | 1.00000       | 0               | 60.25           | 30.000               | 8.000                   |

In this particular case we are probably not warranted in using three decimal places in the values of  $x$  and  $y$ , although the labor is but a very little greater than if but two were used. This is assuming that multiplications are

performed by machine or a multiplication-table. After a little practice Crelle's "Rechentafeln" will be found quite satisfactory for all multiplications and many divisions.

**50. Computation of  $H$  for Unit Loads.**—From (135), Art. 30,

$$H = \frac{\sum_0^a x(y - y_a) + a \sum_0^a (y - y_a)}{\sum y(y - y_a)} P. \quad . \quad . \quad . \quad (135)$$

In this case  $P = 1$ .

We will assume a load unity at point 1, Fig. 20, and compute the value of  $H$  for this load; then a load unity at point 2, Fig. 20, and compute the value of  $H$  for this load alone, and so on, until the values of  $H$  have been found for a unit load at each point respectively.

If the symmetrical points upon the right of the crown be designated 1', 2', 3', etc., the value of  $H$  for a unit load at 1' equals the value of  $H_1$  for a unit load at point 1, etc., since the arch ring is symmetrical in every respect about the crown.

The determination of  $H$  is shown in detail in Table B, p. 77.

When the numerical work is correct the sum of the values of  $y - y_a$  in column 2 should be zero.

**51. Computation of  $M_1$ ,  $V_1$ ,  $y_1$ ,  $y_2$ , and  $y_0$  for Unit Loads.**—From (129), Art. 30,

$$\left. \begin{array}{l} M_1 \\ M_2 \end{array} \right\} = Hy_a - \frac{\sum m_x}{n} \mp \frac{\sum m_x \left( x - \frac{l}{2} \right)}{n \left( \frac{1}{2}l - \frac{\sum x^2}{\sum x} \right)}, \quad . \quad . \quad (129)$$

where for each load

$$\begin{aligned} m_x &= R_1x - P(x - a) \quad . \quad . \quad x > a \\ &= \frac{P(l - a)}{l}x - P(x - a) \quad . \quad . \quad x > a. \end{aligned}$$

TABLE B.  
COMPUTATION OF  $H$  FOR UNIT LOADS.

| Point. | 1              | 2         | 3                 | 4          | 5            | 6                             | 7                             | 8                            | 9                 | 10     | 11             |
|--------|----------------|-----------|-------------------|------------|--------------|-------------------------------|-------------------------------|------------------------------|-------------------|--------|----------------|
|        | Values of $y$  | $y - y_a$ | $y(y - y_a)$      | $x$ or $a$ | $x(y - y_a)$ | $\frac{a}{0} \sum (y - y_a)x$ | $\frac{a}{0} \sum y(y - y_a)$ | $\frac{a}{0} \sum (y - y_a)$ | Columns (6) + (8) | $H$    | Load at Point. |
| 0      | .....          | .....     | .....             | .....      | .....        | .....                         | .....                         | .....                        | .....             | .....  | 0 or 0'        |
| 1      | .765           | - 4.550   | - 3.481           | 1.372      | - 6.243      | - 6.243                       | 4.553                         | 6.247                        | 0.004             | 0      | 1 " 1'         |
| 2      | 2.183          | - 3.132   | - 6.828           | 4.173      | - 13.070     | - 19.313                      | 7.685                         | 32.070                       | 12.757            | 0.1125 | 2 " 2'         |
| 3      | 3.455          | - 1.860   | - 6.426           | 7.044      | - 13.102     | - 32.415                      | 9.545                         | 67.235                       | 34.820            | 0.3075 | 3 " 3'         |
| 4      | 4.575          | - 0.740   | - 3.386           | 9.977      | - 7.383      | - 39.798                      | 10.285                        | 102.614                      | 62.816            | 0.5550 | 4 " 4'         |
| 5      | 5.541          | 0.226     | 1.252             | 12.964     | 2.930        | - 36.868                      | 10.059                        | 130.405                      | 93.537            | 0.8265 | 5 " 5'         |
| 6      | 6.350          | 1.035     | 6.572             | 15.991     | 16.551       | - 20.317                      | 9.024                         | 144.303                      | 123.986           | 1.0955 | 6 " 6'         |
| 7      | 7.000          | 1.685     | 11.795            | 19.070     | 32.133       | 11.816                        | 7.339                         | 139.955                      | 151.771           | 1.3410 | 7 " 7'         |
| 8      | 7.489          | 2.174     | 16.281            | 22.172     | 48.202       | 60.018                        | 5.165                         | 114.508                      | 174.536           | 1.5425 | 8 " 8'         |
| 9      | 7.815          | 2.500     | 19.538            | 25.295     | 63.238       | 123.256                       | 2.665                         | 67.411                       | 190.667           | 1.6850 | 9 " 9'         |
| 10     | 7.979          | 2.665     | 21.267            | 28.430     | 75.766       | 199.022                       | 0                             | 0                            | 199.022           | 1.7585 | 10 or 10'      |
|        | 53.152         | - 10.282  | - 20.121          |            |              |                               |                               |                              |                   |        |                |
|        | 2              | + 10.285  | + 76.705          |            |              |                               |                               |                              |                   |        |                |
|        | 106.304        |           | + 56.584          |            |              |                               |                               |                              |                   |        |                |
|        | $\sum y$       |           | 2                 |            |              |                               |                               |                              |                   |        |                |
|        | $y_a = 5.3152$ |           | 113.168           |            |              |                               |                               |                              |                   |        |                |
|        |                |           | $\sum y(y - y_a)$ |            |              |                               |                               |                              |                   |        |                |

Column 9 gives the value of the numerator of (135) for each load. These values divided by 113.168, the denominator of (135), give the values of  $H$  shown in column 10.



In (129) the values of  $H$  and  $y_a$  are known from Table B, columns 1 and 10. The value of  $H y_a$  for each load unity is given in column 16, Table C, page 81.

The value of  $\Sigma m_x$  may be found as follows: The value of  $m_x$  for symmetrical points which are distant from the left and right supports  $x < a$  and  $x' < a$ , respectively, Fig. 21, may be expressed in the forms  $R_1 x$  and  $R_2 x'$  and their sum in the form  $R_1 x + R_2 x' = (R_1 + R_2)x = Px = x$ , since

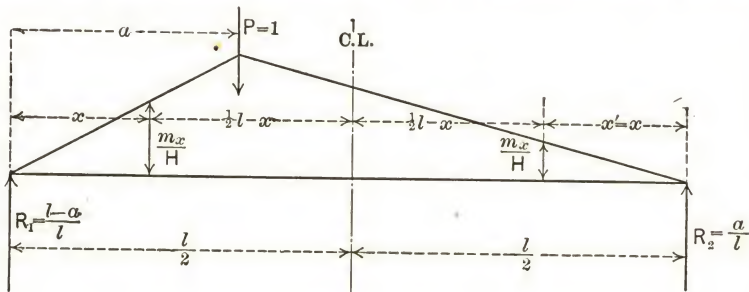


FIG. 21.

$P = \text{unity}$ . The sum of all the symmetrical moments within the limits specified is

$$\sum_{x=0}^{x=a} x.$$

Between the load and the point distant  $a$  from the right support the symmetrical moments are  $R_2(l-x)$  and  $R_2 x'$  and their sum is  $R_2 l$ , since  $x = x'$ .

The total sum of the moments between the limits specified is

$$\sum_{x=a}^{x=l/2} R_2 l = \sum_{x=a}^{x=l/2} \frac{Pa}{l} l = \sum_{x=a}^{x=l/2} Pa = n'a,$$

since  $P = \text{unity}$  and where  $n'$  is the number of divisions between the load and the crown. Therefore,

$$\Sigma m_x = \sum_{x=0}^{x=a} x + n'a = nD'.$$

The computation of  $\Sigma m_x$  is given in detail below.

COMPUTATION OF  $\Sigma m_x = nD'$ .

| 1               | 2          | 3                            | 4    | 5      | 6                               |                      |
|-----------------|------------|------------------------------|------|--------|---------------------------------|----------------------|
| Division Point. | $x$ or $a$ | $x=a$<br>$\Sigma x$<br>$x=0$ | $n'$ | $n'a$  | Col. 5 + Col. 3<br>$\Sigma m_x$ | Load Unity at Point. |
| 1               | 1.372      | 1.372                        | 9    | 12.348 | 13.720                          | 1 or 1'              |
| 2               | 4.173      | 5.545                        | 8    | 33.384 | 38.929                          | 2 " 2'               |
| 3               | 7.044      | 12.589                       | 7    | 49.308 | 61.897                          | 3 " 3'               |
| 4               | 9.977      | 22.566                       | 6    | 59.862 | 82.428                          | 4 " 4'               |
| 5               | 12.964     | 35.530                       | 5    | 64.820 | 100.350                         | 5 " 5'               |
| 6               | 15.991     | 51.521                       | 4    | 63.964 | 115.485                         | 6 " 6'               |
| 7               | 19.070     | 70.591                       | 3    | 57.210 | 127.801                         | 7 " 7'               |
| 8               | 22.172     | 92.763                       | 2    | 44.344 | 137.107                         | 8 " 8'               |
| 9               | 25.295     | 118.058                      | 1    | 25.295 | 143.353                         | 9 " 9'               |
| 10              | 28.430     | 146.488                      | 0    | 0      | 146.488                         | 10 " 10'             |

The values of  $\Sigma m_x$  for points loaded upon the right of the crown are evidently the same as for points symmetrically located upon the left of the crown, that is, the value of  $\Sigma m_x$  for a unit load at point 1' is the same as the value of  $\Sigma m_x$  for a unit load at point 1, etc.

By a similar line of reasoning the values of  $\Sigma m_x \left( x - \frac{l}{2} \right)$  can be found. The value of  $m_x$  upon the left of the crown is  $R_1x$  and for a corresponding point upon the right of the crown the moment is  $R_2x'$ . Multiplying each of these moments by the corresponding  $\left( x - \frac{l}{2} \right)$ , we obtain

$$R_1x \left( x - \frac{l}{2} \right) \quad \text{and} \quad R_2x' \left( l - x' - \frac{l}{2} \right) = R_2x \left( \frac{l}{2} - x \right),$$

adding these values we have

$$-(R_1 - R_2) \left( \frac{l}{2} - x \right) x.$$

Then the sum of all the values of  $m_x \left( x - \frac{l}{2} \right)$  upon the right and left of the crown until  $x=a$  and  $x'=a$  becomes

$$-(R_1 - R_2) \sum_{x=0}^{x=a} x \left( \frac{l}{2} - x \right).$$

For values of  $x > a$  and  $x' > a$  the moments are  $R_2(l-x)$  and  $R_2x'$ . Multiplying each by the corresponding  $\left( x - \frac{l}{2} \right)$ , we have

$$R_2(l-x) \left( x - \frac{l}{2} \right) \quad \text{and} \quad R_2x' \left( l - x' - \frac{l}{2} \right) = R_2x \left( \frac{l}{2} - x \right).$$

Adding these values we obtain

$$R_2(-l+2x) \left( \frac{l}{2} - x \right) = -2R_2 \left( \frac{l}{2} - x \right)^2,$$

and for the sum of all values of  $m_x \left( x - \frac{l}{2} \right)$  between  $x > a$  and  $x < [l - (x' = a)]$  we have

$$-2R_2 \sum_{x=a}^{x=l/2} \left( \frac{l}{2} - x \right).$$

Therefore,

$$\sum m_x \left( x - \frac{l}{2} \right) = -(R_1 - R_2) \sum_{x=0}^{x=a} \left( \frac{l}{2} - x \right) x - 2R_2 \sum_{x=a}^{x=l/2} \left( \frac{l}{2} - x \right).$$

The denominator of the last term in (129) can be replaced by its equivalent, or

$$n \left\{ \frac{l}{2} - \frac{\sum x^2}{\sum x} \right\} = -\frac{4}{l} \sum_{x=0}^{l/2} \left( x - \frac{l}{2} \right).$$

TABLE C.  
COMPUTATION OF  $M_1$ ,  $V_1$ , AND  $y_1$ .

| Point. | 1<br>$R_1$ | 2<br>$R_2$ | 3<br>$x$ | 4<br>$\frac{1}{2}l-x$ | 5<br>$x(\frac{1}{2}l-x)$ | 6<br>$(\frac{1}{2}l-x)^2$ | 7<br>$\frac{a}{0} \sum x(\frac{1}{2}l-x)$ | 8<br>$R_1-R_2$ |
|--------|------------|------------|----------|-----------------------|--------------------------|---------------------------|-------------------------------------------|----------------|
| 1      | .977       | .023       | 1.37     | 28.63                 | 39.223                   | 819.667                   | 39.223                                    | .954           |
| 2      | .931       | .069       | 4.17     | 25.83                 | 107.711                  | 667.189                   | 146.934                                   | .862           |
| 3      | .883       | .117       | 7.04     | 22.96                 | 161.638                  | 527.162                   | 308.572                                   | .766           |
| 4      | .834       | .166       | 9.98     | 20.02                 | 109.800                  | 400.800                   | 508.372                                   | .668           |
| 5      | .784       | .216       | 12.96    | 17.04                 | 220.838                  | 290.362                   | 729.210                                   | .568           |
| 6      | .733       | .267       | 16.00    | 14.00                 | 224.000                  | 196.000                   | 953.210                                   | .466           |
| 7      | .682       | .318       | 19.07    | 10.93                 | 208.435                  | 119.465                   | 1161.645                                  | .364           |
| 8      | .631       | .369       | 22.17    | 7.83                  | 173.591                  | 61.309                    | 1335.236                                  | .262           |
| 9      | .578       | .422       | 25.29    | 4.71                  | 119.116                  | 22.184                    | 1434.352                                  | .156           |
| 10     | .526       | .474       | 28.43    | 1.57                  | 44.935                   | 2.465                     | 1408.087                                  | .052           |

| Point. | 9<br>Column 7<br>times<br>Column 8. | 10<br>$\frac{1}{a} \sum x(\frac{1}{2}l-x)^2$ | 11<br>$2R_2$ | 12<br>Column 10<br>times<br>Column 11. | 13<br>Column 9 plus<br>Column 12,<br>$-\sum m_x(\frac{1}{2}l-x)$ | 14<br>Column 13<br>$\frac{207.1}{E'}$ | 15<br>$\sum m_x \div n$<br>D' | 16<br>$H/a$<br>C' |
|--------|-------------------------------------|----------------------------------------------|--------------|----------------------------------------|------------------------------------------------------------------|---------------------------------------|-------------------------------|-------------------|
| 1      | 37.410                              | 2286.936                                     | .046         | 105.199                                | 142.618                                                          | .688                                  | .686                          | 0                 |
| 2      | 126.657                             | 1619.747                                     | .138         | 223.525                                | 350.182                                                          | 1.991                                 | 1.946                         | .598              |
| 3      | 236.366                             | 1092.585                                     | .234         | 255.055                                | 492.031                                                          | 2.376                                 | 3.094                         | 1.634             |
| 4      | 339.592                             | 691.785                                      | .332         | 229.673                                | 569.265                                                          | 2.749                                 | 4.121                         | 2.950             |
| 5      | 414.191                             | 401.423                                      | .432         | 173.415                                | 587.606                                                          | 2.837                                 | 5.017                         | 4.393             |
| 6      | 444.196                             | 205.423                                      | .534         | 109.690                                | 553.802                                                          | 2.674                                 | 5.774                         | 5.823             |
| 7      | 422.839                             | 85.958                                       | .636         | 54.669                                 | 477.508                                                          | 2.305                                 | 6.390                         | 7.127             |
| 8      | 349.832                             | 24.649                                       | .738         | 18.101                                 | 368.023                                                          | 1.777                                 | 6.855                         | 8.198             |
| 9      | 226.879                             | 2.465                                        | .844         | 2.080                                  | 228.959                                                          | 1.105                                 | 7.166                         | 8.956             |
| 10     | 77.947                              | .....                                        | .948         | .....                                  | 77.947                                                           | 0.376                                 | 7.324                         | 9.346             |



Equation (129) now becomes, for unit load,

$$\left. \begin{matrix} M_1 \\ M_2 \end{matrix} \right\} = H_1 y_a - \frac{\sum_0^a x + n'a}{n} \mp \frac{(R_1 - R_2) \sum_0^a \left( \frac{l}{2} - x \right) x + 2R_2 \sum_a^{l/2} \left( \frac{l}{2} - x \right)^2}{\frac{4}{l} \sum_0^{l/2} \left( x - \frac{l}{2} \right)^2}$$

or

$$\left. \begin{matrix} M_1 \\ M_2 \end{matrix} \right\} = \mathbf{C}' - \mathbf{D}' \mp \mathbf{E}' \dots \dots \dots (136)$$

This equation is in shape for quick and easy substitutions. The work in detail is given in Table C, page 81. If the axial stress is neglected use  $H$  for  $H_1$ .

TABLE C.  
COMPUTATION OF  $M_1$ ,  $V_1$ , AND  $y_1$ —(Concluded).

|                               | 17     | 18     | 19     | 20    | 21    | 22    | 23      | 24    |
|-------------------------------|--------|--------|--------|-------|-------|-------|---------|-------|
| Load<br>Unity<br>at<br>Point. | $M_1$  | $M_2$  | $M_1$  | $M_2$ | $V_1$ | $V_2$ | $y_1$   | $y_0$ |
| 1                             | -1.374 | + .002 | -1.374 | .003  | 1.000 | 0     |         |       |
| 2                             | -3.039 | + .343 | -3.037 | .345  | .987  | .013  | -26.996 | 9.588 |
| 3                             | -3.836 | + .916 | -3.834 | .918  | .962  | .038  |         |       |
| 4                             | -3.920 | +1.578 | -3.916 | 1.582 | .926  | .074  | -7.056  | 9.595 |
| 5                             | -3.461 | +2.213 | -3.456 | 2.218 | .879  | .121  |         |       |
| 6                             | -2.625 | +2.723 | -2.619 | 2.730 | .822  | .178  | -2.391  | 9.614 |
| 7                             | -1.568 | +3.042 | -1.562 | 3.049 | .759  | .241  |         |       |
| 8                             | -.428  | +3.114 | -.427  | 3.127 | .690  | .310  | -.277   | 9.639 |
| 9                             | +.685  | +2.895 | +.692  | 2.903 | .615  | .385  |         |       |
| 10                            | +1.646 | +2.398 | +1.653 | 2.405 | .539  | .461  | +.940   | 9.654 |

\* This value of  $M_1$  should be 1.37, the value of  $x$  for point 1.

$$\left. \begin{matrix} M_1 \\ M_2 \end{matrix} \right\} = \mathbf{C}' - \mathbf{D}' \mp \mathbf{E}' \quad V_1 = \frac{M_2 - M_1}{l} + R_1 \quad y_1 = \frac{M_1}{H_1} \quad y_0 = y_1 + \frac{V_1 a}{H_1}$$

For loads upon the right of the crown interchange  $M_1$  and  $M_2$ , and  $V_1$  and  $V_2$ .

Columns (19) and (20) contain the values of  $M_1$  and  $M_2$  given by the method employed in the first edition of this book, and, as there is so little difference in the values given in columns (17) and (18) and columns (19) and (20) the latter values will be used in the following pages.

For mathematical accuracy the value of  $V_1$  for a load at point 1 should be unity as given in Table C, which evidently is not the actual condition. When the first point is quite near the support, however, the value of  $V_1$  approaches unity very nearly.

In col. 23 the value of  $y_1$  for point 1 is not given, since it is not possible to obtain its value directly from the formula  $M_1 \div H_1 = y_1$ , as  $H_1$  is zero. The same is true for point 1'. This will be the condition whenever graphical or algebraic summation methods are used. This difficulty does not occur in integration formulas. Fortunately, this peculiarity of the summation methods is of no practical importance if  $\delta s$  is not assumed too great. The defect is quite marked where ribs have a much greater depth at the springing than at the crown, and  $\delta s$  is so taken that everywhere  $\delta s \div I$  is constant.

**52. Depth of Ring and the Dead Load.**—An examination of Table II\* shows that a number of railway bridges have been constructed with spans of about 60 feet with arch rings 3 feet deep. Let this be assumed as the depth of the ring.

The load at point 10 can be found as follows: Divide the vertical projection of the arch as shown in Fig. 22, and carefully scale the distances  $ab$ ,  $bc$ , and  $de$ . Then the weight of the ring at point 10 is  $[(bc)(de) = (3.00)(3.14)]170 = 1601$  pounds,† taking granite at 170 lbs. per cubic foot. Assume the fill to be made of material weighing 95 lbs. per cubic foot, then the weight at 10 is  $(3.02)(3.14)95 = 905$  lbs., say. The weight from the track is  $(3.14)30 = 94$  lbs.

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\* Appendix A.

† It will be more accurate to take  $(h)(\delta s)$  170 as the weight of the ring at each point.

The total dead load at 10 now is  $1601 + 905 + 94 = 2600$  pounds. In order that the equilibrium polygon shall pass

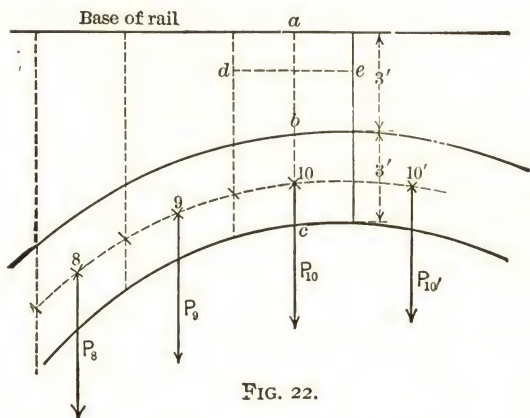
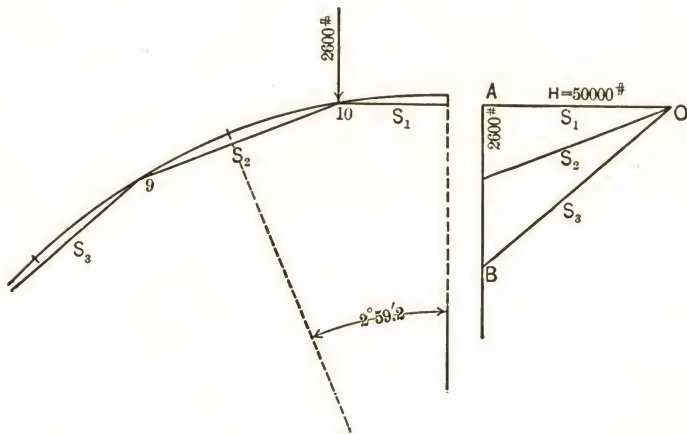


FIG. 22.

through 10 and 9, Fig. 23, the pole distance must be

$$H = \frac{2600}{\tan 2^\circ 59'.2} = \frac{2600}{0.052} = 50000.$$



For the polygon to pass through point 8, load 9 must equal  $[50000 \tan 2(2^\circ 59'.2) = 50000(0.1046)] - 2600 = 5250$

— 2600 = 2650 pounds. In like manner all loads may be computed, or obtained by drawing strings parallel to the chords connecting the points of division as indicated in Fig. 23.

COMPUTATION OF DEAD LOAD.

| Point. | $\phi$ .    | $\tan \phi$ . | 50000 $\tan \phi$ . | Dead Load $P$ . | Unit $H$ , Table B. | Computed $H$ . |
|--------|-------------|---------------|---------------------|-----------------|---------------------|----------------|
| 1      | 29° 51'.760 | 0.574         | 28700               | 3350            | 0                   | 0              |
| 2      | 26 52 .584  | 0.507         | 25350               | 3200            | 0.113               | 362            |
| 3      | 23 53 .408  | 0.443         | 22150               | 3050            | 0.308               | 939            |
| 4      | 20 54 .232  | 0.382         | 19100               | 2950            | 0.555               | 1637           |
| 5      | 17 55 .056  | 0.323         | 16150               | 2800            | 0.827               | 2316           |
| 6      | 14 55 .880  | 0.267         | 13350               | 2800            | 1.096               | 3069           |
| 7      | 11 56 .704  | 0.211         | 10550               | 2700            | 1.341               | 3621           |
| 8      | 8 57 .528   | 0.157         | 7850                | 2600            | 1.543               | 4012           |
| 9      | 5 58 .352   | 0.105         | 5250                | 2650            | 1.685               | 4465           |
| 10     | 2 59 .176   | 0.052         | 2600                | 2600            | 1.759               | 4573           |
|        |             |               |                     | 28700           |                     | 24994          |

The above table gives the computations necessary for obtaining the proper dead loads and also the corresponding values of  $H$ . The value of  $H$  for the entire dead load is in round figures  $2(25000) = 50000$ .

The next step will be the separating of the above dead loads into parts, the ring, filling, and track. The ring and track are fixed, so that their combined weight taken from the total will leave the weight of fill required. The tabular statement on page 86 shows the process in detail.

No great degree of accuracy has been attempted in this table, as a hard rain may change the weight of the fill a considerable amount. The last column gives the average weight per cubic foot of the fill which is necessary to just fulfill the requirement that the equilibrium polygon coincides with the arch axis. It will be noticed that the weight of the arch ring is very nearly uniform for each section.



The lack of uniformity in the variation of the values given is due to inaccuracies of scaling *ab*, *bc*, and *de* from a drawing.

## FINAL DEAD LOADS.

| Point. | Fig. 22.   |            |            | 170 lbs.<br>per cu.<br>ft.<br>Ring. | 30 lbs.<br>per sq.<br>ft.<br>Track. | Ring<br>and<br>Track. | Fill. | Area<br>of Fill. | Average<br>Weight of<br>Fill per<br>Cubic<br>Foot. |
|--------|------------|------------|------------|-------------------------------------|-------------------------------------|-----------------------|-------|------------------|----------------------------------------------------|
|        | <i>ab.</i> | <i>bc.</i> | <i>de.</i> |                                     |                                     |                       |       |                  |                                                    |
| 1      | 10.10      | 3.40       | 2.77       | 1601                                | 83                                  | 1684                  | 1666  | 28.5             | 59                                                 |
| 2      | 8.72       | 3.28       | 2.83       | 1578                                | 85                                  | 1663                  | 1537  | 24.7             | 62                                                 |
| 3      | 7.45       | 3.25       | 2.91       | 1608                                | 87                                  | 1695                  | 1355  | 21.7             | 63                                                 |
| 4      | 6.36       | 3.16       | 2.96       | 1590                                | 89                                  | 1679                  | 1271  | 18.8             | 68                                                 |
| 5      | 5.42       | 3.13       | 3.01       | 1602                                | 90                                  | 1692                  | 1108  | 16.3             | 68                                                 |
| 6      | 4.62       | 3.08       | 3.05       | 1597                                | 92                                  | 1689                  | 1111  | 14.1             | 79                                                 |
| 7      | 4.00       | 3.05       | 3.09       | 1602                                | 93                                  | 1695                  | 1005  | 12.4             | 81                                                 |
| 8      | 3.52       | 3.00       | 3.11       | 1586                                | 93                                  | 1679                  | 921   | 10.9             | 85                                                 |
| 9      | 3.20       | 3.00       | 3.13       | 1596                                | 94                                  | 1690                  | 960   | 10.0             | 96                                                 |
| 10     | 3.02       | 3.00       | 3.14       | 1601                                | 94                                  | 1695                  | 905   | 9.5              | 95                                                 |

**53. Live Load and Loads Producing Maximum Moments.\*—**

The live load is 400 lbs. per linear foot and hence the load at each point is obtained by multiplying *de*, Fig. 22, by 400. These products are given in Table D.

In order to select the loads which produce maximum moments draw the equilibrium polygons for a load unity at each point respectively, as shown on Plate I. One-half of the polygons are shown. These reversed will be the polygons for loads upon the right of the crown.

By inspection we see that loads 1-8 inclusive produce negative moments at the left support, and the remaining loads produce positive moments.

At the crown loads 1-7 and 7'-1' inclusive produce negative moments, and loads 8-8' inclusive positive moments.

For point 6', between  $\frac{1}{3}$  and  $\frac{1}{4}$  point of the span, loads

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\* See Art. 20.

TABLE D.  
LIVE LOADS.

400 POUNDS PER FOOT.

| Pt. | Load. | $\sum$<br>Loads. | H.     | $\sum$ H. | $M_1$ . | $\sum M_1$ . | $V_1$ . | $\sum V_1$ . |
|-----|-------|------------------|--------|-----------|---------|--------------|---------|--------------|
| 0   |       |                  |        |           |         |              |         |              |
| 1   | 1108  | 1108             | 00.0   | 00.0      | -1522.4 | -1522.4      | 1108.0  | 1108.0       |
| 2   | 1132  | 2240             | 127.4  | 127.4     | -3437.9 | -4960.3      | 1117.3  | 2225.3       |
| 3   | 1164  | 3404             | 357.9  | 485.3     | -4462.8 | -9423.1      | 1119.8  | 3345.1       |
| 4   | 1184  | 4588             | 657.1  | 1142.4    | -4636.6 | -14059.7     | 1096.4  | 4441.5       |
| 5   | 1204  | 5792             | 995.1  | 2137.5    | -4161.0 | -18220.7     | 1058.3  | 5499.8       |
| 6   | 1220  | 7012             | 1336.5 | 3474.0    | -3195.2 | -21415.9     | 1002.8  | 6502.6       |
| 7   | 1236  | 8248             | 1657.5 | 5131.5    | -1930.6 | -23346.5     | 938.1   | 7440.7       |
| 8   | 1244  | 9492             | 1918.9 | 7050.4    | -531.2  | -23877.7     | 858.4   | 8299.1       |
| 9   | 1252  | 10744            | 2109.6 | 9160.0    | 866.4   | -23011.3     | 770.0   | 9069.1       |
| 10  | 1256  | 12000            | 2208.7 | 11368.7   | 2076.2  | -20935.1     | 677.0   | 9746.1       |
| 10' | 1256  | 13256            | 2208.7 | 13577.4   | 3020.7  | -17914.4     | 579.0   | 10325.1      |
| 9'  | 1252  | 14508            | 2109.6 | 15687.0   | 3634.6  | -14279.8     | 482.0   | 10807.1      |
| 8'  | 1244  | 15752            | 1918.9 | 17605.9   | 3800.0  | -10389.8     | 385.6   | 11192.7      |
| 7'  | 1236  | 16988            | 1657.5 | 19263.4   | 3768.6  | -6621.2      | 297.9   | 11490.6      |
| 6'  | 1220  | 18208            | 1336.5 | 20599.9   | 3330.6  | -3290.6      | 217.2   | 11707.8      |
| 5'  | 1204  | 19412            | 995.1  | 21595.0   | 2670.6  | -620.0       | 145.7   | 11853.5      |
| 4'  | 1184  | 20596            | 657.1  | 22252.1   | 1873.1  | 1253.1       | 87.6    | 11941.1      |
| 3'  | 1164  | 21760            | 357.9  | 22610.0   | 1068.6  | 2321.7       | 44.2    | 11985.3      |
| 2'  | 1132  | 22892            | 127.4  | 22737.4   | 390.5   | 2712.2       | 14.7    | 12000.0      |
| 1'  | 1108  | 24000            | 00.0   | 22737.4   | 3.4     | 2715.6       | 0.0     | 12000.0      |
| 0'  |       |                  |        |           |         |              |         |              |

1-8' produce negative moments, and loads 7'-1' positive moments.

If the ring is safe at these three points it will be safe at all other points.

54.  $M_1$ ,  $V_1$ , and  $H$  for Live Loads.—These values are obtained by multiplying the values for a load unity given in Tables B and C by the live load. The results are given in Table D. For convenience these values are summed from 0 to  $x$ , as shown.

55. Maximum Moments at Point 0 Produced by the Live Load.—For loads 1-8 inclusive

$$M_1 = -23878 \text{ (Table D).}$$

For loads 9-1' inclusive

$$M_1 = 2716 - (-23878) = +26594.$$

For a full load

$$M_1 = -23878 + 26594 = +2716.$$

For a load up to the crown

$$M_1 = -20935 \text{ (Table D).}$$

For load 10'-1' inclusive

$$M_1 = +23651 \text{ (Table D).}$$

Evidently loading one half the span does not produce maximum moments at point o. The difference between the moment for a load extending from one support up to the crown and the maximum moments will not make any serious difference in the fiber stresses, as the dead load contributes a large portion of these stresses. If temperature effects are considered, the live-load effect becomes almost insignificant.

**56. Maximum Moments at the Crown Produced by the Live Load.**—For loads 1-7 inclusive

$$M_1 = -23347 \text{ (Table D),}$$

$$V_1 = 7441 \quad \text{“} \quad \text{“}$$

$$H = 5132 \quad \text{“} \quad \text{“}$$

$$M_x = M_1 + V_1x - Hy - \sum^x P(x-a). \quad x=30 \text{ and } y=8.$$

$\sum^x P(x-a)$  can be found graphically by means of the ordinary equilibrium polygon. In this instance we will compute its value as shown in the following table.

$$\sum^x P(x-a).$$

| Point | P.      | $\frac{1}{2}l-a.$ | $P(x-a).$                 |
|-------|---------|-------------------|---------------------------|
| 1     | 1108    | 28.63             | 31722                     |
| 2     | 1132    | 25.83             | 29239                     |
| 3     | 1164    | 22.96             | 26725                     |
| 4     | 1184    | 20.02             | 23703                     |
| 5     | 1204    | 17.04             | 20516                     |
| 6     | 1220    | 14.00             | 17080                     |
| 7     | 1236    | 10.93             | 13509                     |
|       | Table D | Table C           | 162494<br>$\sum^x P(x-a)$ |

$$V_1x = 7441(30) = 223230,$$

$$Hy = 5132(8) = 41056,$$

$$\sum^x P(x-a) = 162494;$$

$$M_x = -23347 + 223230 - 41056 - 162494 = -3667.$$

If this is the moment for loads 1-7, then for loads 1-7 and 7'-1' inclusive,  $M_x = 2(-3667) = -7334$ .

If our coefficients are absolutely correct, the moment for loads 7'-1' inclusive should be the same as for loads 1-7 inclusive, as assumed. For loads 7'-1' inclusive

$$M_x = M_1 + V_1x - H_1y,$$

$$M_1 = +13105 \text{ (Table D),}$$

$$V_1x = 807(30) = 24210,$$

$$Hy = 5132(8) = 41056,$$

and

$$M_x = +13105 + 24210 - 41056 = -3741.$$

This is  $3741 - 3667 = 74$  larger than obtained by above method, an error of about 2%.



**57. Moment at the Crown Produced by Live Loads 1-10 Inclusive.**—From Table D,

$$M_2 = +23651, \quad V_2x' = 2254(30) = 67620,$$

$$Hy = 11369(8) = 90952.$$

$$\therefore M_x = +23651 + 67620 + 90952 = +319.$$

For a load over all,

$$M_x = 2(319) = +638.$$

**58. Moment at the Crown Produced by Loads 8-8' Inclusive.**  
—We will first compute the moment for loads 10', 9', and 8' by the formula

$$M_x = M_1 + V_1x - Hy.$$

From Table D,

$$M_1 = +10545, \quad V_1x = 1447(30) = 43410,$$

$$Hy = 6237(8) = 49896.$$

$$\therefore M_x = +10545 + 43410 - 49896 = +4059.$$

Check:

From Art. 56,  $M_x = -3741$  for loads 1-7 inclusive.

$$\text{“ “ 57, } M_x = + 319 \text{ “ “ 1-10 “}$$

$$\therefore M_x = +4060 \text{ “ “ 8-10 “}$$

or practically the same as found above.

The above computations show that the moment at the crown produced by a load covering the half-span is hardly *one tenth* the maximum moment.

59. Maximum Moment at Point 6' Produced by Live Loads 1-8' Inclusive.—Use the formula

$$M_x = M_2 + V_2x' - Hy.$$

From Table D,

$$M_2 = +2715 - (-23347) = +26062,$$

$$V_2x' = 4559(16) = 72944,$$

$$Hy = 17606(6.35) = 111798.$$

$$\therefore M_x = +26062 + 72944 - 111798 = -12792.$$

60. Maximum Moment at Point 6' Produced by Live Loads 7'-1' Inclusive.—From Table D,

$$M_1 = +13105, \quad V_1x = 807(44) = 35508,$$

$$Hy = 5132(6.35) = 32588,$$

$$\sum^x P(x-a) = 1236(3.07) = 3795,$$

$$M_x = M_1 + V_1x - Hy - \sum^x P(x-a)$$

$$= +13105 + 35508 - 32588 - 3795 = +12230.$$

61. Moment at Point 6' Produced by Live Loads 1-10 Inclusive.

$$M_x = M_2 + V_2x' - Hy.$$

From Table D,

$$M_2 = +23651, \quad V_2x' = 2254(16) = 36064,$$

$$Hy = 11369(6.35) = 72193.$$

$$\therefore M_x = +23651 + 36064 - 72193 = -12478,$$

which is about  $2\frac{1}{2}\%$  less than the maximum moment found in Art. 59.

62. Moments at All Points Produced by Live Loads 1-8' Inclusive Determined Graphically.—The constructions are given on Plate II. Lay off a load line in the usual way and scale off  $V_1$  downward. Horizontally opposite the point found, at a distance  $H$ , take a pole and draw the strings  $S_1, S_2, S_3$ , etc. At the left support lay off the distance  $AA''$  equal to  $y_1$  and at the right support the distance  $BB''$  equal to  $y_2$ . Beginning at  $A''$  construct the equilibrium polygon in the usual manner and if the work is correct the last string of the polygon will pass through  $B''$ .

The moment at any point in the arch axis equals the ordinate in the shaded area at the point multiplied by  $H$ .

TABLE E.  
LIVE LOADS, 1-8' INCLUSIVE.

| Point. | $A'B'$ .<br>See Plate II. | $A'C'$ . | $\frac{M_z}{H} =$<br>$A'C' - A'B'$ . | $M_z$ . |
|--------|---------------------------|----------|--------------------------------------|---------|
| 0      | -5.905                    | -5.315   | .590                                 | -10387  |
| 1      | -5.09                     | -4.550   | .54                                  | -9506   |
| 2      | -3.48                     | -3.132   | .348                                 | -6127   |
| 3      | -2.01                     | -1.860   | .150                                 | -2641   |
| 4      | -.74                      | -.740    | .000                                 | -0000   |
| 5      | .40                       | .226     | -.174                                | +3066   |
| 6      | 1.32                      | 1.035    | -.285                                | +5017   |
| 7      | 2.06                      | 1.685    | -.375                                | +6602   |
| 8      | 2.60                      | 2.174    | -.426                                | +7509   |
| 9      | 2.88                      | 2.500    | -.380                                | +6690   |
| 10     | 2.96                      | 2.665    | -.295                                | +5194   |
| 10'    | 2.80                      | 2.665    | -.135                                | +2377   |
| 9'     | 2.43                      | 2.500    | .070                                 | -1232   |
| 8'     | 1.83                      | 2.174    | .344                                 | -6056   |
| 7'     | 1.05                      | 1.685    | .635                                 | -11180  |
| 6'     | .27                       | 1.035    | .705                                 | -13468  |
| 5'     | -.50                      | .226     | .726                                 | -12781  |
| 4'     | -1.28                     | -.740    | .540                                 | -9506   |
| 3'     | -2.05                     | -1.860   | .190                                 | -3345   |
| 2'     | -2.78                     | -3.132   | -.352                                | +6198   |
| 1'     | -3.52                     | -4.550   | -1.030                               | +18134  |
| 0'     | -3.834                    | -5.315   | -1.481                               | +26075  |

The ordinates and corresponding values of  $M_x$  are given in Table E.

The line  $CD$  is a horizontal line drawn at a distance above  $A''B''$  at the center of the span equal to the sum of the ordinates  $C'B'$  divided by their number or,  $\Sigma m_x \div n$ . Using this line for reference the ordinates may be scaled to the equilibrium polygon and to the arch axis and the difference will be the shaded ordinates. These values are given in Table E.

The point of maximum moment is at 6', where  $M_x = -13468$ . From Art. 59, by computation,  $M_x = -12792$ , showing a difference of 676 or an error of about 5 per cent, corresponding to an ordinate of 0.033 feet. The scale employed was 3 feet to the inch, hence 0.033 feet corresponds to 0.011 of an inch on the drawing. This shows that the greatest care must be employed when graphical methods are applied and all possible checks applied.

**63. Maximum Moment at Point 6 Produced by Loads 7'-1' Inclusive. Graphical Determinations.**—Plate II shows the construction, and Table F the computation of  $M_x$  in detail. Here again there is a difference in the results obtained by the two methods. From Art. 60,  $M_x = +12230$ , while by graphics  $M_x = +11520$ , a difference of 710, or about 6%.

**64. Fiber Stresses Produced by Dead and Live Loads.**—From Art. 31,

$$p = \frac{N_x}{F} \pm \frac{M_x z_1}{I}.$$

For this problem,  $N_x = (V_1 - \Sigma P) \sin \phi + H \cos \phi$ .

$$F = 3 \text{ sq. ft.}, \quad z_1 = 1.5 \text{ ft.}, \quad I = \frac{1}{12} b h^3 = \frac{3^3}{12} = \frac{9}{4}.$$



TABLE F.  
LIVE LOADS, 7'-1' INCLUSIVE.

| Point. | $A'B'$ .<br>See Plate II. | $A'C'$ | $\frac{M_x}{H_1} =$<br>$A'C' - A'B'$ . | $M_x$ . |
|--------|---------------------------|--------|----------------------------------------|---------|
| o      | -2.761                    | -5.315 | -2.554                                 | +13105  |
| 1      | -2.59                     | -4.550 | -1.960                                 | +10058  |
| 2      | -2.16                     | -3.132 | -.972                                  | +4987   |
| 3      | -1.71                     | -1.860 | -.150                                  | +770    |
| 4      | -1.26                     | -.740  | .520                                   | -2668   |
| 5      | -.80                      | .226   | 1.026                                  | -5262   |
| 6      | -.33                      | 1.035  | 1.365                                  | -7009   |
| 7      | .15                       | 1.685  | 1.535                                  | -7882   |
| 8      | .60                       | 2.174  | 1.574                                  | -8077   |
| 9      | 1.10                      | 2.500  | 1.400                                  | -7184   |
| 10     | 1.55                      | 2.665  | 1.115                                  | -5722   |
| 10'    | 2.05                      | 2.665  | .615                                   | -3158   |
| 9'     | 2.53                      | 2.500  | -.030                                  | +154    |
| 8'     | 3.00                      | 2.174  | -.826                                  | +4238   |
| 7'     | 3.50                      | 1.685  | -1.815                                 | +9318   |
| 6'     | 3.28                      | 1.035  | -2.245                                 | +11520  |
| 5'     | 2.30                      | .226   | -2.074                                 | +10643  |
| 4'     | .65                       | -.740  | -1.390                                 | +7133   |
| 3'     | -1.62                     | -1.860 | -.240                                  | +1231   |
| 2'     | -4.45                     | -3.132 | 1.318                                  | -6763   |
| 1'     | -7.90                     | -4.550 | 3.350                                  | -17188  |
| o'     | -9.864                    | -5.135 | 4.549                                  | -23341  |

Then

$$\frac{z_1}{I} = \frac{1.5 \times 4}{9} = \frac{2}{3}$$

and

$$p = \frac{1}{3}N_x \pm \frac{2}{3}M_x.$$

Point o.

Dead Load.

From Art. 52,  $\frac{1}{2}$  the total load =  $V_1 = 28700$ , say 29000, and  $H = 50000$ ;  $M_x = M_1 = 0$ ; then

$$N_x = 29000(0.498) + 50000(0.867) = 14442 + 43350 = 57792.$$

$\therefore p = \frac{1}{3}(57792) + \frac{2}{3}(0) = 19264$ , say 19300 comp. for both the upper and lower extreme fibers.

*Live Loads.*

From Art. 55 . . .  $M_x = M_1 = -23878$  for loads 1-8 incl.

“ “ “ . . .  $M_x = M_1 = +26594$  for loads 9-1' “

$N_x = 8299(0.498) + 7050(0.867) = 10245$  for loads 1-8 “

$N_x = 3701(0.498) + 15687(0.867) = 15443$  for loads 9-1' “

Then

$p = \frac{1}{3}(10245) - \frac{2}{3}(23878) = 3415 - 15919 = -12500$  tension in  
upper fiber and  $3415 + 15919 = +19300$  compression in  
the lower fiber for loads 1-8 inclusive.

For loads 9-1' inclusive

$p = \frac{1}{3}(15443) + \frac{2}{3}(26594) = 5143 + 17729 = 22900$  compression  
in the upper fiber and

$p = 5143 - 17729 = 12600$  tension in the lower fiber.

*Combined Stresses.*

Combining the above results we have for the maximum fiber stresses produced by the dead and live loads the following:

| Load.                     | Upper Fiber.      | Lower Fiber.      |
|---------------------------|-------------------|-------------------|
| Dead Load . . . . .       | 19300 compression | 19300 compression |
| L.L. 1-8 . . . . .        | 12500 tension     | 19300 “           |
| L.L. 9-1' . . . . .       | 22900 compression | 12600 tension     |
| Maximum compression . .   | 42200             | 38600             |
| Maximum tension . . . . . | 0                 | 0                 |

These intensities are pounds per square foot.

For pounds per square inch we have, 293 and 268 as the maximum compression in the upper and lower fibers respectively.

Considering that granite has an ultimate crushing

strength of from 13000 to 17000 pounds per square inch, the above fiber stresses are of little consequence if the mortar joints have an equal strength, or even one fourth the strength of the granite. The fiber stresses at other points are obtained in the manner followed for point o. A tabulated statement for points o, 6', and the crown is given below:

FIBER STRESSES.

| Load.                 | $N_z$ . | $M_z$ . | $\frac{1}{2}N_z$ . | $\frac{1}{2}M_z$ . | $p$ .  |        | Point. |
|-----------------------|---------|---------|--------------------|--------------------|--------|--------|--------|
|                       |         |         |                    |                    | Upper. | Lower. |        |
| Dead load.....        | 57792   | o       | 19300              | o                  | +19300 | +19300 | o      |
| L.L. 1-8.....         | 10245   | -23878  | 3415               | -15919             | -12500 | 19300  | o      |
| L.L. 9-1'.....        | 15443   | +26594  | 5148               | +17729             | +22900 | -12600 | o      |
| Max. compression..... |         |         |                    |                    | 42200  | 38600  | o      |
| Max. tension.....     |         |         |                    |                    | o      | o      | o      |
| Dead load.....        | 50000   | o       | 16666              | o                  | +16666 | +16666 | Crown  |
| L.L. 1-7 and 7'-1'... | 10264   | -7482   | 3421               | -4988              | -1567  | +8409  | "      |
| L.L. 8-8'.....        | 12474   | +8118   | 4158               | +5412              | +9570  | -1254  | "      |
| Max. compression..... |         |         |                    |                    | 26200  | 25100  | "      |
| Max. tension.....     |         |         |                    |                    | o      | o      | "      |
| Dead load.....        | 50500   | o       | 16833              | o                  | +16833 | +16833 | 6'     |
| L.L. 1-8'.....        | 18170   | -12792  | 6057               | -8528              | -2471  | +14585 | 6'     |
| L.L. 7'-o'.....       | 5093    | +12230  | 1698               | +8154              | +9852  | -6546  | 6'     |
| Max. compression..... |         |         |                    |                    | 26700  | 31400  | 6'     |
| Max. tension.....     |         |         |                    |                    | o      | o      | 6'     |

The + sign indicates compression.

In this table all stresses are given in pounds per *square foot*.

From the above table we see that there is no tension at the three points considered, and that the maximum compression is well within the safe strength of the material assumed. Also, that the greatest fiber stress is at the supports.

**65. Effect of Temperature Changes.**—Our knowledge of the effect of changes of temperature upon stone arches is

very meager. The coefficients of expansion for different stones are known, but how long it takes for a stone bridge to become of uniform temperature we do not know. Probably all portions of the arch ring are never of the same temperature. The range of the average temperature is probably small. (See Arts. 33 and 34.)

In this case we will assume that the temperature changes  $40^\circ$  above or below the temperature of the arch when built. This is without doubt an excessive range. The horizontal thrust is (Art. 27)

$$H_t = \frac{et^\circ lE}{4\Sigma y(y-y_a)} = \frac{et^\circ lE}{1.4(113.168)},$$

where  $4 = \delta s \div I = 3.14 \div 2.25 = 1.4.$

For Quincy granite, Appendix A, Table I.

$$e = 0.00000381,$$

$$E = 6776000.$$

Then

$$H_t = \frac{(0.00000381)(40)(60)(6776000)(144)}{158.4} = 56100.$$

From Art. 30,

$$M_1 = H_t y_a = 56100(5.315) = 298200,$$

$$M_x = M_1 - H_t y = H_t(y_a - y).$$

The above values of  $H_t$  and  $M_1$  will have signs depending upon whether the change of temperature is an increase or a decrease.

For *falling* temperature the *upper fibers* at the support are in *tension*, and at the *crown* in *compression*.



The following table gives the fiber stresses at the support and the crown.

FIBER STRESSES DUE TO CHANGES OF TEMPERATURE.

POINT O.

| Tempera-<br>ture. | $N_z$ . | $M_z$ . | $\frac{1}{2}N_z$ . | $\frac{1}{2}M_z$ . | $p$ .   |         |
|-------------------|---------|---------|--------------------|--------------------|---------|---------|
|                   |         |         |                    |                    | Upper.  | Lower.  |
| -40°              | -48638  | -298200 | -16213             | -198800            | -215000 | +182000 |
| +40°              | +48638  | 298200  | +16213             | 198800             | +215000 | -182000 |

CROWN.

|      |        |         |        |        |         |         |
|------|--------|---------|--------|--------|---------|---------|
| -40° | -56100 | 149600  | -18700 | 99700  | + 81000 | -118400 |
| +40° | +56100 | -149600 | +18700 | -99700 | - 81000 | +118400 |

Combining the above values with those obtained for the dead load and live load we have

|                          | Upper Fibers. | Lower Fibers. |
|--------------------------|---------------|---------------|
| For point o:             |               |               |
| Maximum compression..... | 257200        | 220600        |
| “ tension.....           | 208200        | 175300        |
| For the crown:           |               |               |
| Maximum compression..... | 107200        | 143500        |
| “ tension.....           | 65900         | 103000        |

The above values correspond to a maximum compression of 1786 pounds per square inch and a maximum tension of 1446 pounds per square inch. In compression the factor of safety is from 8 to 10, but in tension the ultimate strength of the joints is exceeded. As a large number of railway bridges have been built upon practically the dimensions we assumed and no indications of failure having been found, we must conclude that the range of temperature change assumed in this example is very much too great.

Furthermore, it requires a drop in temperature of only *four* degrees to completely balance the compression produced by the dead load in the upper fibers at the support. Without question, then, our assumptions about the effect of temperature changes are not correct. Until we know more about the subject it is useless to make calculations according to the ordinary assumptions. (See Art. 33.)

**66. Effect of the Axial or Direct Stress upon the Fiber Stresses.**—In all of the work above the effect of the axial stress has been neglected. From (128) the horizontal thrust produced by the axial stress is

$$H_a = \frac{-\Sigma \frac{\partial_x \cos \phi}{F_x J}}{\Sigma y(y - y_a) + \Sigma \frac{\partial_x \cos \phi}{F_x J}} \left\{ \begin{matrix} H \\ H_i \end{matrix} \right\} \cdot \cdot \cdot \quad (128)$$

From (129),

$$M_1 = M_2 = H_a y_a,$$

and hence

$$M_x = M_1 - H_a y = H_a (y_a - y).$$

For the axial stress alone,

$$N_x = H_a \cos \phi.$$

Then

$$p = \frac{H_a \cos \phi}{F_x} \pm \frac{H_a (y_a - y) z_1}{I},$$

or, in this case,

$$p = \frac{1}{3} H_a \cos \phi \pm \frac{2}{3} H_a (y_a - y).$$

If the value of  $H_a$  is found in terms of  $H$  or  $H_i$  the fiber stresses can be found and added algebraically to

those already found. In (128) all quantities are known from previous calculations, with the exception of the last term in the denominator. The computation of this term is given in detail below.

| Point. | $\delta x$ . | $\cos \phi$ . | $\delta x \cos \phi$ . |
|--------|--------------|---------------|------------------------|
| 1      | 2.77         | 0.880         | 2.44                   |
| 2      | 2.83         | .903          | 2.56                   |
| 3      | 2.90         | .925          | 2.68                   |
| 4      | 2.96         | .943          | 2.79                   |
| 5      | 3.01         | .959          | 2.89                   |
| 6      | 3.05         | .973          | 2.97                   |
| 7      | 3.08         | .983          | 3.03                   |
| 8      | 3.11         | .992          | 3.09                   |
| 9      | 3.13         | .997          | 3.12                   |
| 10     | 3.14         | .999          | 3.14                   |
|        |              |               | 28.71                  |

$$\Sigma \delta x \cos \phi = 2(28.71) = 57.42, \quad F = 3, \quad A = 1.4, \quad FA = 4.2,$$

$$\frac{\Sigma \delta x \cos \phi}{FA} = \frac{57.42}{4.2} = 13.67, \quad \Sigma y(y - y_a) = 113.168.$$

Then

$$\frac{113.168}{126.84} = 0.892. \quad \therefore H_a = 0.108H_1 = 11\% H_1, \text{ say.}$$

The value of  $H$  for the dead load is 50000; then the corresponding axial stress produces a thrust, opposite in character, of 5500. The horizontal thrust produced by a drop of  $40^\circ$  in temperature is 56100; therefore the effect of the axial stress equals  $\frac{5500}{56100} = .091$  of the stresses due to this drop of temperature. At joint zero the upper fiber stress due to  $-40^\circ$  is 215000 tension.  $215000(0.091) = 19600$  tension.

## FIBER STRESSES DUE TO THE AXIAL STRESS.

|                      | H.    | $H_a$ . | $\frac{H_a}{H_t}$ . | Point o. |        | Crown. |        |
|----------------------|-------|---------|---------------------|----------|--------|--------|--------|
|                      |       |         |                     | Upper.   | Lower. | Upper. | Lower. |
| Dead load. ....      | 50000 | 5500    | 0.091               | -19600   | +16600 | +7400  | -10800 |
| L.L. 1-8. ....       | 7050  | 776     | 0.014               | -3000    | +2500  | +1100  | -1700  |
| " 9-1'. ....         | 15687 | 1726    | 0.031               | -6700    | +5600  | +2500  | -3700  |
| " 1-7 and 7'-1'. ... | 10263 | 1128    | 0.020               | -4300    | +3600  | +1600  | -2400  |
| " 8-8'. ....         | 12474 | 1372    | 0.024               | -5200    | +4400  | +1900  | -2900  |
| " 1-8'. ....         | 17006 | 1936    | 0.034               | -7300    | +6200  | +2800  | -4000  |
| " 7'-1'. ....        | 5132  | 565     | 0.001               | -220     | +182   | +81    | -118   |
| -40°. ....           | 51600 | 5676    | 0.110               | +24000   | -20000 | -8900  | +13000 |

Combining these stresses with the dead- and live-load stresses previously obtained, we have (see Art. 64):

## FINAL STRESSES, INCLUDING EFFECT OF AXIAL STRESS.

| Loads.       |                         | Upper Fibers.         | Maximums.          |
|--------------|-------------------------|-----------------------|--------------------|
| Point o.     | Dead load. ....         | +19300-19600=-300     | Max. comp.=12200   |
|              | L.L. 1-8. ....          | -12500-3000=-15500    | " ten. =18500      |
|              | " 9-1'. ....            | +22900-6700=+15200    |                    |
|              | Lower Fibers.           |                       |                    |
|              | Dead load. ....         | +19300+16600=+35900   | Max. comp.=57700   |
|              | L.L. 1-8. ....          | +19300+2500=+21800    | " ten. 0           |
| Crown.       | " 9-1'. ....            | -12600+5600=-7000     |                    |
|              | Upper Fibers.           |                       |                    |
|              | Dead load. ....         | +16666+7400=+24100    | Max. comp.=35600   |
|              | L.L. 1-7 and 7'-1'. ... | -1567+1600=+33        | " ten. 0           |
|              | " 8-8'. ....            | +9570+1900=+11470     |                    |
|              | Lower Fibers.           |                       |                    |
|              | Dead load. ....         | +16666-10800=+5900    | Max. comp.=11900   |
|              | L.L. 1-7 and 7'-1'. ... | +8409-2400=+6000      | " ten. 0           |
|              | " 8-8'. ....            | -1254-2900=-4200      |                    |
| Temperature. |                         |                       |                    |
| ±40°         |                         | ±215000±24000=±191000 | Upper fibers at o. |
| ±40°         |                         | ±182000±20000=±162000 | Lower " " o.       |
| ±40°         |                         | ±81000±8900=±72100    | Upper " " crown    |
| ±40°         |                         | ±118400±13000=±105400 | Lower " " "        |



These stresses show that the effect of the axial stress is considerable, and also that the fiber stresses at the support are reversed in one case so that the upper fibers are in tension about 128 lbs. per square inch. As this tension is not large and exists for but a short distance, the ring may be considered safe. This assumes that no temperature effects are considered. The maximum compression is 400 lbs. per square inch in the lower fibers at the support.

The effect of the axial stress is to *lower* the equilibrium polygon at the *support* and *raise* it at the *crown*, or it *increases* the *compression* and *decreases* the *tension* in the lower fibers and *decreases* the *compression* and *increases* the *tension* in the upper fibers at the support. While at the crown the reverse is true.

If this arch ring had been assumed free, then the above tension could not have been allowed (see Arts. 31 and 33).

**67. Direct Computation of  $H_1$  and  $M_1$ , Including the Effect of the Axial Stress.**—In case the effect of the axial stress is to be included, it is a simple matter to correct the values of  $H$  and  $M_1$  before computing the fiber stresses.

COMPUTATION OF  $H_1$  AND  $M_1$ .

|    | 1               | 2                             | 3                                    | 4                | 5                | 6      | 7     |
|----|-----------------|-------------------------------|--------------------------------------|------------------|------------------|--------|-------|
|    | $H$<br>Table B. | $H_1$<br>$0.892H$<br>Art. 66. | $H_1 \gamma a$<br>$C'$<br>$5.315H_1$ | $D'$<br>Table C. | $E'$<br>Table C. | $M_1$  | $M_2$ |
| 1  | 0               | 0                             | 0                                    | 0.686            | 0.688            | -1.374 | 0     |
| 2  | 0.1125          | 0.100                         | 0.531                                | 1.946            | 1.691            | -3.106 | 0.276 |
| 3  | 0.3075          | 0.274                         | 1.456                                | 3.094            | 2.376            | -4.014 | 0.738 |
| 4  | 0.5550          | 0.495                         | 2.631                                | 4.121            | 2.749            | -4.239 | 1.259 |
| 5  | 0.8265          | 0.737                         | 3.917                                | 5.017            | 2.837            | -3.937 | 1.737 |
| 6  | 1.0955          | 0.977                         | 5.193                                | 5.774            | 2.674            | -3.255 | 2.093 |
| 7  | 1.3410          | 1.195                         | 6.357                                | 6.390            | 2.305            | -2.338 | 2.272 |
| 8  | 1.5425          | 1.376                         | 7.313                                | 6.855            | 1.777            | -1.319 | 2.235 |
| 9  | 1.6850          | 1.503                         | 7.988                                | 7.166            | 1.105            | -0.283 | 1.927 |
| 10 | 1.7585          | 1.568                         | 8.334                                | 7.324            | 0.376            | +0.634 | 1.386 |

The table on the bottom of page 102 shows the work in detail. The values of  $V_1$  and  $V_2$  remain unchanged.

The fiber stresses can now be computed in the manner explained in the previous articles where the effect of the axial stress is neglected.

**68. Effect of Making Spandrel Filling of Uniform Material Weighing 100 Pounds per Cubic Foot.**

COMPUTATION OF  $H_1$ .

| Point. | Ring and Track. | Fill<br>100 Lbs.<br>per Cu. Ft. | Total<br>Dead<br>Load. | Common<br>$H$ Load<br>Unity. | Common<br>$H$ . | $H_1$ with<br>Effect of<br>Axial Stress. |
|--------|-----------------|---------------------------------|------------------------|------------------------------|-----------------|------------------------------------------|
| 1      | 1684            | 2850                            | 4500                   | 0.0                          | 0               | $54200(1 - 0.108) = 48300$               |
| 2      | 1663            | 2470                            | 4100                   | 0.1125                       | 461             |                                          |
| 3      | 1605            | 2170                            | 3900                   | 3075                         | 1199            |                                          |
| 4      | 1679            | 1880                            | 3600                   | 5550                         | 1998            |                                          |
| 5      | 1602            | 1630                            | 3300                   | 8265                         | 2727            |                                          |
| 6      | 1689            | 1410                            | 3100                   | 1.0955                       | 3396            |                                          |
| 7      | 1695            | 1240                            | 2900                   | 1.3410                       | 3889            |                                          |
| 8      | 1679            | 1090                            | 2800                   | 1.5425                       | 4319            |                                          |
| 9      | 1690            | 1000                            | 2700                   | 1.6850                       | 4550            |                                          |
| 10     | 1695            | 950                             | 2600                   | 1.7585                       | 4572            |                                          |
|        |                 |                                 | 33500<br>2             | Table B                      | 27111<br>2      |                                          |
|        |                 |                                 | 67000                  |                              | 54200           |                                          |

If the values of  $M_1$  from Table C are combined in pairs of values for symmetrical points the value of  $M_1$  is quickly found from the table on page 104.

The fiber stresses will be found considering the effect of the axial stress:

$$H_1 = 48300, \quad M_1 = -40000, \quad V_1 = 33500.$$

POINT 0.

$$N_x = 33500(0.498) + 48300(0.867) = 58600,$$

$$p = \frac{1}{3}(58600) \pm \frac{2}{3}(-40000) = 19500 \mp 26700,$$

COMPUTATION OF  $M_1$ .

|                 | 1                        | 2              | 3                  | 4                                                                                      |
|-----------------|--------------------------|----------------|--------------------|----------------------------------------------------------------------------------------|
| Loads at Points | $M_1$<br>For Unit Loads. | Dead Load.     | $M_1$<br>For D. L. | Remarks.                                                                               |
| 1 and 1'        | - 1.371                  | 4500           | - 6170             | $M_1$ for the dead load equals -8600 when the effect of the axial stress is neglected. |
| 2 " 2'          | - 2.692                  | 4100           | - 11040            |                                                                                        |
| 3 " 3'          | - 2.916                  | 3900           | - 11370            | $H_a = 48300 - 54200 = -5900$                                                          |
| 4 " 4'          | - 2.334                  | 3600           | - 8400             |                                                                                        |
| 5 " 5'          | - 1.238                  | 3300           | - 4080             | $M_{1a} = H_a y_a = -5900(5.315)$                                                      |
| 6 " 6'          | + 0.111                  | 3100           | + 340              |                                                                                        |
| 7 " 7'          | + 1.487                  | 2900           | + 4310             | $= -31400$                                                                             |
| 8 " 8'          | + 2.700                  | 2800           | + 7560             |                                                                                        |
| 9 " 9'          | + 3.595                  | 2700           | + 9700             | Therefore the value of                                                                 |
| 10 " 10'        | + 4.058                  | 2600           | + 10550            |                                                                                        |
|                 | - 11.951                 | 33500<br>$V_1$ | - 41060            | $M_1 = -31400 - 8600$                                                                  |
|                 | + 10.551                 |                | + 32460            |                                                                                        |
|                 | - 1.400                  |                | - 8600<br>$M_1$    | when the effect of the axial stress is considered.                                     |

and the tension in the upper fibers is 7200 and the compression in the lower fibers 46200.

## CROWN.

$$N_x = 48300,$$

$$M_c = M_1 + V_1 \frac{l}{2} - H_1 f - \sum_0^x (x - a)$$

$$= -40000 + 1005000 - 386400 - 566400 = +12200,$$

$$p = \frac{1}{3}(48300) \pm \frac{2}{3}(+12200) = 16100 \pm 8100,$$

and the compression in the upper fibers is 24200 and the compression in the lower 8000.

Comparing these results with those found in Art. 66 it appears that the fiber stresses at the support are quite different. At the crown, the difference in the fiber stresses is slight.

69. The Radial Shear.—From Art. 21,

$$T_x = V_x \cos \phi - H_x \sin \phi.$$

For point zero, or the support, this becomes

$$T_x = V_1(0.867) - H_1(0.498).$$

For dead load (see Art. 52)

$$T_x = 28700(0.867) - 50000(0.498) = 24880 - 24900 = 0.$$

For live load over all (see Table D)

$$T_x = 12000(0.867) - 22737(0.498) = 10400 - 11320 = -920.$$

At the crown  $\phi$  is zero, hence

$$T_x = V_x = V_1 - \sum^x P.$$

For the dead load  $T_x = 0$ .

For a live load 10'-1' inclusive  $T_x = V_1 = 2254$ .

In like manner any other point may be considered. When equilibrium polygons are drawn a glance is sufficient to determine if there is danger of slipping at the joints. Usually the radial shear requires but little attention in *stone arches*.

**70. Second Example. Data.**—For this example we will take a reinforced-concrete rib of the Thatcher type.\* Clear span 50 ft. and rise 10 ft. The thickness at the crown is taken as 12 inches, and at the spring line 4 feet 6 inches. Plate IV gives all data concerning dimension and reinforcement. The dead weight of the entire structure is assumed at 140 pounds per cubic foot, and the live load

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\* Essentially the arch taken by Professor Cain in "Theory of Concrete Arches and of Vaulted Structures."



112 pounds per square foot. The first step in the solution of a problem of this type is to obtain all the data shown in Plate IV either by algebraic or graphical methods. In the present instance many of the data were obtained from a carefully constructed drawing as indicated in the figure. The modulus of elasticity of the concrete is assumed to be  $\frac{1}{20}$  that of steel, and hence the area of the steel is equivalent to twenty times that area in concrete.

**71. Subdivision of the Arch Axis.**—Contrary to the usual custom we will not attempt to so divide the arch ring that  $\delta s \div I$  will be constant, but simply divide the span into twenty equal parts and determine all quantities necessary for points at the centers of these divisions. This is clearly shown in Plate IV.

The moment of inertia at each point is found as shown in Table I, page 118. Prof. Cain in his book referred to above gives a very complete exposition of the method for dividing the axis so that  $\delta s \div I$  shall be constant.

**72. Computation of the Horizontal Thrust Produced by Vertical Unit Loads. The Effect of the Axial Stress Neglected.**  
—From (79)

$$H = \frac{\sum_0^a zB - k \sum_0^a B}{\sum yB} \left( P \frac{\delta x}{2} \right). \quad . \quad . \quad . \quad . \quad (79)$$

Table II shows the work in detail for the determination of the numerator of (79) and, finally, the value of  $H$  for each unit load. Since the loads are vertical and the arch symmetrical, a unit load at point 1 has the same  $H$  as a unit load at point 1', and likewise for any pair of symmetrical points. Therefore, it is necessary to consider

only the points upon the left of the crown. The influence line for  $H$  is shown on Plates III and V.

**73. Computation of the Bending Moments  $M_1$  and  $M_2$  Produced by Vertical Unit Loads. The Effect of the Axial Stress Neglected.**—From Art. 16

$$\left. \begin{matrix} M_1 \\ M_2 \end{matrix} \right\} = \mathbf{C} - \mathbf{D} \mp \mathbf{E}; \quad . . . . . (80)$$

$$\mathbf{C} = H \frac{\Sigma y \Delta}{\Sigma \Delta};$$

$$\mathbf{D} = \left\{ \frac{a}{\Sigma z \Delta} + k \frac{l^{1/2}}{\Sigma \Delta} \right\} \frac{\delta x}{2 \Sigma \Delta} P; \quad . . . . . (81)$$

$$\mathbf{E} = \left[ (n-k) \frac{a}{\Sigma} (z-n) z \Delta - k \frac{l^{1/2}}{\Sigma} (z-n)^2 \Delta \right] \frac{\delta x}{-4 \frac{l^{1/2}}{\Sigma} (z-n)^2 \Delta}. \quad (84)$$

The values of  $\mathbf{C}$  are given in column 1 of Table V. The values of  $H$  used are given in column 8 of Table II, and the value of  $\Sigma y \Delta \div \Sigma \Delta$  is from Table I.

The various steps in computing the terms  $\mathbf{D}$  and  $\mathbf{E}$  are clearly shown in Tables III and IV and the combinations of the terms  $\mathbf{C}$ ,  $\mathbf{D}$ , and  $\mathbf{E}$  are given in Table V. The influence line for  $M_1$  is given in Plate V.

**74. Computation of the Values of  $V_1$  and  $V_2$  for Vertical Unit Loads.**—From (5)

$$V_1 = \frac{M_2 - M_1}{l} + R_1 \quad \text{and} \quad V_2 = P - V_1.$$

In Table VI are given the necessary steps in the derivation of  $V_1$  and  $V_2$ , and also the values of  $M_1$ ,  $M_2$ , and  $H$  for convenience of reference.

**75. The Horizontal Thrusts and the Bending Moments at the Supports Produced by Vertical Unit Loads Placed at All Points.**—As a check on the values of  $H$ ,  $M_1$  and  $M_2$  for individual loads a direct determination of  $H$  and  $M_1$  should be made for a unit load placed at each point. Table VII shows the necessary work to obtain the value of  $H$  and Table VIII that for  $M_1$  and  $M_2$ .

The sum of the values of  $H$  in Table II is 16.772 and the value of  $H$  in Table VII is 16.768. These values should agree when all of the numerical work is correct.

The algebraic sum of the values of  $M_1$  in Table V is +20.354 and the value in Table VIII is +20.348. The agreement is sufficiently close for practical purposes.

The tables just explained probably have considerably more decimal places than is warranted by the data upon which they are based.

**76. The Effect of the Axial Stress.**—As shown in Art. 9, the effect of the axial stress upon the horizontal thrust is a constant percentage for any given arch. This percentage is readily found from (63). The calculations in detail are shown in Table IX, from which it appears that the thrust produced by the axial stress is about 6.65% of the common horizontal thrust, as found in Table II. This is quite small. The effect upon the fiber stresses is not constant and in some cases it is quite large, yet the final maximum stresses are not seriously affected.

**77. Stresses in the Extreme Fibers Produced by a Unit Horizontal Thrust Acting toward the Center of the Span.**—As a matter of convenience in determining the effects of the axial stress and changes in temperature upon the stresses in the extreme fibers of the sections of the arch con-



sidered, the effect of a unit horizontal thrust will first be found. Except in important structures, where each radial section should be considered, it is usually sufficient to determine the maximum fiber stresses for sections 0, 6, and the crown. Table X gives the fiber stresses produced by a unit horizontal thrust at these sections.

**78. The Extreme Fiber Stresses Produced by a Unit Load Moving Across the Span.**—As stated in Art. 20, the best method of obtaining the maximum fiber stresses at any section is to compute the fiber stresses produced by a unit load moving across the span and then drawing an influence diagram from which the stresses, produced by any combination of loading, can be found.

Table XI gives the fiber stresses for section 0, Table XII those for section 6, and Table XIII those for the section at the crown.

In computing the values of  $M_x$  the usual formula (2) has been replaced by the form given in Table XII which is more accurate owing to the fact that a number of the terms are constants.

Plate VI shows the influence diagrams for the stresses in the extreme fibers at sections 0, 6, and the crown. These diagrams show at a glance the fields of loading which produce like stresses.

**79. The Horizontal Thrust and Bending Moments Produced by the Dead Load.**—The dead load is composed of the paving earth fill and the concrete in the arch ring. Strictly the weight of all the material for the full width and clear span of the arch should be computed. This would include spandrel walls, copings, balustrades, etc.; but a longitudinal strip one unit wide through the center of the





$$N_x = V_x \sin \phi + H \cos \phi;$$

$$V_x \sin \phi = (22100 - 17040)(0.226) = 1144$$

$$H \cos \phi = (23100)(0.974) = 22499$$

$$\text{and } N_x = \dots\dots\dots 23643$$

Stress in the upper fiber

$$\begin{aligned} &= \frac{N_x}{F_x} + \frac{M_x}{S} = \frac{23643}{1.5} + (-790)(2.82) \\ &= +15760 - 2128 = +13632 \text{ (compression).} \end{aligned}$$

Stress in the lower fiber

$$= 15760 + 2128 = +17888 \text{ (compression).}$$

The stresses are stated in pounds per square foot. The fiber stresses at any other section are computed in a similar manner.

**81. Extreme Fiber Stresses at Section 6 Produced by the Live Load.**—The live load is assumed at 200 pounds per square foot of roadway, which is equivalent to  $200 \times 2.68 = 536$  pounds per point. From Table XII it is seen that the maximum compression in the upper fiber is produced by loading points 1-8 and 3'-1'. For a unit load at each of these points the fiber stress is +14.5493 and for a load of 536 pounds or units, the stress is  $+14.5493 \times 536 = +7798$ . The maximum tension in the upper fiber is  $-10.8540 \times 536 = -5818$ . In like manner the maximum tension in the lower fiber is -5558 and the maximum compression is +15898.

These stresses with the dead load stresses are given in Table XVI, columns 2 and 3. Combining the stresses it is found that the entire section is in compression, the

maximum stress being 34000. All stresses are stated in pounds per square foot.

**82. The Stresses in the Extreme Fibers at Section 6 Produced by the Dead and Live Loads, Considering the Effect of the Axial Stress.**—The horizontal thrust used in Art. 80, was 23100. From Table IX the horizontal thrust produced by the axial thrust is 6.65% of this, or 1536. This thrust is opposite in character to 23100, hence is used with the negative sign, that is,  $H = -1536$ .

Referring to Table X the stress in the upper fiber is  $(-1536)(1.74) = -2673$  (tension) and in the lower fiber  $(-1536)(-0.444) = +682$  (compression). These stresses are added algebraically to those found when the axial stress was neglected to determine the true fiber stresses.

In a similar manner are considered the horizontal thrusts produced by the live-load axial stresses. Table XVI shows these stresses and the results obtained by combining them with the stresses found by neglecting the axial thrust. The maximum fiber stress is increased about 3%.

**83. The Effect of Temperature Changes upon the Stresses in the Extreme Fibers.**—Section 6. From (62),

$$H_t = \frac{et^{\circ}lE}{\Sigma yB} \quad \dots \quad (62)$$

For  $t = +1^{\circ}$ ,  $e = 0.000006$ , and  $E = 1500000(144)$ ,

$$H_t = \frac{0.000006(53.6)(1500000)(144)}{432.533} = 162.$$

For a rise of  $20^{\circ}$ ,  $H_t = 162(20) = 3240$ .

Referring to Table X the stress in the upper fiber is



$3240(+1.74) = +5638$  (compression) and in the lower fiber the stress is  $3240(-0.444) = -1439$  (tension). In a like manner the stresses produced by a drop of  $40^\circ$  are found.

Table XVI contains the fiber stresses produced by the temperature changes and also the temperature axial stresses.

The final maximum compressive stresses are about the same when the axial stress is considered and when it is neglected. The maximum tensile stress, when the axial stress is considered, is about twice that found when the axis stress is neglected. The magnitude of the stress is small, however.

If the axial stress is to be considered, its effect should be introduced when computing the horizontal thrusts and bending moments produced by unit loads. The true fiber stresses can then be found.

In Table XIX are given the values of  $H$ ,  $M_1$ , etc., including the effect of the axial stress, for all loadings used in computing the fiber stresses at section 6.

**84. The Stresses in the Extreme Fibers at the Crown and Section o.**—It is not necessary to compute these stresses in detail, as the routine is the same for all sections. Table XV gives the fiber stresses for section o and Table XVII the stresses at the crown.

**85. The Maximum Stresses in the Extreme Fibers.**—Inspection of Tables XV, XVI, and XVII shows that the maximum compression is in the upper fibers at the crown, or 68500, and the maximum tension in the lower fibers at the same section, or 34600. The tension is equivalent to about 240 pounds per square inch, which exceeds the ultimate strength of the concrete. This tension is pro-



duced almost entirely by the change in temperature of  $-40^{\circ}$ .

Without considering the effect of temperature changes, the fiber stresses are about 310 pounds per square inch compression and 31 pounds per square inch tension, values well within the safe values; in fact, these values are about one-half the allowable.

Neglecting the effect of the axial stress and including the effect of the temperature changes, the maximum compression is about 435 pounds per square inch and the maximum tension about 115 pounds per square inch. The tensile stress is about one-half the ultimate strength of the concrete.

If the concrete cracks the fiber stresses are indeterminate. The steel may be doubled in amount at the crown without sensibly affecting the computations. This will provide against the cracking of the concrete if the somewhat improbable combination of stresses obtain.

**86. Direct Determination of the Horizontal Thrust Produced by a Symmetrical Loading.**—The horizontal thrust produced by any symmetrical loading can be found very quickly after  $B$  and  $\Sigma yB$  have been computed. The formula used is

$$H = \frac{\Sigma m_x B}{\Sigma y B}, \quad . . . . . (61)$$

where  $m_x$  is the bending moment for the loading on a beam supported at the ends. Table XX gives the work in detail. The value of  $H$  is 23158. From Table XIV, using the unit load coefficients,  $H$  was found to equal 23100.

87. Direct Determination of the Bending Moments  $M_1$  and  $M_2$  Produced by a Symmetrical Loading.—For symmetrical load, from (86),

$$M_1 = M_2 = H \frac{\Sigma y \Delta}{\Sigma \Delta} - \frac{\Sigma m_x \Delta}{\Sigma \Delta} \quad . \quad . \quad . \quad (86)$$

Table XXI shows the work in detail, where the value of  $M_1 = M_2 = -4171$ . From Table XIV, using unit load coefficients, the moment was found to be  $-4186$ . The two values are the same when all mathematical work is correct.

88. The Bending Moments  $M_1$  and  $M_2$  Produced by Unit Loads at Points 1-10 Inclusive.—For a uniform moving load it is necessary to determine the values of  $M_1$  and  $M_2$  for unsymmetrical loading. To illustrate the method let points 1-10 inclusive be loaded with a unit load each. From (85),

$$\left. \begin{matrix} M_1 \\ M_2 \end{matrix} \right\} = \left\{ H_1 \frac{\Sigma y \Delta}{\Sigma \Delta} - \frac{\Sigma m_x \Delta}{\Sigma \Delta} \mp \frac{n \Sigma m_x \Delta (z-n)}{-2 \sum_0^{l/2} (z-n)^2 \Delta} \right\} \quad . \quad . \quad (85)$$

If the effect of the axial stress is neglected,  $H_1$  is replaced by  $H$ .

The computations for  $H \frac{\Sigma y \Delta}{\Sigma \Delta}$  and  $\frac{\Sigma m_x \Delta}{\Sigma \Delta}$  are shown in Table XXII and for the last term of (85) the work is shown in Table XXIII.

The advantage of dividing the span into equal parts is quite evident when computing the values of  $m_x$  for unit loads. Letting  $\delta x \div 2$  be the unit of measure, it can be carried through as a factor and introduced at the end

of the work. In column 1 of Tables XXII and XXIII, the quantities shown must be multiplied by  $\delta x \div 2 = 1.34$  to obtain the actual values.

**89. Forms for the Direct Computation of  $H$ ,  $M_1$ , and  $M_2$  for the Dead Load and Some Combinations of Unit Loads.** For preliminary studies and when only approximate maximum stresses are desired the horizontal thrusts and the moments at the supports can be found by filling in the two forms given at the end of the chapter. Form *A* is for the determination of the horizontal thrusts and Form *B* for the determination of the bending moments at the supports.

Since the values of **B** are symmetrical about the crown the values of  $m_x$  for symmetrical points are added and the product multiplied by their common factor **B**. This can be done when the loading is unsymmetrical. When the live load is uniform the fields of loading which produce maximum fiber stresses at points 0, 6, and the crown are approximately those shown in the forms. The numerical values given for  $m_x \frac{2}{\delta x}$  (symmetrical values added in all cases but one) are constant for any arch where the span is divided into twenty equal parts.

In the determination of  $\Sigma m_x(z-n)$  the symmetrical values of  $m_x$  are subtracted, since the values of  $z-n$  for symmetrical points are of different signs but of equal magnitudes.

When Forms *A* and *B* are used, a good slide rule gives satisfactory results. Having found  $H$ ,  $M_1$ ,  $M_2$ , etc., the fiber stresses are computed in the usual manner.

**89a. Determination of the Horizontal Thrusts and Bending Moments at the Supports Produced by Unit Horizontal Loads Acting from Right to Left.**—The computation for unit horizontal loads is not very difficult, as shown by Tables I Q to V Q inclusive. Many of the quantities employed are known from the computations made for vertical loads.



TABLE I.  
DATA AND CONSTANTS. $n=20.$ 

| Point | 1     | 2                | 3                 | 4             | 5    | 6        | 7                   | 8                               | 9          | 10                            | 11   | 12               | 13                                         | 14                                                                 | 15          |
|-------|-------|------------------|-------------------|---------------|------|----------|---------------------|---------------------------------|------------|-------------------------------|------|------------------|--------------------------------------------|--------------------------------------------------------------------|-------------|
| $h$   | $h^3$ | $\frac{h^3}{12}$ | $\frac{h^3}{I_c}$ | $\frac{h}{2}$ | $h'$ | $(h')^2$ | $\frac{1}{8}(h')^2$ | $(\frac{3}{7} + \frac{7}{I_x})$ | $\delta s$ | $\frac{\delta s}{I_x \Delta}$ | $y$  | $y\Delta$        | $y - \frac{\Sigma y\Delta}{\Sigma \Delta}$ | $\Delta \left\{ y - \frac{\Sigma y\Delta}{\Sigma \Delta} \right\}$ | $yB$        |
| 1     | 3.80  | 54.87            | 4.57              | 1.90          | 1.73 | 2.99     | 0.598               | 5.17                            | 3.41       | 0.66                          | 1.10 | 0.726            | -7.0576                                    | -4.658                                                             | -5.1238     |
| 2     | 2.80  | 21.95            | 1.83              | 1.40          | 1.23 | 1.51     | 0.302               | 2.13                            | 3.21       | 1.51                          | 3.05 | 9.606            | -5.1076                                    | -7.712                                                             | -23.5256    |
| 3     | 2.08  | 9.00             | 0.75              | 1.04          | 0.87 | 0.77     | 0.154               | 0.90                            | 3.08       | 3.42                          | 4.66 | 15.937           | -3.4976                                    | -11.962                                                            | -55.7412    |
| 4     | 1.54  | 3.65             | 0.30              | 0.77          | 0.60 | 0.36     | 0.072               | 0.37                            | 2.92       | 7.89                          | 5.95 | 96.946           | -2.2076                                    | -17.418                                                            | -103.6379   |
| 5     | 1.38  | 2.63             | 0.22              | 0.69          | 0.52 | 0.27     | 0.054               | 0.27                            | 2.81       | 10.41                         | 6.98 | 72.662           | -1.1776                                    | -12.259                                                            | -85.5667    |
| 6     | 1.30  | 2.20             | 0.18              | 0.65          | 0.48 | 0.23     | 0.046               | 0.23                            | 2.75       | 11.96                         | 7.77 | 92.929           | -0.3876                                    | -4.636                                                             | -36.0192    |
| 7     | 1.19  | 1.69             | 0.14              | 0.59          | 0.42 | 0.18     | 0.036               | 0.18                            | 2.74       | 15.22                         | 8.30 | 126.326          | +0.1424                                    | +2.167                                                             | +17.9888    |
| 8     | 1.10  | 1.33             | 0.11              | 0.55          | 0.38 | 0.14     | 0.028               | 0.14                            | 2.70       | 19.28                         | 8.71 | 167.929          | +0.5524                                    | +10.650                                                            | +92.7639    |
| 9     | 1.03  | 1.09             | 0.09              | 0.51          | 0.34 | 0.12     | 0.024               | 0.11                            | 2.68       | 24.36                         | 8.98 | 218.753          | +0.8224                                    | +20.034                                                            | +179.9024   |
| 10    | 1.00  | 1.00             | 0.08              | 0.50          | 0.33 | 0.11     | 0.022               | 0.10                            | 2.68       | 26.80                         | 9.12 | 244.416          | +0.9624                                    | +25.792                                                            | +235.2259   |
|       |       |                  |                   |               |      |          |                     |                                 |            | 121.51                        |      | 991.23           |                                            | -58.645                                                            | -309.6144   |
|       |       |                  |                   |               |      |          |                     |                                 |            | 2                             |      | 2                |                                            | +58.643                                                            | +525.8810   |
|       |       |                  |                   |               |      |          |                     |                                 |            | 243.02                        |      | 1982.46          |                                            | -0.002                                                             | +216.2666   |
|       |       |                  |                   |               |      |          |                     |                                 |            | $\Sigma \Delta$               |      | $\Sigma y\Delta$ |                                            | 2                                                                  | 2           |
|       |       |                  |                   |               |      |          |                     |                                 |            |                               |      |                  |                                            | -0.004                                                             | 432.5333    |
|       |       |                  |                   |               |      |          |                     |                                 |            |                               |      |                  |                                            | $\Sigma B$                                                         | $\Sigma yB$ |

$$h' = \frac{h}{2} = -0.17 \quad \frac{\Sigma y\Delta}{\Sigma \Delta} = 8.1576$$

REMARKS.—When the numerical work is correct  $\Sigma B=0$ .  $\Sigma yB$ =the denominator of the expression for  $H$ . All of the quantities in this table are independent of the loading and are constants for this arch.

TABLE II.  
HORIZONTAL THRUSTS PRODUCED BY UNIT VERTICAL  
LOADS.

$n = 20$

| Point. | 1  | 2                                          | 3                 | 4   | 5                                                                                             | 6                   | 7                | 8                                                   | Point. |
|--------|----|--------------------------------------------|-------------------|-----|-----------------------------------------------------------------------------------------------|---------------------|------------------|-----------------------------------------------------|--------|
| $z$    |    | $zB$                                       | $\sum_{z=0}^a zB$ | $k$ | $\sum_{z=0}^a zB$                                                                             | $k \sum_{z=0}^a zB$ | Cols.<br>(3)-(6) | $\frac{\text{Col. (7)} \delta x}{2 \sum yB}$<br>$H$ |        |
| 1      | 1  | - 4.658                                    | - 9.658           | 1   | - 4.658                                                                                       | - 4.658             | 0                | 0                                                   | 1'     |
| 2      | 3  | - 23.136                                   | - 27.794          | 3   | - 12.370                                                                                      | - 37.110            | 9.316            | 0.029                                               | 2'     |
| 3      | 5  | - 59.810                                   | - 87.604          | 5   | - 24.332                                                                                      | - 121.660           | 34.056           | 0.106                                               | 3'     |
| 4      | 7  | - 121.926                                  | - 209.530         | 7   | - 41.750                                                                                      | - 292.250           | 82.720           | 0.256                                               | 4'     |
| 5      | 9  | - 110.331                                  | - 319.861         | 9   | - 54.009                                                                                      | - 486.081           | 166.220          | 0.515                                               | 5'     |
| 6      | 11 | - 50.996                                   | - 370.857         | 11  | - 58.645                                                                                      | - 645.095           | 274.238          | 0.850                                               | 6'     |
| 7      | 13 | + 28.171                                   | - 342.686         | 13  | - 56.478                                                                                      | - 734.214           | 391.528          | 1.213                                               | 7'     |
| 8      | 15 | + 159.750                                  | - 182.936         | 15  | - 45.828                                                                                      | - 687.420           | 504.484          | 1.563                                               | 8'     |
| 9      | 17 | + 340.578                                  | + 157.642         | 17  | - 25.794                                                                                      | - 438.498           | 596.140          | 1.847                                               | 9'     |
| 10     | 19 | + 490.048                                  | + 647.690         | 19  | * -0.002                                                                                      | * 0.038             | 647.690          | 2.007                                               | 10,    |
|        |    | - 370.857<br>+ 1018.547                    |                   |     | $\sum_{z=0}^a zB - k \sum_{z=0}^a zB$                                                         |                     |                  | 8.386<br>2                                          |        |
|        |    | + 647.690<br>$\frac{1}{2} \sum_{z=0}^a zB$ |                   |     | $H = \frac{\sum_{z=0}^a zB - k \sum_{z=0}^a zB}{\sum yB} \left( P \frac{\delta x}{2} \right)$ |                     |                  | 16.772<br>$\sum H$                                  |        |

\* These quantities should be zero.

TABLE III.  
BENDING MOMENTS  $M_1$  AND  $M_2$  PRODUCED BY UNIT  
VERTICAL LOADS. TERM D.

$n = 20$

| Point. | 1  | 2         | 3                      | 4   | 5                      | 6                        | 7               | 8                                                    |
|--------|----|-----------|------------------------|-----|------------------------|--------------------------|-----------------|------------------------------------------------------|
| $z$    |    | $z\Delta$ | $\sum_{z=0}^a z\Delta$ | $k$ | $\sum_{z=0}^a z\Delta$ | $k \sum_{z=0}^a z\Delta$ | Col.<br>(3)+(6) | $\text{Col. (7)} \frac{\delta x}{2 \sum \Delta} = D$ |
| 1      | 1  | 0.66      | 0.66                   | 1   | 120.85                 | 120.85                   | 121.51          | 0.670*                                               |
| 2      | 3  | 4.53      | 5.19                   | 3   | 119.34                 | 358.02                   | 363.21          | 2.003                                                |
| 3      | 5  | 17.10     | 22.29                  | 5   | 115.92                 | 579.60                   | 601.89          | 3.318                                                |
| 4      | 7  | 55.23     | 77.52                  | 7   | 108.03                 | 756.21                   | 833.73          | 4.597                                                |
| 5      | 9  | 93.69     | 171.21                 | 9   | 97.62                  | 878.58                   | 1049.79         | 5.788                                                |
| 6      | 11 | 131.56    | 302.77                 | 11  | 85.66                  | 942.26                   | 1245.03         | 6.865                                                |
| 7      | 13 | 197.86    | 500.63                 | 13  | 70.44                  | 915.72                   | 1416.35         | 7.810                                                |
| 8      | 15 | 289.20    | 789.83                 | 15  | 51.16                  | 767.40                   | 1557.23         | 8.586                                                |
| 9      | 17 | 414.12    | 1203.95                | 17  | 26.80                  | 455.60                   | 1659.55         | 9.150                                                |
| 10     | 19 | 509.20    | 1713.15                | 19  | 0                      | 0                        | 1713.15         | 9.446                                                |

$$D = \frac{\sum m_x \Delta}{\sum \Delta} = \left\{ \sum_{z=0}^a z\Delta + k \sum_{z=0}^a z\Delta \right\} \frac{\delta x}{2 \sum \Delta} P = \frac{\delta x}{2 \sum \Delta} = \frac{2.68}{2(243.02)} = 0.005514.$$

\* When the numerical work is correct this equals  $\delta x \div 4$ .

TABLE IV.  
BENDING MOMENTS  $M_1$  AND  $M_2$  PRODUCED BY UNIT  
VERTICAL LOADS.—TERM E.  $n=20$

| Point. | 1     | 2   | 3     | 4   | 5        | 6              | 7                               | 8                                    |
|--------|-------|-----|-------|-----|----------|----------------|---------------------------------|--------------------------------------|
|        | $n-k$ | $k$ | $z-n$ | $z$ | $z(z-n)$ | $z(z-n)\Delta$ | $\sum_{x=0}^{x=a} z(z-n)\Delta$ | $(n-k)\sum_{x=0}^{x=a} (z-n)z\Delta$ |
| 1      | 19    | 1   | -19   | 1   | -19      | -12.54         | -12.54                          | -238.26                              |
| 2      | 17    | 3   | -17   | 3   | -51      | -77.01         | -89.55                          | -1522.35                             |
| 3      | 15    | 5   | -15   | 5   | -75      | -256.50        | -346.05                         | -5190.75                             |
| 4      | 13    | 7   | -13   | 7   | -91      | -717.99        | -1064.04                        | -13832.52                            |
| 5      | 11    | 9   | -11   | 9   | -99      | -1030.59       | -2094.63                        | -23040.93                            |
| 6      | 9     | 11  | -9    | 11  | -99      | -1184.04       | -3278.67                        | -29508.03                            |
| 7      | 7     | 13  | -7    | 13  | -91      | -1385.02       | -4663.69                        | -32645.83                            |
| 8      | 5     | 15  | -5    | 15  | -75      | -1446.00       | -6109.69                        | -30548.45                            |
| 9      | 3     | 17  | -3    | 17  | -51      | -1242.36       | -7352.05                        | -22056.15                            |
| 10     | 1     | 19  | -1    | 19  | -19      | -509.20        | -7861.25                        | -7861.25                             |

When  $n=20$  and the unit loads are placed at points 1, 2, 3, etc., columns 1, 2, 3, 4, 5, and 9 are constant.

TABLE IV.—Continued.  $n=20$

| Point. | 9          | 10               | 11                                          | 12                                            | 13             | 14                             |
|--------|------------|------------------|---------------------------------------------|-----------------------------------------------|----------------|--------------------------------|
|        | $-(z-n)^2$ | $-(z-n)^2\Delta$ | $\sum_{x=a}^{x=\frac{1}{2}l} (z-n)^2\Delta$ | $-k\sum_{x=a}^{x=\frac{1}{2}l} (z-n)^2\Delta$ | Cols. (8)+(12) | Col. (13)<br>(-0.0001034)<br>E |
| 1      | -361       | -238.26          | -6241.49                                    | -6241.49                                      | -6479.75       | 0.670*                         |
| 2      | -289       | -436.39          | -5805.10                                    | -17415.30                                     | -18937.65      | 1.958                          |
| 3      | -225       | -769.50          | -5035.60                                    | -25178.00                                     | -30368.75      | 3.140                          |
| 4      | -169       | -1333.41         | -3702.19                                    | -25915.33                                     | -39747.85      | 4.110                          |
| 5      | -121       | -1259.61         | -2442.58                                    | -21983.22                                     | -45024.15      | 4.656                          |
| 6      | -81        | -968.76          | -1473.82                                    | -16212.02                                     | -45720.25      | 4.728                          |
| 7      | -49        | -745.78          | -728.04                                     | -9464.52                                      | -42110.35      | 4.354                          |
| 8      | -25        | -482.00          | -246.04                                     | -3690.60                                      | -34238.05      | 3.540                          |
| 9      | -9         | -219.24          | -26.80                                      | -455.60                                       | -22511.75      | 2.328                          |
| 10     | -1         | -26.80           | 0                                           | 0                                             | -7861.25       | 0.813                          |
|        |            | -6479.75         |                                             |                                               |                |                                |

$$E = \left\{ (n-k) \sum_{x=0}^{x=a} (z-n)z\Delta - k \sum_{x=a}^{x=\frac{1}{2}l} (z-n)^2\Delta \right\} \frac{\delta x}{-4 \sum_0^{\frac{1}{2}l} (z-n)^2\Delta} P$$

Col. (8)                      Col. (12)

\* When the numerical work is correct this quantity equals  $\delta x + 4$ .



TABLE V.  
BENDING MOMENTS  $M_1$  AND  $M_2$  PRODUCED BY UNIT  
VERTICAL LOADS.

| $n = 20$                                      |        |       |       |                                                       |        |        |                   |
|-----------------------------------------------|--------|-------|-------|-------------------------------------------------------|--------|--------|-------------------|
| Point.                                        | 1      | 2     | 3     | 4                                                     | 5      | Point. | 6                 |
|                                               | C      | D     | E     | $M_1$                                                 | $M_2$  |        | Remarks.          |
| 1                                             | 0      | 0.670 | 0.670 | -1.340*                                               | 0      | 1'     | $M_1 = C - D - E$ |
| 2                                             | 0.237  | 2.003 | 1.958 | -3.724                                                | +0.192 | 2'     |                   |
| 3                                             | 0.865  | 3.318 | 3.140 | -5.593                                                | +0.687 | 3'     |                   |
| 4                                             | 2.088  | 4.597 | 4.110 | -6.619                                                | +1.601 | 4'     | $M_2 = C - D + E$ |
| 5                                             | 4.201  | 5.788 | 4.656 | -6.243                                                | +3.069 | 5'     |                   |
| 6                                             | 6.934  | 6.865 | 4.728 | -4.659                                                | +4.797 | 6'     |                   |
| 7                                             | 9.895  | 7.810 | 4.354 | -2.269                                                | +6.439 | 7'     |                   |
| 8                                             | 12.750 | 8.586 | 3.540 | +0.624                                                | +7.704 | 8'     |                   |
| 9                                             | 15.067 | 9.150 | 2.328 | +3.589                                                | +8.245 | 9'     |                   |
| 10                                            | 16.372 | 9.446 | 0.813 | +6.113                                                | +7.739 | 10'    |                   |
| $C = H \frac{\Sigma y \Delta}{\Sigma \Delta}$ |        |       |       | $M_2$                                                 | $M_1$  |        |                   |
|                                               |        |       |       | $\Sigma M_1 = -30.445 + 50.799 = 20.354 = \Sigma M_2$ |        |        |                   |

\* When the numerical work is correct this quantity equals  $\delta x \div 2$ .

TABLE VI.  
VALUES OF  $V_1$ ,  $V_2$ ,  $M_1$ ,  $M_2$ , AND  $H$  PRODUCED BY UNIT  
VERTICAL LOADS.

| $n = 20$ |             |                     |       |                             |       |        |        |       |        |
|----------|-------------|---------------------|-------|-----------------------------|-------|--------|--------|-------|--------|
| Point.   | 1           | 2                   | 3     | 4                           | 5     | 6      | 7      | 8     | Point. |
|          | $M_2 - M_1$ | $M_2 - M_1$<br>Span | $R_1$ | $V_1$<br>Cols.<br>(2) + (3) | $V_2$ | $M_1$  | $M_2$  | $H$   |        |
| 1        | 1.340       | 0.025               | 0.975 | 1.000                       | 0     | -1.340 | 0      | 0     | 1'     |
| 2        | 3.916       | 0.073               | 0.925 | 0.998                       | 0.002 | -3.724 | +0.192 | 0.029 | 2'     |
| 3        | 6.280       | 0.117               | 0.875 | 0.992                       | 0.008 | -5.593 | +0.687 | 0.106 | 3'     |
| 4        | 8.220       | 0.153               | 0.825 | 0.978                       | 0.022 | -6.619 | +1.601 | 0.256 | 4'     |
| 5        | 9.312       | 0.173               | 0.775 | 0.948                       | 0.052 | -6.243 | +3.069 | 0.515 | 5'     |
| 6        | 9.456       | 0.176               | 0.725 | 0.901                       | 0.099 | -4.659 | +4.797 | 0.850 | 6'     |
| 7        | 8.706       | 0.162               | 0.675 | 0.837                       | 0.163 | -2.267 | +6.439 | 1.213 | 7'     |
| 8        | 7.080       | 0.132               | 0.625 | 0.757                       | 0.243 | +0.624 | +7.704 | 1.563 | 8'     |
| 9        | 4.656       | 0.087               | 0.575 | 0.662                       | 0.338 | +3.589 | +8.245 | 1.847 | 9'     |
| 10       | 1.626       | 0.030               | 0.525 | 0.555                       | 0.445 | +6.113 | +7.739 | 2.007 | 10'    |
|          |             |                     |       | $V_2$                       | $V_1$ | $M_2$  | $M_1$  | $H$   |        |

$$\Sigma V_1 = 10, \quad \Sigma V_2 = 10, \quad \Sigma M_1 = -20.354, \quad \Sigma M_2 = +20.354, \quad \Sigma H = 16.772$$

$$\Sigma(-M_1) = -30.445, \quad \Sigma(-M_2) = -30.445;$$

$$\Sigma(+M_1) = +50.799, \quad \Sigma(+M_2) = +50.799.$$



TABLE VII.  
THE HORIZONTAL THRUST PRODUCED BY UNIT VERTICAL  
LOADS AT ALL POINTS.  $n = 20$

| Point. | 1                                       | 2                     | 3                                                         | 4                                                                                                                                                                                          |
|--------|-----------------------------------------|-----------------------|-----------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|        | $m_x \left( \frac{2}{\delta x} \right)$ | B                     | $m_x B \left( \frac{2}{\delta x} \right)$                 | Remarks.                                                                                                                                                                                   |
| 1      | 10                                      | - 4.658               | - 46.580                                                  | $H = \frac{\Sigma m_x B}{\Sigma y B} = \frac{5412.460}{432.5333} \cdot \frac{2.68}{2} = 16.768$ <p>This value of <math>H</math> should equal the sum of the values in Table II, Col. 8</p> |
| 2      | 28                                      | - 7.712               | - 215.936                                                 |                                                                                                                                                                                            |
| 3      | 44                                      | - 11.962              | - 526.328                                                 |                                                                                                                                                                                            |
| 4      | 58                                      | - 17.418              | - 1010.244                                                |                                                                                                                                                                                            |
| 5      | 70                                      | - 12.259              | - 858.130                                                 |                                                                                                                                                                                            |
| 6      | 80                                      | - 4.636               | - 370.880                                                 |                                                                                                                                                                                            |
| 7      | 88                                      | + 2.167               | + 190.696                                                 |                                                                                                                                                                                            |
| 8      | 94                                      | + 10.650              | + 1001.100                                                |                                                                                                                                                                                            |
| 9      | 98                                      | + 20.034              | + 1963.332                                                |                                                                                                                                                                                            |
| 10     | 100                                     | + 25.792              | + 2579.200                                                |                                                                                                                                                                                            |
|        | Constant when $n = 20$                  | From Table I, Col. 14 | - 3028.098<br>+ 5734.328<br>+ 2706.230<br>2<br>+ 5412.460 |                                                                                                                                                                                            |

TABLE VIII.  
THE BENDING MOMENTS  $M_1$  AND  $M_2$  PRODUCED BY UNIT  
VERTICAL LOADS AT ALL POINTS.  $n = 20$

| Point. | 1                                       | 2                     | 3                                                             | 4                                                                                                                                                                                                                                                                                                                                                                                         |
|--------|-----------------------------------------|-----------------------|---------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|        | $m_x \left( \frac{2}{\delta x} \right)$ | $\Delta$              | $\left( m_x \Delta \right) \left( \frac{2}{\delta x} \right)$ | Remarks.                                                                                                                                                                                                                                                                                                                                                                                  |
| 1      | 10                                      | 0.66                  | 6.60                                                          | $M_1 = M_2 = H \frac{\Sigma y \Delta}{\Sigma \Delta} - \frac{\Sigma m_x \Delta}{\Sigma \Delta} = C - \frac{\Sigma m_x \Delta}{\Sigma \Delta}$ $C = 16.772(8.1576) = 136.818$ $\frac{\Sigma m_x \Delta}{\Sigma \Delta} = \frac{21122.88}{243.02} \cdot \frac{2.68}{2} = 116.470$ $M_1 = M_2 = 136.818 - 116.470 = +20.348$ <p>This value should equal <math>\Sigma M_1</math>, Table V</p> |
| 2      | 28                                      | 1.51                  | 42.28                                                         |                                                                                                                                                                                                                                                                                                                                                                                           |
| 3      | 44                                      | 3.42                  | 150.48                                                        |                                                                                                                                                                                                                                                                                                                                                                                           |
| 4      | 58                                      | 7.89                  | 457.62                                                        |                                                                                                                                                                                                                                                                                                                                                                                           |
| 5      | 70                                      | 10.41                 | 728.70                                                        |                                                                                                                                                                                                                                                                                                                                                                                           |
| 6      | 80                                      | 11.96                 | 956.80                                                        |                                                                                                                                                                                                                                                                                                                                                                                           |
| 7      | 88                                      | 15.22                 | 1339.36                                                       |                                                                                                                                                                                                                                                                                                                                                                                           |
| 8      | 94                                      | 19.28                 | 1812.32                                                       |                                                                                                                                                                                                                                                                                                                                                                                           |
| 9      | 98                                      | 24.36                 | 2387.28                                                       |                                                                                                                                                                                                                                                                                                                                                                                           |
| 10     | 100                                     | 26.80                 | 2680.00                                                       |                                                                                                                                                                                                                                                                                                                                                                                           |
|        | Constant when $n = 20$                  | 121.51<br>2<br>243.02 | 10561.44<br>2<br>21122.88                                     |                                                                                                                                                                                                                                                                                                                                                                                           |

TABLE IX.  
EFFECT OF THE AXIAL STRESS UPON THE HORIZONTAL THRUST.

| Point. | 1     | 2          | 3              | 4                        | 5                                                                                                                                                                           |
|--------|-------|------------|----------------|--------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|        | $F_x$ | $\delta s$ | $F_x \delta s$ | $\frac{1}{F_x \delta s}$ | Remarks.                                                                                                                                                                    |
| 1      | 4.00  | 3.41       | 13.640         | 0.0733                   | $\frac{H_a}{H} = - \frac{\sum (\delta x)^2}{F_x \delta s}$ $\frac{\sum y B + \sum \frac{(\delta x)^2}{F_x \delta s}}{432.533 + 30.82} = 0.0665$ $\therefore H_a = -0.0665H$ |
| 2      | 3.00  | 3.21       | 9.630          | 0.1038                   |                                                                                                                                                                             |
| 3      | 2.28  | 3.08       | 7.022          | 0.1424                   |                                                                                                                                                                             |
| 4      | 1.74  | 2.92       | 5.081          | 0.1968                   |                                                                                                                                                                             |
| 5      | 1.58  | 2.81       | 4.440          | 0.2252                   |                                                                                                                                                                             |
| 6      | 1.50  | 2.75       | 4.125          | 0.2424                   |                                                                                                                                                                             |
| 7      | 1.39  | 2.74       | 3.809          | 0.2625                   |                                                                                                                                                                             |
| 8      | 1.30  | 2.70       | 3.510          | 0.2849                   |                                                                                                                                                                             |
| 9      | 1.23  | 2.68       | 3.296          | 0.3034                   |                                                                                                                                                                             |
| 10     | 1.20  | 2.68       | 3.216          | 0.3109                   |                                                                                                                                                                             |
|        |       | Table I    |                | 2.1456<br>2              | $F_x = h + 0.20$ . The area of the steel in equivalent concrete is 0.20                                                                                                     |
|        |       |            |                | 4.2912                   |                                                                                                                                                                             |

TABLE X.  
STRESS IN THE EXTREME FIBERS PRODUCED BY A UNIT HORIZONTAL THRUST ACTING TOWARD THE CENTER OF THE SPAN.

| Point. | 1      | 2             | 3               | 4     | 5                 | 6      | 7       | 8                                                                                |
|--------|--------|---------------|-----------------|-------|-------------------|--------|---------|----------------------------------------------------------------------------------|
|        | $M_x$  | $\frac{1}{S}$ | $\frac{M_x}{S}$ | $N_x$ | $\frac{N_x}{F_x}$ | Top.   | Bottom. | Remarks.                                                                         |
| •      | +8.158 | 0.266         | +2.170          | 0.800 | 0.170             | +2.340 | -2.000  | If the thrust acts away from the center, the signs in Cols. 6 and 7 are reversed |
| 6      | +0.388 | 2.820         | +1.093          | 0.974 | 0.649             | +1.742 | -0.444  |                                                                                  |
| C      | -0.962 | 5.000         | -4.812          | 1.000 | 0.833             | -3.979 | +5.645  |                                                                                  |

$$M_1 = H \frac{\sum y \Delta}{\sum \Delta} \quad M_x = M_1 - Hy = H \left\{ \frac{\sum y \Delta}{\sum \Delta} - y \right\} = (8.1576 - y).$$

$$N_x = V_x \sin \phi + H_x \cos \phi = H \cos \phi = \cos \phi.$$

$$F_x = 4.7, 1.5 \text{ and } 1.2.$$

$$\left. \begin{array}{l} \text{Stress in top fiber} \\ \text{Stress in bottom fiber} \end{array} \right\} = \left\{ \frac{N_x}{F_x} \pm \frac{M_x}{S} \right\}$$

TABLE XI.

STRESS IN EXTREME FIBERS PRODUCED BY A UNIT LOAD  
MOVING ACROSS THE SPAN. SECTION No. o. $n = 20$ 

| Point | 1                               | 2               | 3             | 4      | 5                 | 6                        | 7       |
|-------|---------------------------------|-----------------|---------------|--------|-------------------|--------------------------|---------|
|       | $\frac{M_1}{S} =$<br>$0.266M_1$ | $V_1 \sin \phi$ | $H \cos \phi$ | $N_x$  | $\frac{N_x}{F_x}$ | Stress in Extreme Fiber. |         |
|       |                                 |                 |               |        |                   | Top.                     | Bottom. |
| 1     | -0.3564                         | 0.5630          | 0             | 0.5630 | 0.1198            | -0.2366                  | +0.4762 |
| 2     | -0.9906                         | 0.5619          | 0.0232        | 0.5851 | 0.1245            | -0.8661                  | +1.1151 |
| 3     | -1.4877                         | 0.5585          | 0.0848        | 0.6433 | 0.1369            | -1.3308                  | +1.6246 |
| 4     | -1.7606                         | 0.5506          | 0.2048        | 0.7554 | 0.1612            | -1.5994                  | +1.9218 |
| 5     | -1.6606                         | 0.5337          | 0.4120        | 0.9457 | 0.2012            | -1.4594                  | +1.8618 |
| 6     | -1.2393                         | 0.5073          | 0.6800        | 1.1873 | 0.2526            | -0.9867                  | +1.4919 |
| 7     | -0.6030                         | 0.4712          | 0.9704        | 1.4416 | 0.3067            | -0.2963                  | +0.9097 |
| 8     | +0.1660                         | 0.4262          | 1.2504        | 1.6766 | 0.3567            | +0.5227                  | +0.1907 |
| 9     | +0.9547                         | 0.3727          | 1.4776        | 1.8503 | 0.3937            | +1.3484                  | -0.5610 |
| 10    | +1.6260                         | 0.3125          | 1.6056        | 1.9175 | 0.4080            | +2.0340                  | -1.2180 |
| 10'   | +2.0586                         | 0.2505          | 1.6056        | 1.8561 | 0.3950            | +2.4536                  | -1.6636 |
| 9'    | +2.1932                         | 0.1903          | 1.4776        | 1.6679 | 0.3549            | +2.5481                  | -1.8383 |
| 8'    | +2.0493                         | 0.1368          | 1.2504        | 1.3872 | 0.2951            | +2.3444                  | -1.7542 |
| 7'    | +1.7128                         | 0.0917          | 0.9704        | 1.0621 | 0.2260            | +1.9388                  | -1.4868 |
| 6'    | +1.2760                         | 0.0557          | 0.6800        | 0.7357 | 0.1565            | +1.4325                  | -1.1195 |
| 5'    | +0.8163                         | 0.0293          | 0.4120        | 0.4413 | 0.0940            | +0.9103                  | -0.7323 |
| 4'    | +0.4259                         | 0.0124          | 0.2048        | 0.2172 | 0.0462            | +0.4721                  | -0.3797 |
| 3'    | +0.1827                         | 0.0045          | 0.0848        | 0.0893 | 0.0190            | +0.2017                  | -0.1637 |
| 2'    | +0.0511                         | 0.0017          | 0.0232        | 0.0249 | 0.0530            | +0.1041                  | -0.0019 |
| 1'    | 0                               | 0               | 0             | 0      | 0                 | 0                        | 0       |

 $\sin \phi = 0.563, \quad \cos \phi = 0.800, \quad I_x = 8.459, \quad h = 4.50, \quad F_x = 4.70.$ Reciprocal of the section modulus,  $S = 0.266.$ Stress in top fiber =  $\frac{N_x}{F_x} + \frac{M_x}{S}$ .      Bottom fiber =  $\frac{N_x}{F_x} - \frac{M_x}{S}$ .

CHECK.

For a unit load at each point the stress in the top fiber is,

 $-6.7953 + 16.3107 = +9.5154$  from Col. 6. $\frac{M_1}{S} = +20.354(0.266) = 5.4142,$  $V_1 \sin \phi = 10(0.563) = 5.6300.$  $H \cos \phi = 16.772(0.800) = 13.476,$  $\frac{N_x}{F_x} = 4.0526.$ Fiber stress =  $5.4142 + 4.0526 = 9.4668.$ Error is less than  $\frac{1}{2}\%$ .



TABLE XII.

STRESS IN EXTREME FIBERS PRODUCED BY A UNIT LOAD  
MOVING ACROSS THE SPAN. SECTION No. 6.  $n=20$ 

| Point.                                                 | 1      | 2                                         | 3       | 4       | 5       | 6               | 7                                                                                                   |
|--------------------------------------------------------|--------|-------------------------------------------|---------|---------|---------|-----------------|-----------------------------------------------------------------------------------------------------|
|                                                        | .3876H | $\frac{\Sigma m_x \Delta}{\Sigma \Delta}$ | -.45E   | $+m_x$  | $M_x$   | $\frac{M_x}{S}$ | Remarks.                                                                                            |
| 1                                                      | 0      | -0.670                                    | -0.3015 | 0.9715  | 0       | 0               | Col. 4 is constant when $n=20$ and $\delta x=2.68$ .                                                |
| 2                                                      | 0.0112 | -2.003                                    | -0.8811 | 2.9145  | +0.0416 | +0.1173         |                                                                                                     |
| 3                                                      | 0.0411 | -3.318                                    | -1.4130 | 4.8575  | +0.1676 | +0.4726         |                                                                                                     |
| 4                                                      | 0.0992 | -4.597                                    | -1.8495 | 6.8005  | +0.4532 | +1.2780         |                                                                                                     |
| 5                                                      | 0.1996 | -5.788                                    | -2.0952 | 8.7435  | +1.0599 | +2.9889         | Col. 1 is symmetrical about the center of the span                                                  |
| 6                                                      | 0.3295 | -6.865                                    | -2.1276 | 10.6865 | +2.0234 | +5.7060         |                                                                                                     |
| 7                                                      | 0.4702 | -7.810                                    | -1.9593 | 9.9495  | +0.6504 | +1.8341         |                                                                                                     |
| 8                                                      | 0.6058 | -8.586                                    | -1.5930 | 9.2125  | -0.3607 | -1.0169         |                                                                                                     |
| 9                                                      | 0.7159 | -9.150                                    | -1.0496 | 8.4755  | -1.0062 | -2.8375         | Cols. 2 and 3 are symmetrical about the center of the span, but the quantities have opposite signs. |
| 10                                                     | 0.7779 | -9.446                                    | -0.3658 | 7.7385  | -1.2954 | -3.6530         |                                                                                                     |
| 10'                                                    | 0.7779 | +9.446                                    | +0.3658 | 7.0015  | -1.3008 | -3.6682         |                                                                                                     |
| 9'                                                     | 0.7159 | +9.150                                    | +1.0496 | 6.2645  | -1.1220 | -3.1640         |                                                                                                     |
| 8'                                                     | 0.6058 | +8.586                                    | +1.5930 | 5.5275  | -0.8597 | -2.4243         |                                                                                                     |
| 7'                                                     | 0.4702 | +7.810                                    | +1.9593 | 4.7905  | -0.5900 | -1.6638         |                                                                                                     |
| 6'                                                     | 0.3295 | +6.865                                    | +2.1276 | 4.0535  | -0.3544 | -0.9994         |                                                                                                     |
| 5'                                                     | 0.1996 | +5.788                                    | +2.0952 | 3.3165  | -0.1767 | -0.4983         |                                                                                                     |
| 4'                                                     | 0.0992 | +4.597                                    | +1.8495 | 2.5795  | -0.0688 | -0.1940         |                                                                                                     |
| 3'                                                     | 0.0411 | +3.318                                    | +1.4130 | 1.8425  | -0.0214 | -0.0603         |                                                                                                     |
| 2'                                                     | 0.0112 | +2.003                                    | +0.8811 | 1.1055  | -0.0052 | -0.0147         |                                                                                                     |
| 1'                                                     | 0      | +0.670                                    | +0.3015 | 0.3685  | 0       | 0               |                                                                                                     |
| $x=14.74 \quad \frac{1}{S} = \frac{0.65}{0.23} = 2.82$ |        |                                           |         |         | +4.3961 |                 |                                                                                                     |
| $y=7.77$                                               |        |                                           |         |         | -7.1613 |                 |                                                                                                     |
|                                                        |        |                                           |         |         | -2.7652 |                 |                                                                                                     |

$$M_x = H \left\{ \frac{\Sigma y \Delta}{\Sigma \Delta} - y \right\} - \frac{\Sigma m_x \Delta}{\Sigma \Delta} - \frac{n-z}{n} E + m_x. \quad m_x = \left\{ \frac{2n-k}{2n} z - (z-k) \right\} \frac{\delta_x}{2}$$

For Point 6:

$$M_x = 0.3876H - \frac{\Sigma m_x \Delta}{\Sigma \Delta} - 0.45E + m_x. \quad m_x = \left\{ \frac{40-k}{40} 11 - (11-k) \right\} 1.34$$

CHECK.

$M_x = M_1 + V_1 x - H y - \Sigma (x-a)^{x>a}$ . For a unit load at each point the moment at point 6 is  $M_6 = M_1 + V_1(14.74) - H(7.77) - \Sigma(14.74-a) = +20.354 + 147.400 - 130.3184 - 40.20 = -2.7644$ . This is a close agreement with the sum of Col. 5.

all these signs should be -



TABLE XII—Continued

 $n=20$ 

| Point. | 8               | 9               | 10            | 11           | 12      | 13                | 14                        |          | 15      |
|--------|-----------------|-----------------|---------------|--------------|---------|-------------------|---------------------------|----------|---------|
|        | $\frac{M_x}{S}$ | $V_1 \sin \phi$ | $H \cos \phi$ | $-\sin \phi$ | $N_x$   | $\frac{N_x}{F_x}$ | Stress in Extreme Fibers. |          |         |
|        |                 |                 |               |              |         |                   | Top.                      |          | Bottom. |
| 1      | 0               | 0.2260          | 0             | -0.226       | 0       | 0                 | 0                         |          | 0       |
| 2      | +0.1173         | 0.2255          | 0.0282        | -0.226       | 0.0277  | 0.0185            | + 0.1358                  | -        | 0.0982  |
| 3      | +0.4726         | 0.2242          | 0.1032        | -0.226       | 0.1014  | 0.0676            | + 0.5402                  | -        | 0.4050  |
| 4      | +1.2780         | 0.2210          | 0.2493        | -0.226       | 0.2443  | 0.1629            | + 1.4409                  | -        | 1.1151  |
| 5      | +2.9889         | 0.2142          | 0.5016        | -0.226       | 0.4898  | 0.3265            | + 3.3154                  | -        | 2.6624  |
| 6      | +5.7060         | 0.2036          | 0.8279        | -0.226       | 0.8055  | 0.5370            | + 6.2430                  | -        | 5.1690  |
| 7      | +1.8341         | 0.1892          | 1.1814        | .....        | 1.3706  | 0.9137            | + 2.7478                  | -        | 0.9204  |
| 8      | -1.0169         | 0.1711          | 1.5224        | .....        | 1.6934  | 1.1289            | + 0.1120                  | + 2.1458 |         |
| 9      | -2.8375         | 0.1496          | 1.7990        | .....        | 1.9486  | 1.2991            | - 1.5384                  | + 4.1366 |         |
| 10     | -3.6530         | 0.1254          | 1.9548        | .....        | 2.0802  | 1.3867            | - 2.2663                  | + 5.0397 |         |
| 10'    | -3.6682         | 0.1006          | 1.9548        | .....        | 2.0554  | 1.3703            | - 2.2979                  | + 5.0385 |         |
| 9'     | -3.1640         | 0.0764          | 1.7990        | .....        | 1.8754  | 1.2503            | - 1.9137                  | + 4.4143 |         |
| 8'     | -2.4243         | 0.0549          | 1.5224        | .....        | 1.5773  | 1.0515            | - 1.3728                  | + 3.4758 |         |
| 7'     | -1.6638         | 0.0368          | 1.1814        | .....        | 1.2182  | 0.8121            | - 0.8517                  | + 2.4759 |         |
| 6'     | -0.9994         | 0.0224          | 0.8279        | .....        | 0.8503  | 0.5669            | - 0.4325                  | + 1.5663 |         |
| 5'     | -0.4983         | 0.0117          | 0.5016        | .....        | 0.5133  | 0.3422            | - 0.1561                  | + 0.8405 |         |
| 4'     | -0.1940         | 0.0050          | 0.2493        | .....        | 0.2543  | 0.1695            | - 0.0245                  | + 0.3635 |         |
| 3'     | -0.0603         | 0.0018          | 0.1032        | .....        | 0.1050  | 0.0700            | + 0.0097                  | + 0.1303 |         |
| 2'     | -0.0147         | 0.0004          | 0.0282        | .....        | 0.0287  | 0.0192            | + 0.0045                  | + 0.0339 |         |
| 1'     | 0               | 0               | 0             | .....        | 0       | 0                 | 0                         |          | 0       |
|        |                 |                 |               | -1.3560      | 17.2394 | 11.4929           | +14.5493                  | -10.3701 |         |
|        |                 |                 |               |              |         |                   | -10.8539                  | +29.6611 |         |
|        |                 |                 |               |              |         |                   | + 3.6954                  | +19.2910 |         |

$$\sin \phi = 0.226, \quad \cos \phi = 0.974, \quad I = 0.23, \quad h = .130, \quad F_x = .150.$$

$$\text{Stress in top fiber} = \frac{N_x}{F_x} + \frac{M_x}{S}, \quad \text{Bottom fiber} = \frac{N_x}{F_x} - \frac{M_x}{S}.$$

CHECK.

The values of  $N_x$  are checked as follows: For a unit load at each point,

$$V_1 \sin \phi = 10(0.226) = 2.2600$$

$$H_1 \cos \phi = 16.772(0.974) = 16.3359$$

$$-\sum_{x>a} \sin \phi = -6(0.226) = -1.3560$$

$$\text{and} \quad N_x = +17.2399. \quad \text{See sum of Col. 12.}$$

TABLE XIII.—STRESS IN EXTREME FIBERS PRODUCED BY A UNIT LOAD MOVING ACROSS THE SPAN.  
SECTION AT CROWN.  $n = 20$

| Point | 1                                                                                                                                     | 2                                      | 3                        | 4       | 5               | 6                   | 7                 | 8                         |         | 9       |
|-------|---------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------|--------------------------|---------|-----------------|---------------------|-------------------|---------------------------|---------|---------|
|       | $-(0.9624)H$                                                                                                                          | $-\frac{\sum m_x \Delta}{\sum \Delta}$ | $m_x = \frac{\sum x}{4}$ | $M_x$   | $\frac{M_x}{S}$ | $N_x = H \cos \phi$ | $\frac{N_x}{F_x}$ | Stress in Extreme Fibers. |         |         |
|       |                                                                                                                                       |                                        |                          |         |                 |                     |                   | Top.                      | Bottom. |         |
| 1     | 0                                                                                                                                     | -0.670                                 | 0.670                    | 0       | 0               | 0                   | 0                 | 0                         | 0       | 0       |
| 2     | -0.0279                                                                                                                               | -2.003                                 | 2.010                    | -0.0209 | -0.1045         | 0.029               | 0.0242            | -0.0802                   | +0.1287 | +0.1287 |
| 3     | -0.1020                                                                                                                               | -3.318                                 | 3.350                    | -0.0700 | -0.3500         | 0.106               | 0.0883            | -0.2617                   | +0.4383 | +0.4383 |
| 4     | -0.2464                                                                                                                               | -4.597                                 | 4.690                    | -0.1534 | -0.7670         | 0.256               | 0.2133            | -0.5537                   | +0.9803 | +0.9803 |
| 5     | -0.4956                                                                                                                               | -5.788                                 | 6.030                    | -0.2536 | -1.2680         | 0.515               | 0.4291            | -0.8389                   | +1.6971 | +1.6971 |
| 6     | -0.8180                                                                                                                               | -6.865                                 | 7.370                    | -0.3136 | -1.5680         | 0.850               | 0.7083            | -0.8597                   | +2.2763 | +2.2763 |
| 7     | -1.1674                                                                                                                               | -7.810                                 | 8.710                    | -0.2674 | -1.3370         | 1.213               | 1.0108            | -0.3262                   | +2.3478 | +2.3478 |
| 8     | -1.5042                                                                                                                               | -8.586                                 | 10.050                   | -0.0402 | -0.2010         | 1.563               | 1.3025            | +1.1015                   | +1.5035 | +1.5035 |
| 9     | -1.7775                                                                                                                               | -9.150                                 | 11.390                   | +0.4634 | +2.3170         | 1.847               | 1.5391            | +3.8561                   | -0.7779 | -0.7779 |
| 10    | -1.9315                                                                                                                               | -9.446                                 | 12.730                   | +1.3525 | +6.7625         | 2.007               | 1.6725            | +8.4350                   | -5.0900 | -5.0900 |
|       | $M_x = M_c = H \left\{ \frac{\sum y \Delta}{\sum \Delta} - f \right\} - \frac{\sum m_x \Delta}{\sum \Delta} + k \frac{\partial x}{4}$ |                                        |                          |         |                 |                     |                   |                           |         |         |
|       | 6.9881                                                                                                                                |                                        |                          |         |                 |                     |                   |                           |         |         |

$\sin \phi = 0, \cos \phi = 1, I_x = 0.1, h = 1.0, F_x = 1.2, f = 9.12.$

Reciprocal of the section modulus,  $S_1 = 5.$

$$\text{Stress in top fiber} = \frac{N_x}{F_x} + \frac{M_x}{S}$$

$$\frac{N_x}{F_x} - \frac{M_x}{S}$$

CHECK.

Assume a unit load at each point:  $M_c = M_1 + V_1 \frac{l}{2} - Hf - \sum (x-a) = +20.354 + 10(26.8) - 16.772(9.12) - 134 = +1.394.$

$$N_x = H \cos \phi = 16.772. \quad \frac{M_c}{S} = +1.394(5) = 6.970. \quad \frac{N_x}{F_x} = \frac{16.772}{1.2} = 13.893.$$

Stress in top fiber = + 6.970 + 13.893 = +20.863 for full load  
 " " " = +10.432 for one-half load  
 " " " = + 6.923 for full load  
 " " " = + 3.462 for one-half load

Compare results for half load with Cols. 8 and 9.

TABLE XIV.  
DEAD LOADS AND VALUES OF  $V_1$ ,  $H$ ,  $M_1$ , AND  $M_2$  PRODUCED  
BY DEAD LOAD.  $n = 20$

| Point.                               | 1           | 2                        | 3           | 4         | 5            | 6             | 7        | 8        | 9        | Point. |
|--------------------------------------|-------------|--------------------------|-------------|-----------|--------------|---------------|----------|----------|----------|--------|
|                                      | $h\delta s$ | Ring,<br>150 $h\delta s$ | Earth Fill. |           | Pav-<br>ing. | Total<br>D.L. | $H, *$   | $M_1, *$ | $M_2, *$ |        |
|                                      |             |                          | $h''$       | 322 $h''$ |              |               |          |          |          |        |
| 1                                    | 12.958      | 1944                     | 7.35        | 2367      | 402          | 4710          | 0        | - 631.4  | 0        | 1'     |
| 2                                    | 8.988       | 1348                     | 5.95        | 1916      | 402          | 3670          | 106.43   | -13667.1 | + 704.6  | 2'     |
| 3                                    | 6.406       | 961                      | 4.75        | 1529      | 402          | 2890          | 306.34   | -16163.8 | + 1985.4 | 3'     |
| 4                                    | 4.497       | 675                      | 3.71        | 1195      | 402          | 2270          | 581.12   | -15025.1 | + 3634.3 | 4'     |
| 5                                    | 3.878       | 582                      | 2.75        | 885       | 402          | 1870          | 963.10   | -11674.4 | + 5739.0 | 5'     |
| 6                                    | 3.575       | 536                      | 2.15        | 692       | 402          | 1630          | 1385.50  | - 7594.2 | + 7819.1 | 6'     |
| 7                                    | 3.261       | 489                      | 1.70        | 547       | 402          | 1440          | 1746.72  | - 3264.5 | + 9272.2 | 7'     |
| 8                                    | 2.970       | 445                      | 1.36        | 438       | 402          | 1290          | 2016.27  | + 805.0  | + 9938.2 | 8'     |
| 9                                    | 2.760       | 414                      | 1.13        | 364       | 402          | 1180          | 2179.46  | + 4235.0 | + 9729.1 | 9'     |
| 10                                   | 2.680       | 402                      | 1.02        | 328       | 402          | 1130          | 2267.91  | + 6907.7 | + 8745.1 | 10'    |
| $M_1 = M_2 = -61753 + 57567 = -4186$ |             |                          |             |           |              | 22080         | 11552.85 | -73700.4 | +57567.0 |        |
| $V_1 = 22100$                        |             |                          |             |           |              | $V_1 = V_2$   | 2        | +11947.7 |          |        |
| $H_1 = 23100$                        |             |                          |             |           |              |               | 23105.70 | -61752.7 |          |        |
|                                      |             |                          |             |           |              |               | $H$      | $M_2$    | $M_1$    |        |

\*Axial stress neglected.

TABLE XV.  
STRESSES IN THE EXTREME FIBERS AT SECTION O.  $n = 20$

| 1                       | 2                | 3             | 4                | 5                      | 6                | 7 |
|-------------------------|------------------|---------------|------------------|------------------------|------------------|---|
| Axial Stress Neglected  |                  | Axial Stress. |                  | Axial Stress Included. |                  |   |
| Top<br>Fiber.           | Bottom<br>Fiber. | Top<br>Fiber. | Bottom<br>Fiber. | Top<br>Fiber.          | Bottom<br>Fiber. |   |
| Dead load. . . . .      |                  |               |                  |                        |                  |   |
| ± 5466                  | + 7692           | -3595         | +3071            | + 1871                 | +10763           |   |
| Live load 1-7. . . . .  |                  |               |                  |                        |                  |   |
| - 3643                  | .....            | - 248         | .....            | - 3891                 | .....            |   |
| Live load 1-8. . . . .  |                  |               |                  |                        |                  |   |
| + 5141                  | .....            | + 322         | .....            | .....                  | + 5463           |   |
| Live load 8-1'. . . . . |                  |               |                  |                        |                  |   |
| + 8742                  | .....            | -1149         | .....            | + 7593                 | .....            |   |
| Live load 9-1'. . . . . |                  |               |                  |                        |                  |   |
| - 5852                  | .....            | + 872         | .....            | .....                  | - 4980           |   |
| Tension. . . . .        |                  |               |                  |                        |                  |   |
| 0                       | 0                | .....         | .....            | 2021                   | 0                |   |
| Compression. . . . .    |                  |               |                  |                        |                  |   |
| 14208                   | 12833            | .....         | .....            | 9464                   | 16226            |   |
| Rise 20°. . . . .       |                  |               |                  |                        |                  |   |
| + 7582                  | - 6480           | - 503         | + 430            | + 7079                 | - 6050           |   |
| Fall 40°. . . . .       |                  |               |                  |                        |                  |   |
| -15164                  | +12960           | +1006         | - 860            | -14158                 | +12100           |   |
| Tension. . . . .        |                  |               |                  |                        |                  |   |
| 13341                   | 4640             | .....         | .....            | 16178                  | 267              |   |
| Compression. . . . .    |                  |               |                  |                        |                  |   |
| 21790                   | 25793            | .....         | .....            | 16543                  | 28326            |   |

All fiber stresses are expressed in pounds per square foot.

Live load = 200 pounds per square foot of roadway or 536 pounds per point.



TABLE XVI.

STRESSES IN THE EXTREME FIBERS AT SECTION 6.

$n = 20$

| 1                   | 2                       | 3                       | 4                       | 5                     | 6                       | 7                       |
|---------------------|-------------------------|-------------------------|-------------------------|-----------------------|-------------------------|-------------------------|
| Loading.            | Axial Stress Neglected  |                         | Axial Stress.           |                       | Axial Stress Included.  |                         |
|                     | Top Fiber.              | Bottom Fiber.           | Top Fiber.              | Bottom Fiber.         | Top Fiber.              | Bottom Fiber.           |
| Dead load.....      | <sup>18700</sup> +13632 | <sup>28400</sup> +17888 | <sup>-10600</sup> -2673 | <sup>35510</sup> +682 | <sup>75100</sup> +10959 | <sup>58500</sup> +18570 |
| Live load 1-8+3'-1' | + 7798                  | .....                   | - 299                   | .....                 | + 7499                  | .....                   |
| Live load 9-4'..... | - 5818                  | .....                   | - 745                   | .....                 | - 6563                  | .....                   |
| Live load 1-7.....  | .....                   | - 5558                  | .....                   | + 47                  | .....                   | - 5511                  |
| Live load 8-1'..... | .....                   | +15898                  | .....                   | +218                  | .....                   | +16116                  |
| Tension.....        | 0                       | 0                       | .....                   | .....                 | 0                       | 0                       |
| Compression.....    | 21430                   | 33786                   | .....                   | .....                 | 18458                   | 34686                   |
| Rise 20°.....       | + 5638                  | - 1434                  | - 376                   | + 96                  | + 5262                  | - 1333                  |
| Fall 40°.....       | -11276                  | + 2868                  | + 752                   | -192                  | -10524                  | + 2666                  |
| Tension.....        | 3462                    | 0                       | .....                   | .....                 | 6128                    | 0                       |
| Compression.....    | 27068                   | 36654                   | .....                   | .....                 | 23720                   | 37352                   |

TABLE XVII.

STRESSES IN THE EXTREME FIBERS AT THE CROWN.

$n = 20$

| 1                   | 2                      | 3             | 4             | 5             | 6                     | 7             |
|---------------------|------------------------|---------------|---------------|---------------|-----------------------|---------------|
| Loading.            | Axial Stress Neglected |               | Axial Stress. |               | Axial Stress Included |               |
|                     | Top Fiber.             | Bottom Fiber. | Top Fiber.    | Bottom Fiber. | Top Fiber.            | Bottom Fiber. |
| Dead load.....      | +22455                 | +16045        | +6113         | -8663         | +28568                | + 7382        |
| Live load 1-7+7'-1' | - 3130                 | .....         | + 840         | .....         | - 2290                | .....         |
| Live load 8-8'..... | +14356                 | .....         | +1536         | .....         | +15892                | .....         |
| Live load 1-8+8'-1' | .....                  | +10047        | .....         | -1828         | .....                 | + 8219        |
| Live load 9-9'..... | .....                  | - 6290        | .....         | -1545         | .....                 | - 7835        |
| Tension.....        | 0                      | 0             | .....         | .....         | 0                     | 453           |
| Compression.....    | 36811                  | 26092         | .....         | .....         | 44460                 | 15601         |
| Rise 20°.....       | -12895                 | +18274        | + 856         | -1213         | -12039                | +17061        |
| Fall 40°.....       | +25790                 | -36548        | -1712         | +2426         | +24078                | -34122        |
| Tension.....        | 0                      | 16595         | .....         | .....         | 0                     | 34575         |
| Compression.....    | 62600                  | 44366         | .....         | .....         | 68538                 | 32662         |



TABLE XVIII.

THE VALUES OF  $H_1$ ,  $M_1$ ,  $M_2$ ,  $V_1$ , AND  $V_2$  INCLUDING THE EFFECT OF THE AXIAL STRESS.

 $n = 20$ 

| Point. | 1                | 2                  | 3                                                   | 4                                                                                           | 5               | 6                     | 7                     | 8                  | 9                  |
|--------|------------------|--------------------|-----------------------------------------------------|---------------------------------------------------------------------------------------------|-----------------|-----------------------|-----------------------|--------------------|--------------------|
|        | $H$<br>Table VI. | $H_1$<br>$0.9335H$ | $C'$<br>$H_1 \frac{\Sigma y \Delta}{\Sigma \Delta}$ | $D$<br>Table V.                                                                             | $E$<br>Table V. | $M_1$<br>$C' - D - E$ | $M_2$<br>$C' - D + E$ | $V_1$<br>Table VI. | $V_2$<br>Table VI. |
| 1      | 0                | 0                  | 0                                                   | 0.670                                                                                       | 0.670           | -1.340                | 0                     | 1.000              | 0                  |
| 2      | 0.029            | 0.027              | 0.221                                               | 2.003                                                                                       | 1.958           | -3.740                | +0.176                | 0.998              | 0.002              |
| 3      | 0.106            | 0.099              | 0.807                                               | 3.318                                                                                       | 3.140           | -5.651                | +0.629                | 0.992              | 0.008              |
| 4      | 0.256            | 0.239              | 1.949                                               | 4.597                                                                                       | 4.110           | -6.758                | +1.462                | 0.978              | 0.022              |
| 5      | 0.515            | 0.481              | 3.921                                               | 5.788                                                                                       | 4.656           | -6.523                | +2.789                | 0.948              | 0.052              |
| 6      | 0.850            | 0.793              | 6.473                                               | 6.865                                                                                       | 4.728           | -5.120                | +4.336                | 0.901              | 0.099              |
| 7      | 1.213            | 1.132              | 9.237                                               | 7.810                                                                                       | 4.354           | -2.927                | +5.781                | 0.837              | 0.163              |
| 8      | 1.563            | 1.459              | 11.902                                              | 8.586                                                                                       | 3.540           | -0.224                | +6.856                | 0.757              | 0.243              |
| 9      | 1.847            | 1.724              | 14.065                                              | 9.150                                                                                       | 2.328           | +2.587                | +7.243                | 0.662              | 0.338              |
| 10     | 2.007            | 1.874              | 15.283                                              | 9.446                                                                                       | 0.813           | +5.024                | +6.650                | 0.555              | 0.445              |
|        | 8.386<br>2       | 7.828<br>2         | 63.858                                              | $C' = H_1(8.1576) = H(8.1576)(0.9335)$<br>$= H(7.6151)$<br>$H_1 = (1 - 0.0665)H$ , Table IX |                 |                       |                       |                    |                    |
|        | 16.772           | 15.656             |                                                     |                                                                                             |                 |                       |                       |                    |                    |

TABLE XIX.

THE VALUES OF  $H_1$ ,  $M_1$ , AND  $V_1$  FOR LOADINGS WHICH PRODUCE MAXIMUM FIBER STRESSES AT SECTIONS 0, 6, AND THE CROWN, INCLUDING THE EFFECT OF THE AXIAL STRESS.

 $n = 20$ 

| 1                        | 2           | 3       | 4      | 5                        | 6      | 7     |
|--------------------------|-------------|---------|--------|--------------------------|--------|-------|
| Loading.                 | Unit Loads. |         |        | Dead Load and Live Load. |        |       |
|                          | $H_1$       | $M_1$   | $V_1$  | $H_1$                    | $M_1$  | $V_1$ |
| Dead load.....           |             |         |        | 21564                    | -16718 | 22100 |
| Live load 1-7.....       | 2.771       | -32.059 | 6.654  | 1485                     | -17184 | 3566  |
| Live load 8-1'.....      | 12.882      | +43.310 | 3.346  | 6906                     | +23214 | 1793  |
| Live load 1-8+3'-1'..... | 4.356       | -31.478 | 7.411  | 2333                     | -16873 | 3972  |
| Live load 9'-4'.....     | 11.300      | +42.729 | 2.589  | 6057                     | +22903 | 1388  |
| Live load 1-7+7'-1'..... | 5.542       | -16.886 | 7.000  | 2970                     | -9051  | 3752  |
| Live load 8-8'.....      | 10.114      | +28.135 | 3.000  | 5421                     | +15080 | 1608  |
| Live load 1-8+8'-1'..... | 8.460       | -10.254 | 8.000  | 4534                     | -5496  | 4288  |
| Live load 9-9'.....      | 7.196       | +21.503 | 2.000  | 3857                     | +11526 | 1072  |
| Live load 1-1'.....      | 15.656      | +11.251 | 10.000 | 8391                     | +6030  | 5360  |

Live load = 200 pounds per square foot of roadway or 536 pounds per point.

The fields of loading used in this table are the same as found in Tables XI, XII, and XIII, where the effect of the axial stress is neglected. The fields are slightly different when the effect of the axial stress is considered in computing tables corresponding to XI, XII, and XIII.

TABLE XX.

DIRECT DETERMINATION OF THE HORIZONTAL THRUST PRODUCED BY A SYMMETRICAL DEAD LOAD.  $n = 20$

| Point. | 1                    | 2                      | 3                  | 4                                       | 5                                                                                                                                                                                                                                                                                |
|--------|----------------------|------------------------|--------------------|-----------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|        | $P$                  | $\frac{m_x}{\delta x}$ | $B$                | $\frac{m_x}{\delta x} B$                | Remarks.                                                                                                                                                                                                                                                                         |
| 1      | 4710                 | 11100                  | - 4.66             | - 51726                                 | $H = \frac{\sum m_x B}{\sum y B} = \frac{1868793(2.68)(2)}{432.533}$ $H = 23158$ <p>Table I gives the values of <math>B</math> and <math>\sum y B</math>. <math>\delta x = 2.68</math>.</p> <p>If the effect of the axial stress is considered</p> $H_1 = 23158(0.9335) = 21618$ |
| 2      | 3670                 | 28400                  | - 7.71             | - 218964                                |                                                                                                                                                                                                                                                                                  |
| 3      | 2890                 | 42200                  | - 12.00            | - 506400                                |                                                                                                                                                                                                                                                                                  |
| 4      | 2270                 | 53000                  | - 17.40            | - 922200                                |                                                                                                                                                                                                                                                                                  |
| 5      | 1870                 | 61600                  | - 12.30            | - 757680                                |                                                                                                                                                                                                                                                                                  |
| 6      | 1630                 | 68200                  | - 4.64             | + 316448                                |                                                                                                                                                                                                                                                                                  |
| 7      | 1440                 | 73300                  | + 2.17             | - 159061                                |                                                                                                                                                                                                                                                                                  |
| 8      | 1290                 | 76900                  | + 10.70            | + 822830                                |                                                                                                                                                                                                                                                                                  |
| 9      | 1180                 | 79300                  | + 20.00            | + 1586000                               |                                                                                                                                                                                                                                                                                  |
| 10     | 1130                 | 80400                  | + 25.80            | + 2074320                               |                                                                                                                                                                                                                                                                                  |
|        | 22100<br>$V_1 = V_2$ | .....<br>.....         | - 58.71<br>+ 58.67 | - 2773418<br>+ 4642211<br><br>+ 1868793 |                                                                                                                                                                                                                                                                                  |

TABLE XXI.

DIRECT DETERMINATION OF THE MOMENTS  $M_1$  AND  $M_2$  PRODUCED BY A SYMMETRICAL DEAD LOAD.  $n = 20$

| Point. | 1                      | 2           | 3                             | 4                                                                                                                                                                                                                                                                                                                                                                              |
|--------|------------------------|-------------|-------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|        | $\frac{m_x}{\delta x}$ | $\Delta$    | $\frac{m_x}{\delta x} \Delta$ | Remarks.                                                                                                                                                                                                                                                                                                                                                                       |
| 1      | 11100                  | 0.66        | 7326                          | $\delta x = 2.68$ $M_1 = M_2 = H \frac{\sum y \Delta}{\sum \Delta} - \frac{\sum m_x \Delta}{\sum \Delta}$ $H \frac{\sum y \Delta}{\sum \Delta} = 23158(8.1576) = 188913$ $\frac{\sum m_x \Delta}{\sum \Delta} = \frac{17508716}{243.02} \cdot 2.68 = \frac{193084}{M_1 = -4171}$ <p>If the effect of the axial stress is considered</p> $M_1 = M_2 = 176351 - 193084 = -16733$ |
| 2      | 28400                  | 1.51        | 42884                         |                                                                                                                                                                                                                                                                                                                                                                                |
| 3      | 42200                  | 3.42        | 144324                        |                                                                                                                                                                                                                                                                                                                                                                                |
| 4      | 53000                  | 7.89        | 418170                        |                                                                                                                                                                                                                                                                                                                                                                                |
| 5      | 61600                  | 10.41       | 641256                        |                                                                                                                                                                                                                                                                                                                                                                                |
| 6      | 68200                  | 11.96       | 815672                        |                                                                                                                                                                                                                                                                                                                                                                                |
| 7      | 73300                  | 15.22       | 1115626                       |                                                                                                                                                                                                                                                                                                                                                                                |
| 8      | 76900                  | 19.28       | 1482632                       |                                                                                                                                                                                                                                                                                                                                                                                |
| 9      | 79300                  | 24.36       | 1931748                       |                                                                                                                                                                                                                                                                                                                                                                                |
| 10     | 80400                  | 26.80       | 2154720                       |                                                                                                                                                                                                                                                                                                                                                                                |
|        |                        | 121.51<br>2 | 8754358<br>2                  |                                                                                                                                                                                                                                                                                                                                                                                |
|        |                        | 243.02      | 17508716                      |                                                                                                                                                                                                                                                                                                                                                                                |

TABLE XXII.  
THE BENDING MOMENTS  $M_1$  AND  $M_2$  PRODUCED BY UNIT  
LOADS AT POINTS 1-10 INCLUSIVE.  
 $\Sigma m_x \Delta \div \Sigma \Delta$   $n=20$

| Points.  | 1                        | 2           | 3                               | 4                                                                                             |
|----------|--------------------------|-------------|---------------------------------|-----------------------------------------------------------------------------------------------|
|          | $2 \frac{m_x}{\delta x}$ | $\Delta$    | $2 \frac{m_x}{\delta x} \Delta$ | Remarks.                                                                                      |
| 1 and 1' | 10                       | 0.66        | 6.60                            |                                                                                               |
| 2 " 2'   | 28                       | 1.51        | 42.28                           |                                                                                               |
| 3 " 3'   | 44                       | 3.42        | 150.48                          |                                                                                               |
| 4 " 4'   | 58                       | 7.89        | 457.62                          |                                                                                               |
| 5 " 5'   | 70                       | 10.41       | 728.70                          | $\frac{\Sigma m_x \Delta}{\Sigma \Delta} = \frac{10561.44}{243.02} \cdot 1.34 = 58.235.$      |
| 6 " 6'   | 80                       | 11.96       | 956.80                          |                                                                                               |
| 7 " 7'   | 88                       | 15.22       | 1339.36                         |                                                                                               |
| 8 " 8'   | 94                       | 19.28       | 1812.32                         | $H \frac{\Sigma v \Delta}{\Sigma \Delta} = \frac{16.772(8.1576)}{2} = 68.409$                 |
| 9 " 9'   | 98                       | 24.36       | 2387.28                         |                                                                                               |
| 10 " 10' | 100                      | 26.80       | 2680.00                         |                                                                                               |
|          | Constant when $n=20$     | 121.51<br>2 | 10561.44                        | $H \frac{\Sigma y \Delta}{\Sigma \Delta} - \frac{\Sigma m_x \Delta}{\Sigma \Delta} = +10.174$ |
|          |                          | 243.02      |                                 |                                                                                               |

TABLE XXIII.  
THE BENDING MOMENTS  $M_1$  AND  $M_2$  PRODUCED BY UNIT  
LOADS AT POINTS 1-10 INCLUSIVE.  
 $\Sigma m_x \Delta (z-n)n \div (-2 \sum_{l/2}^0 (z-n)^2 \Delta).$   $n=20$

| Points.  | 1                                                                           | 2             | 3                                    | 4                                                    |
|----------|-----------------------------------------------------------------------------|---------------|--------------------------------------|------------------------------------------------------|
|          | $2 \frac{m_x}{\delta x}$                                                    | $(z-n)\Delta$ | $2 \frac{m_x}{\delta x} (z-n)\Delta$ | Remarks.                                             |
| 1 and 1' | 5                                                                           | - 12.54       | - 62.70                              |                                                      |
| 2 " 2'   | 13                                                                          | - 25.67       | - 333.71                             | $\Sigma m_x \Delta (z-n)n = -14647.69(1.34)(20)$     |
| 3 " 3'   | 19                                                                          | - 51.30       | - 974.70                             | $\frac{l/2}{-2 \sum (z-n)^2 \Delta} = -12959.5$      |
| 4 " 4'   | 23                                                                          | -102.57       | - 2359.11                            |                                                      |
| 5 " 5'   | 25                                                                          | -114.51       | - 2862.75                            | $= +30.291. \quad \delta x = 2.68.$                  |
| 6 " 6'   | 25                                                                          | -107.64       | - 2691.00                            | $M_1 = 68.409 - 58.235 - 30.291$                     |
| 7 " 7'   | 23                                                                          | -106.54       | - 2450.42                            | $= -20.117$                                          |
| 8 " 8'   | 19                                                                          | - 96.40       | - 1831.60                            | $M_2 = 68.409 - 58.235 + 30.291$                     |
| 9 " 9'   | 13                                                                          | - 72.90       | - 947.70                             | $= +40.465$                                          |
| 10 " 10' | 5                                                                           | - 26.80       | - 134.00                             | If the effect of the axial stress is considered:     |
|          | Difference of symmetrical values of $2 \frac{m_x}{\delta x} \div \delta x.$ | .....         | -14647.69                            | $M_1 = 63.858 - 58.235 - 30.291$                     |
|          |                                                                             |               |                                      | $= -24.668$                                          |
|          |                                                                             |               |                                      | $M_2 = 63.858 - 58.235 + 30.291$                     |
|          |                                                                             |               |                                      | $= +35.914$                                          |
|          |                                                                             |               |                                      | $V_1 = \frac{M_2 - M_1}{53.6} + R_1 = 1.130 + 7.500$ |
|          |                                                                             |               |                                      | $= 8.630$                                            |



TABLE I Q.  
HORIZONTAL THRUSTS PRODUCED BY UNIT HORIZONTAL LOADS.

| Point. | 1             | 2    | 3               | 4              | 5                | 6                                     | 7                      |
|--------|---------------|------|-----------------|----------------|------------------|---------------------------------------|------------------------|
|        | $\sum B$<br>0 | $b$  | $b \sum B$<br>0 | $\sum yB$<br>0 | Cols.<br>(4)-(3) | Col. (5)<br>$\frac{\sum yB}{\sum yB}$ | $h_1'$<br>1 - Col. (6) |
| 1      | -4.658        | 1.10 | -5.124          | -5.124         | 0                | 0                                     | 0                      |
| 2      | -12.370       | 3.05 | -37.729         | -28.649        | 9.080            | 0.0209                                | 0.9791                 |
| 3      | -24.332       | 4.66 | -113.387        | -84.391        | 28.996           | 0.0670                                | 0.9330                 |
| 4      | -41.750       | 5.95 | -248.413        | -188.029       | 60.384           | 0.1396                                | 0.8604                 |
| 5      | -54.009       | 6.98 | -376.983        | -273.595       | 103.388          | 0.2390                                | 0.7610                 |
| 6      | -58.645       | 7.77 | -455.672        | -309.614       | 146.058          | 0.3377                                | 0.6623                 |
| 7      | -56.478       | 8.30 | -468.767        | -291.626       | 177.141          | 0.4095                                | 0.5905                 |
| 8      | -45.828       | 8.71 | -399.162        | -198.862       | 200.300          | 0.4631                                | 0.5369                 |
| 9      | -25.794       | 8.98 | -231.630        | -18.959        | 212.641          | 0.4916                                | 0.5084                 |
| 10     | -0.002        | 9.12 | 0               | +216.266       | 216.266          | 0.5000                                | 0.5000                 |
|        | See Table I   |      |                 | See Table I    |                  | $h_2'$                                |                        |

$$h_1' = \left\{ 1 - \frac{\frac{a}{y} \sum B - b \sum B}{\sum yB} \right\} \quad \text{The effect of the axial stress neglected. } h_2' = 1 - h_1'.$$

TABLE II Q.  
BENDING MOMENTS  $M_1$  AND  $M_2$  PRODUCED BY UNIT HORIZONTAL LOADS.—TERM G.

| Point. | 1                           | 2    | 3                             | 4                    | 5                | 6                             | 7     |
|--------|-----------------------------|------|-------------------------------|----------------------|------------------|-------------------------------|-------|
|        | $\frac{l/2}{a} \sum \Delta$ | $b$  | $\frac{l/2}{a} b \sum \Delta$ | $\sum y \Delta$<br>0 | Cols.<br>(3)+(4) | $\sum y \Delta -$<br>Col. (5) | G     |
| 1      | 120.85                      | 1.10 | 132.935                       | 0.726                | 133.661          | 1848.799                      | 7.608 |
| 2      | 119.34                      | 3.05 | 363.987                       | 5.332                | 369.319          | 1613.141                      | 6.638 |
| 3      | 115.92                      | 4.66 | 540.187                       | 21.269               | 561.456          | 1421.004                      | 5.847 |
| 4      | 108.03                      | 5.95 | 642.778                       | 68.215               | 710.993          | 1271.467                      | 5.232 |
| 5      | 97.62                       | 6.98 | 681.388                       | 140.877              | 822.264          | 1160.196                      | 4.774 |
| 6      | 85.66                       | 7.77 | 688.888                       | 233.806              | 922.694          | 1059.766                      | 4.361 |
| 7      | 70.44                       | 8.30 | 584.652                       | 360.132              | 944.784          | 1037.676                      | 4.269 |
| 8      | 51.16                       | 8.71 | 445.604                       | 528.061              | 973.664          | 1008.796                      | 4.151 |
| 9      | 26.80                       | 8.98 | 240.664                       | 746.814              | 987.478          | 994.982                       | 4.094 |
| 10     | 0                           | 9.12 | 0                             | 991.230              | 991.230          | 991.230                       | 4.079 |
|        | See Table I                 |      |                               | See Table I          |                  |                               |       |

$$G = \frac{\sum m_x' \Delta}{\sum \Delta} = \frac{\sum y \Delta - b \sum \Delta - \frac{a}{0} \sum y \Delta}{\sum \Delta}.$$

$\sum y \Delta = 1982.46$  (Table I).  $\sum \Delta = 243.02$  (Table I).



TABLE III Q.  
BENDING MOMENTS  $M_1$  AND  $M_2$  PRODUCED BY UNIT HORIZONTAL LOADS.

TERM H.

 $n = 20$ 

| Point. | 1                             | 2    | 3                                                                                                                                                                                                                                                                                                                                                                                                                                   | 4     | 5          | 6               | 7                         |
|--------|-------------------------------|------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|------------|-----------------|---------------------------|
|        | $(n-z)^2 \Delta$              | $b$  | $b \sum_0^{l/2} (n-z)^2 \Delta$                                                                                                                                                                                                                                                                                                                                                                                                     | $z-n$ | $y \Delta$ | $y(z-n) \Delta$ | $\sum_0^a (z-n) y \Delta$ |
| 1      | 238.26                        | 1.10 | 7127.725                                                                                                                                                                                                                                                                                                                                                                                                                            | -19   | 0.726      | -13.794         | -13.974                   |
| 2      | 436.39                        | 3.05 | 19763.238                                                                                                                                                                                                                                                                                                                                                                                                                           | -17   | 4.606      | -78.302         | -92.096                   |
| 3      | 769.50                        | 4.66 | 30195.635                                                                                                                                                                                                                                                                                                                                                                                                                           | -15   | 15.937     | -239.055        | -337.151                  |
| 4      | 1333.41                       | 5.95 | 38544.513                                                                                                                                                                                                                                                                                                                                                                                                                           | -13   | 46.946     | -610.298        | -941.449                  |
| 5      | 1259.61                       | 6.98 | 45228.655                                                                                                                                                                                                                                                                                                                                                                                                                           | -11   | 72.662     | -799.282        | -1740.731                 |
| 6      | 968.76                        | 7.77 | 50347.658                                                                                                                                                                                                                                                                                                                                                                                                                           | -9    | 92.929     | -836.361        | -2577.092                 |
| 7      | 745.78                        | 8.30 | 53781.925                                                                                                                                                                                                                                                                                                                                                                                                                           | -7    | 126.326    | -884.282        | -3461.374                 |
| 8      | 482.00                        | 8.71 | 56438.623                                                                                                                                                                                                                                                                                                                                                                                                                           | -5    | 167.929    | -839.645        | -4301.019                 |
| 9      | 219.24                        | 8.98 | 58188.155                                                                                                                                                                                                                                                                                                                                                                                                                           | -3    | 218.753    | -656.259        | -4957.278                 |
| 10     | 26.80                         | 9.12 | 59095.320                                                                                                                                                                                                                                                                                                                                                                                                                           | -1    | 244.416    | -244.416        | -5201.694                 |
|        | $\sum_0^{l/2} (n-z)^2 \Delta$ |      | $H = \left\{ b \sum_0^{l/2} (n-z)^2 \Delta - n \sum_0^a (z-n) y \Delta + nb \sum_0^a (z-n) \Delta \right\}$ <div style="display: flex; justify-content: space-around; margin-top: -10px;"> <span>Col. 3.</span> <span>Col. 8.</span> <span>Col. 11.</span> </div> $\div \left\{ -2 \sum_0^{l/2} (z-n)^2 \Delta \right\}$ <div style="display: flex; justify-content: space-around; margin-top: -10px;"> <span>Col. 1.</span> </div> |       |            |                 |                           |

TABLE III Q.—Continued.

| Point. | 8                           | 9              | 10                      | 11                         | 12                                  | 13     |
|--------|-----------------------------|----------------|-------------------------|----------------------------|-------------------------------------|--------|
|        | $n \sum_0^a (z-n) y \Delta$ | $(z-n) \Delta$ | $\sum_0^a (z-n) \Delta$ | $nb \sum_0^a (z-n) \Delta$ | Cols.<br>(3)-(8)+(11)<br>Numerator. | H      |
| 1      | -275.880                    | -12.54         | -12.54                  | -275.880                   | 7127.725                            | -0.550 |
| 2      | -1841.920                   | -25.67         | -38.21                  | -2330.810                  | 19274.348                           | -1.487 |
| 3      | -6623.020                   | -51.30         | -89.51                  | -8342.332                  | 28476.323                           | -2.197 |
| 4      | -18828.980                  | -102.57        | -192.08                 | -22857.520                 | 34525.973                           | -2.664 |
| 5      | -34814.620                  | -114.51        | -306.59                 | -42799.964                 | 37243.311                           | -2.874 |
| 6      | -51541.840                  | -107.64        | -414.23                 | -64371.342                 | 37518.156                           | -2.895 |
| 7      | -69227.480                  | -106.54        | -520.77                 | -86447.820                 | 36561.585                           | -2.821 |
| 8      | -86020.380                  | -96.40         | -617.17                 | -107511.014                | 34947.989                           | -2.697 |
| 9      | -99145.560                  | -72.90         | -690.07                 | -123936.572                | 33397.143                           | -2.577 |
| 10     | -104033.880                 | -26.80         | -716.87                 | -130757.088                | 32372.112                           | -2.498 |

REMARKS.—The values given in Col. 13 are symmetrical about the center of the span.

The value of H for point 1 equals one-half the ordinate y for this point.

TABLE IV Q.  
BENDING MOMENTS  $M_1$  AND  $M_2$  PRODUCED BY UNIT  
HORIZONTAL LOADS.

| Point. | 1      | 2                                            | 3                                          | 4     | 5      | 6      | 7      | Point. | 8     | 9      |
|--------|--------|----------------------------------------------|--------------------------------------------|-------|--------|--------|--------|--------|-------|--------|
|        | $h_1'$ | $\frac{h_1' \Sigma y \Delta}{\Sigma \Delta}$ | $\frac{\Sigma m_x' \Delta}{\Sigma \Delta}$ | F-G   | H      | $M_1$  | $M_2$  |        | $M_1$ | $M_2$  |
|        |        | F                                            | G                                          |       |        |        |        |        |       |        |
| 1      | 1.000  | 8.158                                        | 7.608                                      | 0.550 | -0.550 | *1.100 | 0      | 1'     | 0     | -1.100 |
| 2      | 0.997  | 7.986                                        | 6.638                                      | 1.348 | -1.487 | 2.835  | -0.139 | 2'     | 0.139 | -2.835 |
| 3      | 0.933  | 7.611                                        | 5.847                                      | 1.764 | -2.197 | 3.961  | -0.433 | 3'     | 0.433 | -3.961 |
| 4      | 0.860  | 7.016                                        | 5.232                                      | 1.784 | -2.664 | 4.448  | -0.880 | 4'     | 0.880 | -4.448 |
| 5      | 0.761  | 6.208                                        | 4.774                                      | 1.434 | -2.874 | 4.308  | -1.440 | 5'     | 1.440 | -4.308 |
| 6      | 0.662  | 5.400                                        | 4.361                                      | 1.039 | -2.895 | 3.934  | -1.856 | 6'     | 1.856 | -3.934 |
| 7      | 0.591  | 4.821                                        | 4.269                                      | 0.552 | -2.821 | 3.373  | -2.269 | 7'     | 2.269 | -3.373 |
| 8      | 0.537  | 4.381                                        | 4.151                                      | 0.230 | -2.697 | 2.927  | -2.467 | 8'     | 2.467 | -2.927 |
| 9      | 0.508  | 4.144                                        | 4.094                                      | 0.050 | -2.577 | 2.627  | -2.527 | 9'     | 2.527 | -2.627 |
| 10     | 0.500  | 4.078                                        | 4.079                                      | 0     | -2.498 | 2.498  | -2.498 | 10'    | 2.498 | -2.498 |

$$M_1 = F - G - H, \quad M_2 = F - G + H.$$

The values of **F-G** in Col. 4 are symmetrical in magnitude about the center of the span, but the values at symmetrical points are of opposite signs.

\* This value of  $M_1$  equals the ordinate  $y$  for point 1. The effect of the axial stress is neglected.

TABLE V Q.  
VALUES OF  $V_1$  AND  $V_2$  PRODUCED BY UNIT HORIZONTAL  
LOADS.

| Point. | 1     | 2               | 3       | 4        | Point. | 5                                                   |
|--------|-------|-----------------|---------|----------|--------|-----------------------------------------------------|
|        | $b$   | $M_2 - M_1 + b$ | $V_1$   | $V_2$    |        | Remarks.                                            |
| 1      | 1.100 | 0               | 0       | 0        | 1'     | $V_1 = \frac{M_2 - M_1 + b}{l} = -V_2$ $l = 53'.6.$ |
| 2      | 3.050 | 0.076           | 0.00142 | -0.00142 | 2'     |                                                     |
| 3      | 4.660 | 0.266           | 0.00496 | -0.00496 | 3'     |                                                     |
| 4      | 5.950 | 0.622           | 0.0116  | -0.0116  | 4'     |                                                     |
| 5      | 6.980 | 1.232           | 0.0230  | -0.0230  | 5'     |                                                     |
| 6      | 7.770 | 1.980           | 0.0370  | -0.0370  | 6'     |                                                     |
| 7      | 8.300 | 2.658           | 0.0496  | -0.0496  | 7'     |                                                     |
| 8      | 8.710 | 3.316           | 0.0619  | -0.0619  | 8'     |                                                     |
| 9      | 8.980 | 3.826           | 0.0714  | -0.0714  | 9'     |                                                     |
| 10     | 9.120 | 4.124           | 0.0770  | -0.0770  | 10'    |                                                     |
|        |       |                 | $V_2$   | $V_1$    |        |                                                     |

## FORM A.

THE DIRECT COMPUTATION OF THE HORIZONTAL THRUST  
H. EFFECT OF AXIAL STRESS NEGLECTED.

| Point. | 1  | 2             | 3              | 4                                      | 5                             | 6     | 7                                       | 8                            | 9                                            | 10                           | 11                                            | 12                                |
|--------|----|---------------|----------------|----------------------------------------|-------------------------------|-------|-----------------------------------------|------------------------------|----------------------------------------------|------------------------------|-----------------------------------------------|-----------------------------------|
|        |    |               | 1 × 2          |                                        | 2 × 4                         | 3 × 4 |                                         | 5 × 7                        |                                              | 5 × 9                        |                                               | 5 × 11                            |
|        | y  | Δ             | yΔ             | $y - \frac{\sum y\Delta}{\sum \Delta}$ | B                             | yB    | Dead Load<br>$m_x \frac{2}{\partial x}$ | $m_x B \frac{2}{\partial x}$ | Unit Loads 1-8<br>$m_x \frac{2}{\partial x}$ | $m_x B \frac{2}{\partial x}$ | Unit Loads 1-11<br>$m_x \frac{2}{\partial x}$ | $m_x B \frac{2}{\partial x}$      |
| 1      | .. | ..            | ..             | ..                                     | ..                            | ..    | ..                                      | ..                           | 8                                            | ..                           | 20                                            | ..                                |
| 2      | .. | ..            | ..             | ..                                     | ..                            | ..    | ..                                      | ..                           | 22                                           | ..                           | 56                                            | ..                                |
| 3      | .. | ..            | ..             | ..                                     | ..                            | ..    | ..                                      | ..                           | 34                                           | ..                           | 88                                            | ..                                |
| 4      | .. | ..            | ..             | ..                                     | ..                            | ..    | ..                                      | ..                           | 44                                           | ..                           | 116                                           | ..                                |
| 5      | .. | ..            | ..             | ..                                     | ..                            | ..    | ..                                      | ..                           | 52                                           | ..                           | 140                                           | ..                                |
| 6      | .. | ..            | ..             | ..                                     | ..                            | ..    | ..                                      | ..                           | 58                                           | ..                           | 160                                           | ..                                |
| 7      | .. | ..            | ..             | ..                                     | ..                            | ..    | ..                                      | ..                           | 62                                           | ..                           | 176                                           | ..                                |
| 8      | .. | ..            | ..             | ..                                     | ..                            | ..    | ..                                      | ..                           | 64                                           | ..                           | 188                                           | ..                                |
| 9      | .. | ..            | ..             | ..                                     | ..                            | ..    | ..                                      | ..                           | 64                                           | ..                           | 196                                           | ..                                |
| 10     | .. | ..            | ..             | ..                                     | ..                            | ..    | ..                                      | ..                           | 64                                           | ..                           | 200                                           | ..                                |
|        |    | 2             | 2              |                                        | Algebraic sum should be zero. |       | 2                                       |                              |                                              |                              |                                               |                                   |
|        |    | $\sum \Delta$ | $\sum y\Delta$ |                                        |                               |       | $\sum yB$                               |                              | $\sum m_x B \frac{2}{\partial x}$            |                              | $\sum m_x B \frac{2}{\partial x}$             | $\sum m_x B \frac{2}{\partial x}$ |

$$H = \frac{\sum_x mB}{\sum_y B} \cdot \frac{\delta x}{2} = \frac{\text{span}}{2n} \quad \text{Cols. 9 and 11 are constant for } n=20.$$

For Unit Loads:

$H$  for loads 1-8 and  $8'-1' = 2H$  for loads 1-8.

$H$  for loads 9-11 =  $H$  for loads 1-11 less  $H$  for loads 1-8.

$H$  for loads 9-9' =  $H$  for loads 1-11 less  $H$  for loads 1-8 and 8'-1'.

NOTE.—In columns 7, 9, and 11 symmetrical values of  $m_x \frac{2}{\partial x}$  are combined by addition.

NOTE.—If the effect of the axial stress is not neglected correct the values of  $H$  found by the relation:

$$H_1 = \left\{ 1 - \frac{\sum \frac{(\delta x)^2}{F_x \delta s}}{\sum yB + \sum \frac{(\delta x)^2}{F_x \delta s}} \right\} \quad H = \text{horizontal thrust including the effect of the axial stress.}$$



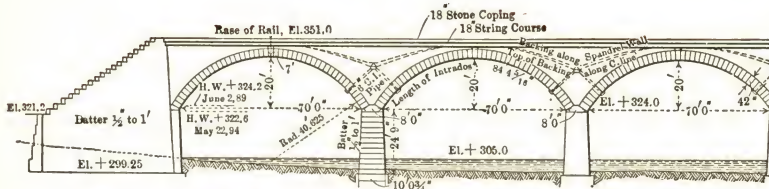
FORM B.—THE DIRECT COMPUTATION OF THE BENDING MOMENTS  $M_1$  AND  $M_2$ .  $n = 20$



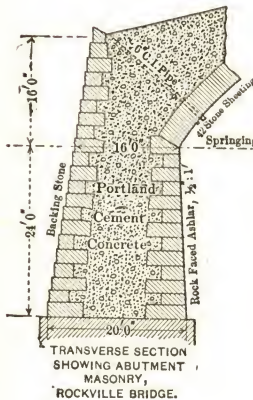
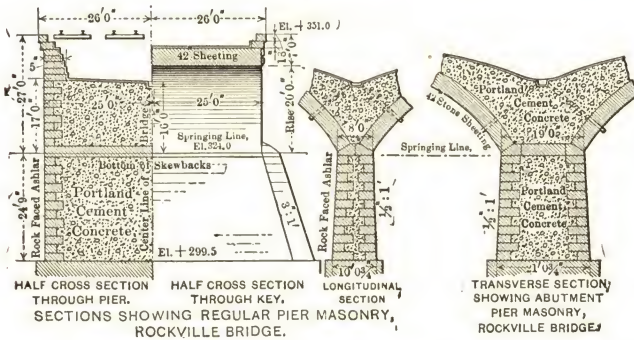
## CHAPTER V.

### TYPICAL ARCHES.

A FEW typical bridges will be illustrated in this chapter which will clearly show that, as ordinarily constructed, the arch ring proper is heavily reinforced either by masonry or concrete backing or masonry side walls. Since this masonry does not readily follow the arch ring if it sinks, the actual dead load is never more than the dead weight of the material above the ring; and since the passive resistance of this masonry against moving upward is large in case the arch ring has such a tendency, it is evident that any ring which is stable under the elastic theory must be stable in the structures as built. Furthermore, experience teaches that temperature stresses may be ignored in stone arches well backed, as is customary. A recording thermometer placed in the ring of a reinforced-concrete bridge having earth filling indicated that the total range of temperature change did not exceed about  $20^{\circ}$  F. in some ten or twelve months. All rings without backing should be designed to resist a change of temperature of about  $\pm 35^{\circ}$  F. In rings like that of the Luxemburg bridge full account of temperature must be considered, the range approaching that for steel.



PART SIDE ELEVATION OF 3,820 FT. STONE ARCH BRIDGE FOR THE PENNSYLVANIA R.R. AT ROCKVILLE, PA.

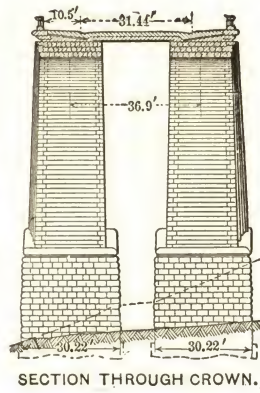
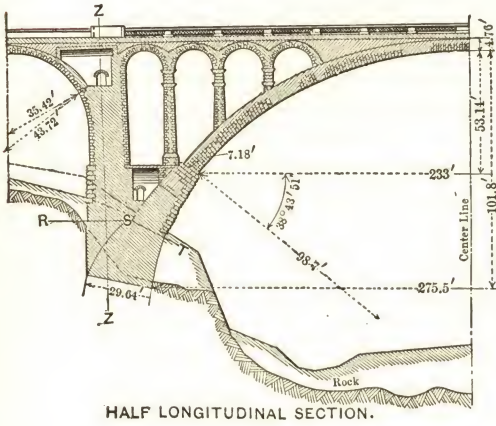
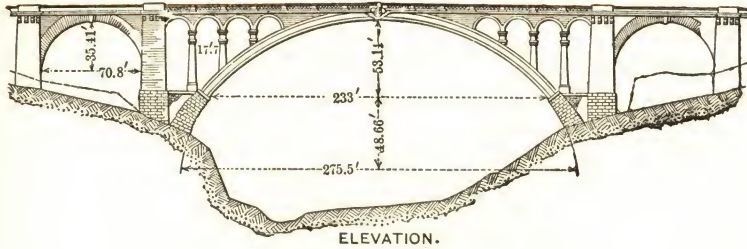


**90. The Rockville Stone Arch Bridge.**—This is typical of a large number of arches recently constructed by the Pennsylvania R.R. The arch ring is backed with Portland-cement concrete to such an extent that it is increased in thickness nearly three times near the springing line. (Eng. News, May 10, 1900.)

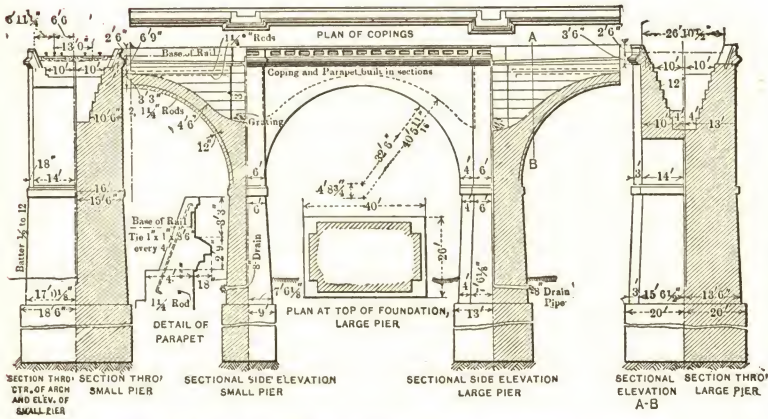
**91. The Bellefield Stone Arch Bridge, Pittsburg, Pa.**—In this bridge the outside spandrels are of solid masonry. Inside there are six longitudinal walls reinforced with three lateral walls. The lateral walls do not support any vertical load, as they stop at the springing of the





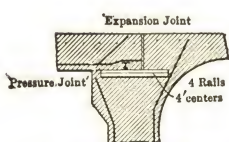
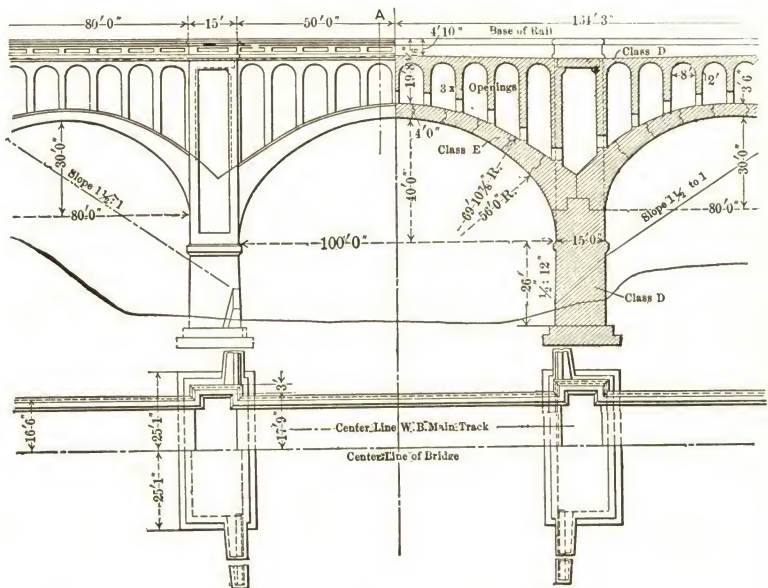


LUXEMBURG BRIDGE

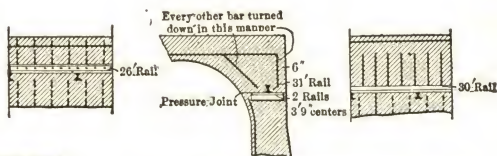




94. **Vermillion River Plain-concrete Arch Bridge.**—This bridge is composed of three spans. The entire loading above the ring is supported by lateral walls which makes the application of the elastic theory quite rational.



DETAIL OF EXPANSION JOINT OVER PIERS



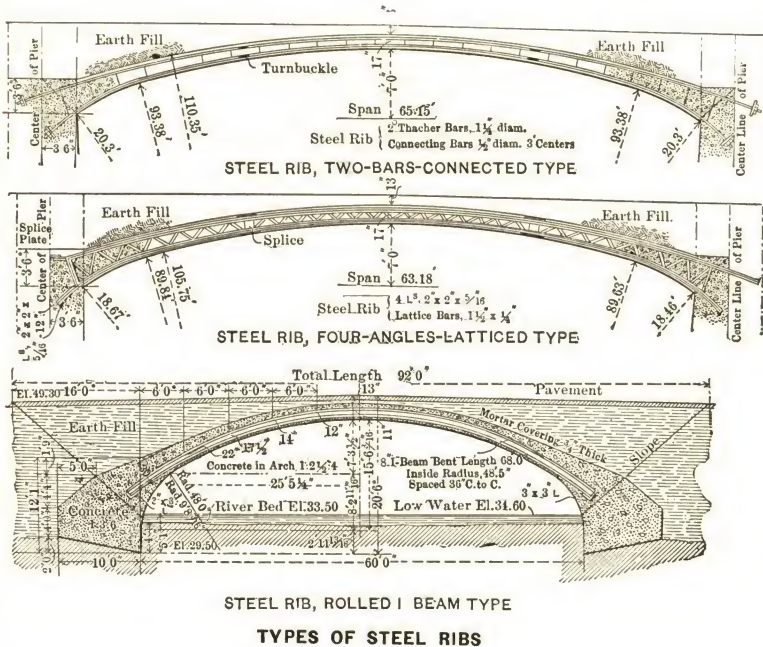
DETAIL OF EXPANSION JOINT OVER ABUTS

#### CONCRETE ARCH BRIDGE OVER SALT FORK OF THE VERMILION RIVER

The ring was designed without reinforcement, but when built the entire concrete work was reinforced with steel bars. This reinforcement is shown in the *Railroad Gazette*, Oct. 27, 1905.

95. **Steel Reinforcement in the Form of Ribs.**—Where the steel reinforcement has been concentrated in concrete

rings the three types shown below have been successfully used. The steel is usually assumed to take the entire bending moment. The two upper types were used



inforced with rolled rods, some plain and some "deformed."

**97. Area of Steel Reinforcement.**—The amount of steel is seldom less than 0.6% or greater than 1% of the area of the ring at the crown, regardless of the type of reinforcement employed.

## APPENDIX A.

TABLE I.

## PHYSICAL PROPERTIES OF STONE AND CONCRETE.

## BUILDING STONES.

C=Ultimate crushing strength in pounds per square inch. Test Specimens: Cubes.

$R$  = Cross-bending fiber stress in pounds per square inch.

S=Ultimate shearing strength in pounds per square inch.

$e$  = Expansion per degree F. Determined Wet.

$E$  = Young's modulus in pounds per square inch. Compression.

 $\tau$  = Limits between which  $E$  was determined.

$W$  = Weight in pounds per cubic foot.

Compiled from Tests of Metals and Other Materials, etc. made at Watertown Arsenal, Mass.

GRANITE.

|                 | Color, Name, Source, etc.                        | C<br>10 <sup>2</sup> . | R<br>10 <sup>2</sup> . | S<br>10 <sup>2</sup> . | e *<br>10 <sup>-8</sup> . | E<br>10 <sup>5</sup> . | r<br>10 <sup>2</sup> . | W   |
|-----------------|--------------------------------------------------|------------------------|------------------------|------------------------|---------------------------|------------------------|------------------------|-----|
| Dark.<br>Light. | Braddock, near Little Rock, Ark.....             | 242                    | 12                     | 24                     | 324                       | 71                     |                        |     |
|                 | Exeter, Tulare Co., Cal. ....                    | 196                    | 16                     | 21                     | 341                       | 71                     | 1-50                   | 156 |
| Red.            | Platte Cañon, Colorado.....                      | 226                    | 19                     | 24                     | 401                       |                        |                        |     |
|                 | Branford, Conn. ....                             | 146                    |                        |                        |                           |                        |                        | 164 |
|                 | Pine Mt., Quarry, Lithonia, Ga. . . . .          | 157                    | 12                     |                        | 398                       | 57                     | 1-50                   | 161 |
|                 | Stone Troy, N. H. ....                           | 277                    |                        | 18                     | 398                       | 83                     | 10-30                  | 162 |
|                 | " " "                                            | 209                    | 26                     |                        | 375                       |                        |                        |     |
|                 | " " "                                            |                        | 22                     |                        | 337                       | 48                     | 1-50                   |     |
| Black.          | Maine. ....                                      | 262                    |                        | 22                     | 337                       | 45                     | 10-30                  | 165 |
| Light.          | Mt. Waldo Quarry, Frankfort, Me. . . . .         | 210                    |                        |                        |                           |                        |                        |     |
|                 | White Rock Mt., Millbridge, Me. ....             | 322                    |                        |                        |                           |                        |                        |     |
|                 | Cape Ann, Rockport, Mass. ....                   | 199                    | 20                     | 28                     | 400                       | 98                     | 1-50                   |     |
|                 | " " " " .....                                    | 242                    | 24                     | 25                     |                           | 65                     | 10-30                  |     |
|                 | " " " " .....                                    | 203                    |                        |                        |                           |                        |                        |     |
|                 | Pigeon Hill, Rockport, Mass. ....                | 173                    |                        |                        |                           |                        |                        |     |
|                 | Quincy, Mass. ....                               | 107                    | 24                     | 15                     |                           | 67                     | 10-20                  | 162 |
| Pink.           | Milford, Mass. ....                              | 130                    | 20                     |                        | 381                       | 68                     | 10-30                  |     |
|                 | " " " " .....                                    | 238                    |                        | 26                     | 418                       | 77                     | 10-20                  | 163 |
|                 | " " " " .....                                    | 190                    | 17                     | 18                     |                           | 51                     | 10-20                  | 162 |
|                 | " " " " .....                                    |                        | 15                     |                        | 417                       | 53                     | 1-50                   | 163 |
|                 | " " " " .....                                    | 90                     | 21                     | 24                     | 408                       | 76                     | 10-30                  |     |
|                 | " " " " .....                                    | 252                    |                        |                        |                           |                        |                        |     |
| Pink.           | Ortonville, Minn. ....                           | 204                    |                        |                        |                           |                        |                        |     |
| Mottled.        | Rockville, Stearns Co., Minn. ....               | 181                    | 13                     | 19                     | 397                       | 94                     | 1-50                   |     |
| Red.            | Sioux Falls, Minn. ....                          | 182                    | 12                     | 21                     | 380                       | 60                     | 1-50                   |     |
| Light.          | Korah Station, Va. ....                          | 234                    | 16                     | 27                     | 431                       |                        |                        |     |
|                 | Broad Rock Quarry, Chesterfield Co., Va. . . . . | 154                    | 17                     | 21                     | 402                       |                        |                        |     |
|                 | Snoqualmie District, Wash. ....                  | 198                    |                        |                        |                           |                        |                        |     |

\* The quantities in this column are to be divided by 100,000,000.





TABLE I.—PHYSICAL PROPERTIES OF STONE AND CONCRETE—(Continued).

## LIMESTONE.

| Color, Name, Source, etc. |                                          | C<br>10 <sup>2</sup> . | R<br>10 <sup>2</sup> . | S<br>10 <sup>2</sup> . | e   | E<br>10 <sup>5</sup> . | r<br>10 <sup>2</sup> . | W   |
|---------------------------|------------------------------------------|------------------------|------------------------|------------------------|-----|------------------------|------------------------|-----|
| Light Buff.               | Ft. Riley, Kansas. . . . .               | 32                     | 5                      | 10                     | 300 |                        |                        |     |
|                           | Junction City, Kansas. . . . .           | 32                     |                        |                        |     |                        |                        |     |
|                           | Mt. Vernon, Ky. . . . .                  | 60                     | 13                     | 12                     | 464 | 40                     | 1-20                   |     |
|                           | Bowling Green, Ky. . . . .               | 60                     | 11                     | 12                     | 89  | 93                     | 1-20                   |     |
|                           | Oolitic. . . . .                         | 62                     |                        |                        |     |                        |                        |     |
|                           | Hopkinsville, Ky., Oolitic. . . . .      | 78                     |                        |                        |     |                        |                        |     |
|                           | Bowling Green, Ky., Oolitic. . . . .     | 150                    |                        |                        |     |                        |                        |     |
|                           | Spring Ledge, Mt. Vernon, Ky. . . . .    | 76                     |                        | 17                     | 464 | 32                     | 10-20                  | 139 |
|                           | Kelly Island, Lake Erie. . . . .         | 122                    |                        |                        |     |                        |                        |     |
|                           | Kasota, Minn. . . . .                    | 108                    |                        |                        |     |                        |                        |     |
| Pink.<br>Gray.<br>Buff.   | Wasioja, Dodge Co., Minn. . . . .        | 50                     | 3                      | 12                     | 217 |                        |                        |     |
|                           | Carthage, Mo., Quality No. 1. . . . .    | 40                     |                        | 11                     |     |                        |                        |     |
|                           | 2. . . . .                               | 137                    |                        |                        |     |                        |                        |     |
|                           | Yammerthal Flint, Buffalo, N. Y. . . . . | 162                    |                        |                        |     |                        |                        |     |
|                           | Isle La Motte, Vt. . . . .               | 237                    | 17                     | 21                     |     |                        |                        |     |
|                           |                                          | 146                    | 16                     | 22                     | 219 | 147                    | 1-40                   |     |
|                           |                                          |                        |                        |                        |     |                        |                        |     |

## MARBLE.

|                          |                                              |     |    |    |     |       |       |     |
|--------------------------|----------------------------------------------|-----|----|----|-----|-------|-------|-----|
| Fossiliferous.           | St. Joe, Searcy Co, Ark. . . . .             | 103 | 11 | 10 | 219 | 82    | 1-40  |     |
|                          | Chocolate. . . . .                           | 123 | 16 | 10 | 107 | 107   | 1-30  |     |
| Marble Hill, Ga. . . . . |                                              | 115 | 10 | 13 | 202 | 91    | 10-30 | 169 |
|                          |                                              |     | 10 | 13 | 103 | 55    | 1-40  | 168 |
|                          | Tate, Ga., Creole Quarry. . . . .            |     | 8  |    | 41  | 41    | 1-40  | 169 |
|                          | " " Cherokee Quarry. . . . .                 | 135 |    | 14 | 69  | 10-30 | 170   |     |
|                          | " " Etowah Quarry. . . . .                   | 126 | 4  | 12 | 441 | 40    | 1-40  | 168 |
|                          | " " Kennesaw Quarry. . . . .                 |     | 18 |    | 441 | 91    | 10-30 | 168 |
|                          | " " " . . . . .                              | 141 |    | 14 |     | 78    | 10-30 | 170 |
|                          | Lee Co., Mass. . . . .                       | 96  | 13 | 12 |     | 76    | 10-30 | 168 |
|                          |                                              | 160 | 16 |    | 562 | 67    | 10-20 |     |
|                          |                                              | 181 |    | 21 | 454 |       |       |     |
| Drab.                    | Faribault, Minn. . . . .                     | 178 |    |    |     |       |       |     |
|                          | Tuckahoe, N. Y. . . . .                      | 162 |    | 15 | 441 | 136   | 10-30 | 178 |
| White.                   | Richville, Dekalb, St. L. Co., N. Y. . . . . | 125 | 8  |    | 634 | 120   | 1-40  |     |
|                          | Vermont. . . . .                             | 90  | 10 |    | 361 | 66    | 10-30 |     |
| White.<br>Blue.          | Rutland, Vt. . . . .                         | 120 |    | 10 |     |       |       |     |
|                          | " " " . . . . .                              | 110 | 7  | 10 | 312 | 45    | 1-40  | 168 |
|                          | " " " . . . . .                              | 139 | 21 | 12 |     | 65    | 1-40  |     |
|                          | " " " . . . . .                              |     |    |    |     | 78    | 1-40  |     |
| Mt. Dark.                | " " " . . . . .                              |     |    |    |     | 75    |       |     |
|                          | Sutherland Falls, Vt. . . . .                | 128 | 18 | 15 | 433 | 93    | 1-40  |     |
|                          | " " " . . . . .                              | 162 | 23 | 16 | 550 | 126   | 1-40  |     |
|                          | " " " . . . . .                              | 113 |    |    |     |       |       |     |
|                          | " " " . . . . .                              | 102 |    |    |     |       |       |     |
|                          | Roche Harbor, San Juan Co., Wash. . . . .    | 90  |    |    |     |       |       |     |
|                          | Snoqualmie District Wash. . . . .            | 47  |    |    |     |       |       |     |

## CONCRETE.

The physical properties of concrete depend upon so many variable factors that it is useless to attempt to give more than approximate values.

Mr. Edwin Thacher, C.E., has deduced formulas, based upon a large number of experiments made at the Watertown Arsenal,

showing the effect of age and composition upon the ultimate strength. Values according to these formulas are given below. For tests of concrete of all kinds reference is made to *Tests of Metals and other Materials, etc., made at the Watertown Arsenal, Mass.*

ULTIMATE STRENGTH IN POUNDS PER SQUARE INCH.

| Mixture. | Age.    |          |           |           | Remarks.                                                                                                                  |
|----------|---------|----------|-----------|-----------|---------------------------------------------------------------------------------------------------------------------------|
|          | 7 Days. | 1 Month. | 3 Months. | 6 Months. |                                                                                                                           |
| I-1-3    | 1600    | 2750     | 3360      | 4300      | $S = 1800 - 200x$ ; 7 days<br>$S = 3100 - 350x$ ; 1 month<br>$S = 3820 - 460x$ ; 3 months<br>$S = 4900 - 600x$ ; 6 months |
| I-2-4    | 1400    | 2400     | 2900      | 3700      |                                                                                                                           |
| I-2½-5   | 1300    | 2225     | 2670      | 3400      |                                                                                                                           |
| I-3-6    | 1200    | 2050     | 2440      | 3100      |                                                                                                                           |
| I-3½-7   | 1100    | 1875     | 2210      | 2800      | $x = \text{parts of sand to one part cement.}$<br>$S = \text{ultimate strength for 12-inch cubes.}$                       |
| I-4-8    | 1000    | 1700     | 1980      | 2500      |                                                                                                                           |
| I-5-10   | 800     | 1350     | 1520      | 1900      |                                                                                                                           |
| I-6-12   | 600     | 1000     | 1060      | 1300      |                                                                                                                           |

E=MODULUS OF ELASTICITY IN THOUSANDS OF POUNDS PER SQUARE INCH.

(Compiled by E. Thacher.)

| Mixture. | Age 7 Days. |              |               | Age 1 Month. |              |               | Age 3 Months. |              |               | Age 6 Months. |              |               |
|----------|-------------|--------------|---------------|--------------|--------------|---------------|---------------|--------------|---------------|---------------|--------------|---------------|
|          | 100 to 600. | 100 to 1000. | 1000 to 2000. | 100 to 600.  | 100 to 1000. | 1000 to 2000. | 100 to 600.   | 100 to 1000. | 1000 to 2000. | 100 to 600.   | 100 to 1000. | 1000 to 2000. |
| I-1-3    | 2450        | 2050         | 1380          | 2830         | 2580         | 1910          | 3500          | 3140         | 2120          | 3850          | 3580         | 2700          |
| I-2-4    | 2580        | 2050         | 1350          | 2660         | 2450         | 1460          | 3670          | 3160         | 2160          | 3670          | 3570         | 2580          |
| I-2½-5   | 2220        | 1800         | .....         | 2550         | 2300         | 1350          | 3320          | 2900         | 1980          | 3630          | 3540         | 2220          |
| I-3-6    | 1860        | 1540         | .....         | 2440         | 2130         | 1220          | 2970          | 2650         | 1800          | 3600          | 3500         | 1860          |
| I-4-8    | .....       | .....        | .....         | 2100         | 1800         | .....         | 2530          | 2220         | .....         | 3020          | 2840         | .....         |
| I-5-10   | .....       | .....        | .....         | 1740         | 1460         | .....         | 2100          | 1780         | .....         | 2420          | 2200         | .....         |
| I-6-12   | .....       | .....        | .....         | 1380         | 1140         | .....         | 1640          | 1360         | .....         | 1820          | 1520         | .....         |

These values are means of tests made upon 12-inch cubes made with four brands of cement respectively. A statement of the data upon which the above tables are based is given in an article by Mr. E. Thacher, entitled *Effect of Age and Composition on the Strength and Modulus of Elasticity of Concrete, Cement*, May 1902.



## EXPANSION OF CONCRETE.

Prof. Pence gives 0.0000055 as the coefficient of expansion for one degree F. for 1-2-4 concrete composed of Lehigh Portland cement and limestone. With the limestone replaced with gravel the coefficient becomes 0.0000054. This makes the coefficient of expansion of concrete and steel essentially the same.

## WEIGHT OF CONCRETE.

The weight of concrete will vary somewhat according to the materials used and the methods of mixing. In "Tests of Metals, etc.," for 1898, the weights of a large number of 12-inch cubes are given, the proportions varying from 1-1 to 1-4, with 33 and 40 per cent of the stone used as mortar. The mortar was made "dry," "plastic," and "wet." The weights per cubic foot ranged from 138.9 to 143.7 pounds. For all ordinary purposes 140 pounds per cubic foot may be used. Some specifications state that concrete shall be taken at 150 pounds per cubic foot.

## WEIGHT OF FILLING MATERIAL.

This will vary according to the kind of material and the method of depositing it. For average conditions, when the spandrels are filled with sand or gravel, 100 pounds per cubic foot may be assumed. For gravel deposited in thin layers and rolled, some specifications state that the fill shall be taken as weighing 120 pounds per cubic foot.



TABLE

DATA FOR ABOUT 500 ARCH BRIDGES

MASONRY

| Number. | Name.             | Place.                             | Over.            | Date.           | Engineer.            | No. of Spans. |
|---------|-------------------|------------------------------------|------------------|-----------------|----------------------|---------------|
| 1       | Taff Vale Viad.   | Near Quaker's Yard, S. Wales       | Taff R.          |                 | Brunel               | 6             |
| 2       | Queretaro         | Near Queretaro, Mexico             | Valley           | 1726-35         | Antonio Avana        | 74            |
| 3       | Malaunay          | Near Rouen, France                 | Malaunay Val.    | 1840-44         | Locke                | 8             |
| 4       | Magnolia St.      | Elizabeth, N. J., U. S. A.         | Magnolia St.     | 1894            | Brown                | 1             |
| 5       | L. Juniata No. 8  | Penn., U. S. A.                    | L. Juniata R.    |                 | Brown                | 3             |
| 6       |                   | Morpeth, England                   | Wansbeck R.      | 1831            | Telford              | 3             |
| 7       | Mass. Ave.        | Washington, D. C., U. S. A.        | Rock Crk.        | 1900-1          | Douglas              | 1             |
| 8       |                   |                                    | Cree R.          |                 | Rennie               | 1             |
| 9       | Chateau Thierry   | France                             | Marne.           | 1786            | Perronet             | 2             |
| 10      | Charles           | Nuremberg, Bavaria                 |                  | 1728            |                      | ?             |
| 11      | Starrucca Viad    | Nr. Lanesboro', Pa., U.S.A.        | Starrucca Crk.   | 1847            | Adams                | 17            |
| 12      | Pont Neuf         | Paris, France                      | Seine R.         | 1578-1604       | Cerceau and Marchand | 12            |
| 13      | Enz               | Wildbad, Germany                   | Enz R.           | 1886            | Leibbrand            | 1             |
| 14      | Pont au Change    | Paris, France                      | Seine R.         | 1639-47         |                      | 7             |
| 15      |                   |                                    |                  |                 |                      |               |
| 16      | Court St.         | Rochester, N. Y., U.S.A.           | Genesee R.       | 1893            | McClintock           | 8             |
| 17      |                   | Rochester, N. Y., U. S. A.         |                  |                 |                      | 7             |
| 18      |                   | Bet. Norwood-Bromley, England      | Lon. Croydon Ry. |                 | Gibbs                | 1             |
| 19      |                   | Dôle, France                       | Doubs R.         | 1760-64?        | Gueret               | 7?            |
| 20      |                   | England                            | Mouse R.         | 1822            | Telford              | 3             |
| 21      |                   | Gien, France                       | Loire, R. Val.   | 1888-9          | Lethier              | 15            |
|         |                   |                                    |                  |                 |                      | 70            |
| 22      | Dinan Viad.       | Dinan, France                      | Rance R.         | 1845-50         | Fessard              | 10            |
| 23      | Guétin            | Bet. Digoin and Mains-bray, France | Valley           | 1890-98         |                      | 18            |
| 24      | Stura             | N. of Turin, Italy                 | Stura R.         |                 | Bella                | 5             |
| 25      |                   | Montalierie, Italy                 | Po R.            | 1840            | Barbavara            | 7             |
| 26      | Digoin            | Digoin, France                     | Valley           | Changed 1890-98 |                      | 11            |
| 27      | Roquefavour       | Vic. of Marseilles, France         | Arc R. and Val.  | 1841-47         | de Montricher        | 15            |
|         |                   |                                    |                  |                 |                      | 13            |
| 28      | Strasbourg Sta.   | Paris, France                      | Station Platform |                 |                      | 51            |
| 29      | Croydon           | Near Croydon, England              | Lon. & C. Ry.    |                 | Gibbs                | 3             |
| 30      | Abattoir St.      | Paris, France                      |                  |                 |                      | 1             |
| 31      |                   | Nemours, France                    | Loing R.         | 1805            | Perronet             | 3             |
| 32      |                   | Stirling, Scotland                 | Forth R.         | 1400†           |                      |               |
| 33      |                   | Moret, France                      | Loing R.         | 1771            | Perronet             | 3             |
| 34      |                   | Moulins, France                    | Allier R.        | 1758-60         | Regemortes           | 13            |
| 35      | Park St.          | Hartford, Conn., U.S.A.            |                  | 1898            | Graves               | 1             |
| 36      | Pathhead          | Pathhead, Scotland, U.S.A.         | Tyne R           | 1830            | Telford              | 5             |
| 37      | Mill Creek        | W. of Bird-in-Hand, Pa.,           | Mill Creek       | 1880-90         | Brown                | 4             |
| 38      | L. Juniata No. 13 | 1 mi. W. of Tyrone, Pa., U.S.A.    | L. Juniata R.    | 1892-93         | Brown                | 3             |

\* Maximum.

REMARKS.—1. Piers 14' octagon. On curve. Skew 40°. 1320' Radius. 2. Max. H.=95' Av.=75'-80'. 3. Found upon piles. 4. 4 tracks. Intrados to base of rail, 5'6". 5. Middle Div. Penn. Ry. 7. Skew 17°. Cost \$132,000. 11. 2 tracks. Max. H.=110'. N. Y., L. E., & W. 12. Repaired 1886. 13. Middle 3d joints at crown and springing filled with lead. 18. Ribbed skew. 20. H.=134.5'. Piers hollow. 21. Approaches to metal spans. 22. Max.

## II.

## ARRANGED ACCORDING TO SPAN.

## ARCHES.

| Span.        | Rise. | Thickness of Arch Ring at Crown to. | At Springing, <i>ts.</i> | Curve.         | Radius at Crown. | $\frac{I_0}{R}$ | Width, Face to Face at Crown. | Thickness of Piers at Springing. | Material.   | Class of Bridge. | Reference.       | Number. |
|--------------|-------|-------------------------------------|--------------------------|----------------|------------------|-----------------|-------------------------------|----------------------------------|-------------|------------------|------------------|---------|
| 50.0         | 25.0  |                                     |                          | C              | 25.0             |                 | 14.0                          |                                  | Blue Grit   | Ry.              | A. March, 1850   | 1       |
| 50.0         |       |                                     |                          |                |                  |                 |                               |                                  |             |                  |                  |         |
| 50.0         | 25.0  | 3.1                                 | 3.1                      | C              | 25.0             | .124            | 27.7                          | 8.8                              |             | Aqued't          | B. June 2, 1888  | 2       |
| 50.0         | 7.4   | 2.8                                 | 2.8                      | C              | 38.5             | .073            | 50.0                          |                                  |             | Ry.              | C. 1851-2        | 3       |
| 50.0         | 12.5  |                                     |                          |                |                  |                 |                               |                                  |             | Ry.              | D.               | 4       |
| *50.0        | 15.0  |                                     |                          | E              | 48.0             |                 | 31.5                          | 8.0                              |             | Ry.              | E.               | 5       |
| 50.0         | 25.0  | 3.0                                 | 7.0                      | C <sub>2</sub> | 25.0             | .150            | 200.                          |                                  | Brick       | H.W.             | F. 1852, p. 290  | 6       |
| 50.0         | 6.6   | 1.3                                 |                          | C              |                  |                 |                               |                                  |             |                  | B. Dec. 25, 1902 | 7       |
| 46.0         | 5.9   | 1.0                                 |                          |                |                  |                 |                               | 8.0 & 7.6                        |             |                  | S.               | 8       |
| 39.0         | 4.9   | 1.0                                 |                          |                |                  |                 |                               |                                  |             |                  |                  |         |
| 51.0         | 17.0  | 4.0                                 |                          | E              |                  |                 |                               |                                  |             |                  |                  |         |
| 51.1         | 16.0  | 4.3                                 |                          | E              | 37.2             | .115            | 19.2                          | 13.8                             |             | H.W.             | Woodbury, 1858   | 9       |
| 51.0         | 20.0  | 2.5                                 | 2.5                      | C              | 26.3             | .095            | 24.8                          | 7.0                              | Sandstone   | Ry.              | F. 1852, p. 278  | 10      |
| *51.1        | 21.9  | 2.3                                 | 3.6                      | C              | 25.8             | .089            | 72.5                          |                                  |             | H.W.             | B. Sept. 1, 1888 | 11      |
|              |       |                                     |                          |                |                  |                 |                               |                                  |             |                  | G. 1891, p. 887  | 12      |
| 51.2         | 10.7  | 1.6                                 | 3.9                      | C?             | 35.0             | .045            |                               |                                  |             | H.W.             | G. 1891, p. 918  | 13      |
| 51.2 to 35.2 | 25.6  | 5.3                                 |                          |                | 25.6             | .207            |                               | 16.0                             |             | H.W.             | F. 1852, p. 276  | 14      |
| *52.0        | 20.5  |                                     |                          | C              | 26.7             |                 | 764.0                         | 6.0                              |             | H.W.             | B. Feb. 2, 1893  | 15      |
| *52.0        | 10.0  | 2.5                                 |                          | C              | 38.8             | .064            |                               |                                  | Limestone   | Aqued't          | H.               | 16      |
| 52.0         | 12.0  | 2.3                                 | 2.3                      | C              | 34.2             | .067            | 730.0                         |                                  | Brick       | H.W.             | C. 1855-6        | 17      |
|              |       |                                     |                          |                |                  |                 |                               |                                  |             |                  |                  | 18      |
| 52.0         | 17.5  | 3.8                                 |                          | E              |                  |                 |                               |                                  |             |                  |                  |         |
| 52.0         | 26.0  |                                     |                          | C <sub>2</sub> |                  |                 |                               | 8.5                              |             | H.W.             | I.               | 19      |
| 52.5         | 26.2  | 3.0                                 | 3.3                      | C <sub>2</sub> | 26.2             | .114            | 17.7                          | 3.3                              |             | Ry.              | F. 1852, p. 197  | 20      |
| 52.5 to 42.6 | 17.5  |                                     |                          |                |                  |                 |                               |                                  |             |                  | G. VI, 1893      | 21      |
| *52.5        | 14.2  |                                     |                          |                |                  |                 |                               |                                  |             |                  |                  |         |
| *52.5        | 26.2  |                                     |                          | C <sub>2</sub> | 26.2             |                 | 22.1                          | 713.1                            |             | H.W.             | G. 1888, p. 363  | 22      |
| 52.5         | 23.0  | 3.6                                 | 3.6                      | 3C             | 30.7             | .117            | 31.9                          | 10.0                             | Canal       | G.               | G. 1899          | 23      |
| 52.5         | 5.1   | 3.0                                 |                          | E              | 102.7            | .042            | 35.1                          | 9.8                              | Brick(?)    | H.W.             | F. 1852, p. 296  | 24      |
| 52.5         | 13.6  | 3.0                                 |                          | C              | 32.1             | .093            | 29.5                          | 7.5                              | Brick       | Ry.              | F. 1852, p. 296  | 25      |
| 52.5         | 23.0  | 3.9                                 | 3.9                      | 3C             | 30.7             | .127            | 31.8                          | 9.8                              |             | Canal            | F. 1852, p. 286  | 26      |
| 52.5         | 26.3  |                                     |                          | C <sub>2</sub> | 26.3             |                 |                               |                                  |             |                  | A. 1855, p. 65   | 27      |
| 49.2         | 24.6  | 3.3                                 |                          |                | 24.6             | .134            |                               |                                  |             |                  |                  |         |
| 16.4         | 8.2   | 3.4                                 |                          |                | 8.2              |                 |                               |                                  |             |                  |                  |         |
| *52.7        | 5.0   | 3.0                                 |                          | E              | 71.6             | .041            | 42.6                          | 8.2                              | Mill'e Grit | H.W.             | F. 1852, p. 296  | 28      |
| 53.0         | 12.0  | 3.0                                 | 3.0                      | E              |                  |                 | 720.0                         |                                  | Brick       | H.W.             | C. 1855-6        | 29      |
| 53.0         | 5.1   | 3.0                                 |                          | C              | 71.3             | .042            |                               |                                  |             | Ry.              | I.               | 30      |
| 53.0         | 3.2   | 3.2                                 |                          | C              | 99.1             | .032            |                               | 6.4                              | Freestone   | H.W.?            | F. 1852, p. 286  | 31      |
| 53.0         | 10.3  | 2.8                                 |                          | C              | 39.2             | .071            |                               |                                  |             |                  | I.               | 32      |
| 53.3         | 6.1   | 4.3                                 |                          | C              | 61.2             | .072            |                               |                                  | Freestone   | H.W.?            | J.               | 33      |
| 53.0         | 21.3  | 3.2                                 |                          | E              |                  |                 | 34.0                          | 12.8                             |             | H.W.             | F. 1852, p. 276  | 34      |
| 54.0         | 7.3   | 3.3                                 | 4.3                      |                |                  |                 | 70.0                          |                                  | Brick       | H.W.             | B. Jan. 12, 1899 | 35      |
| 54.0 & 50.0  | 8.0   |                                     |                          |                |                  |                 | 24.0                          | 8.0 & 4.0                        |             | H.W.             | F. 1852, p. 195  | 36      |
| 54.0         | 13.5  | 2.7                                 | 2.7                      | C              | 33.7             | .081            | 32.0                          |                                  |             | Ry.              | E.               | 37      |
| 54.0         | 13.5  | 3.2                                 | 3.2                      | C              | 33.7             | .095            | 49.4                          |                                  |             | Ry.              | E.               | 38      |

† About.

H. = 130'.5. Nat. Route No. 176. 23. Max. H. = 39'.4. 24. Route Turin-Milan. 25. Turin-Genoa. 27. 3 tiers of arches. Max. H. = 271'.0. 29. Ribbed skew. 35. Stone facing. 36. Max. H. = 75'.0. The 54'.0 spans are under roadway. 37. Three tracks—1° curve. 38. 4 tracks. Mid. Div. Penn. Ry. ribbed skew.

TABLE II.—DATA FOR ABOUT 500 ARCH BRIDGES  
MASONRY

| Number. | Name.             | Place.                               | Over.           | Date.   | Engineer.   | No. of Spans. |
|---------|-------------------|--------------------------------------|-----------------|---------|-------------|---------------|
| 39      | Kennet            | Near Caversham, England              | Kennet R.       |         |             | 3             |
| 40      | Monocacy Viad.    | Monocacy, Penn., U.S.A.              | Sudbury R.      | 1883    | Fisk        | 1             |
| 41      | Nashawtuc         | Concord, Mass., U.S.A.               | Rieners R.      | 1770-90 | Wheeler     | 3             |
| 42      | Ouctoine          |                                      |                 |         | Garipuy     | 3             |
| 43      | Big Conestoga     | E. of Lancaster, Pa., U.S.A.         |                 |         | Brown       | 5             |
| 44      | Peas              | Bet. Berwick and Edinburgh, Scotland | Deep Dingle     |         | Henderson   | 4             |
| 45      | Oder              | Kunnersdorf, Saxony                  | Oder R.         |         |             | 7             |
| 46      | Bachthal          | Dermitz, Saxony                      | Bach R.         | 1844-5  |             | 11            |
| 47      | Dauphin           |                                      | Romanche R.     | 1842-4  | Potie       | 1             |
| 48      | Carmes            | France                               | Beauvoir Ravine | 1843-47 | Cunit       | 3             |
| 49      |                   | Löbau, Saxony                        | Spree R.?       | 1845-46 |             | 7             |
| 50      |                   | Königstein, Saxony                   |                 |         |             | 2             |
| 51      | Spreethal         | Saxony                               | Spree R.        | 1845-46 |             | 32            |
| 52      |                   | Neunck, Germany                      | Glatt R.        | 1886    | Leibbrand   | 15            |
| 53      | No. 28            | 26.5 m. Pittsburg, Pa., U.S.A.       | Raccoon Crk.    | 1887-88 |             | 1             |
| 54      | Washington        | New York, N. Y., U.S.A.              | Harlem R.       | 1886-89 | Hutton      | 1             |
| 55      | Nôtre Dame        | Paris, France                        | Seine R.        | 1507    | Jaconde     | 6             |
| 56      |                   | Chateau Thierry, France              | Marne R.        | 1765    | Pérronet    | 1             |
| 57      |                   | Pontlieu, France                     | Huisine R.?     | 1773    | Voglie      | 2             |
| 58      | Gravant           | France                               | Yonne R.        | 1760    | Adwine      | 3             |
| 59      | Zempoala Aq.      | 7 m. south of Huauachinango, Mexico  | Valley          | 1553-70 | Tembleque   | 68            |
| 60      | Monford           | England                              | Severn R.       | 1790-92 | Telford     | 1             |
| 61      | Johnstown         | Johnstown, Penn., U.S.A.             | Conemaugh R.    | 1888    | Brown       | 2             |
| 62      |                   | Llanrwst, Wales                      | Conway R.       | 1634-36 | Inigo Jones | 1             |
| 63      | Carrolton Viad.   | Nr. Baltimore, Md., U.S.A.           | Patapsco R.     | 1833-35 | Latrobe     | 3             |
| 64      | Jamaica St.       | Glasgow, Scotland                    | Clyde R.        | 1833-36 | Telford     | 8             |
| 65      | Brives            | France                               | Loire R.        | 1772    | Grangent    | 1             |
| 66      | Tournelle         | Paris, France                        | Seine R.        | 1630-56 | Marie       | 2             |
| 67      | Marie             | Paris, France                        | Seine R.        | 1635-58 | Marie       | 5             |
| 68      | Aelius            | Rome, Italy                          | Tiber R.        | 136     | Hadrian     | 3             |
| 69      | Sèvres            | Near Paris, France                   | Seine R.        | 1820    | Beaupre     | 9             |
| 70      | Rahway Ave.       | Elizabeth, N. J., U.S.A.             | Rahway Ave.     |         | Brown       | 2             |
| 71      | Washington        | New York, N. Y., U.S.A.              | Harlem R.       | 1886-89 | Hutton      | 1             |
| 72      | Ingersheim        | France                               | Tech R.         | 1773    | Clinchamp   | 6             |
| 73      | Trenton           | Near Trenton, N. J., U.S.A.          | Delaware R.     | 1902    | Brown       | 7             |
| 74      | L. S. & M. S. Ry. | Elyria, Ohio, U.S.A.                 | W. Br. Black R. |         |             | 18            |
|         |                   |                                      |                 |         |             | 3             |

\* Maximum.

REMARKS.—39. Skew. S. E. Ry. Co. 40. Chesapeake & Ohio Canal. 41. Granite from Fitchburg, Mass. 43. Penn. Ry. 44. Max. H.=124'.o. 45. Löbau-Zitlau, H.=62.3. 46. Pile found. H.=59. L.=725. 49. Saxony-Silesia, H.=95'.o. 50. Prague-Dresden, H.=33.6. 51. Saxony-Silesia, H.=66.9. 52. Sheet-lead "Hinges," 3. 53. 2 tracks. Rail 27'.5 above key. 54. Approach to metal spans. 56. See No. 9. 59. Waterway 84"×12". H.=124'.o. On two tangents. Max. span is highest. 61. Skew=55°. Ribbed. Stood through Johnstown Flood.



## ARRANGED ACCORDING TO SPAN—(Continued).

## ARCHES.

| Span.   | Rise. | Thickness of Arch Ring at Crown $\frac{1}{2}$ . | At Springing, $\frac{1}{2}$ . | Curve.         | Radius at Crown. | $\frac{1}{2}$ R. | Width Face to Face at Crown. | Thickness of Piers at Springing. | Material.       | Class of Bridge. | Reference.                  | Number. |
|---------|-------|-------------------------------------------------|-------------------------------|----------------|------------------|------------------|------------------------------|----------------------------------|-----------------|------------------|-----------------------------|---------|
| *54.0   | 11.0  | 2.6                                             |                               |                |                  |                  | 24.0                         |                                  | Brick           | Ry. Canal        | K. Dec. 20, 1895            | 39      |
| 54.0    | 9.0   | 2.5                                             |                               | E              | 50.0             | .050             |                              |                                  | Granite         | H.W.             | I. Wm. Wheeler              | 40      |
| 54.0    | 6.0   | 1.2                                             | 1.2                           | E              | 63.75            | .019             | 25.0                         |                                  |                 |                  | F. 1852, p. 280             | 41      |
| 54.5    | 17.1  | 2.1                                             |                               | E <sub>2</sub> | 27.3             |                  |                              | 12.8                             |                 | Ry. H.W.         | E. L.                       | 42      |
| *54.5   | 27.3  |                                                 |                               |                |                  |                  |                              |                                  |                 |                  |                             | 43      |
| *55.0   |       |                                                 |                               |                |                  |                  |                              |                                  |                 |                  |                             | 44      |
| *55.8   |       |                                                 |                               | E              |                  |                  |                              | 7.4                              |                 | Ry.              | F. 1852, p. 229             | 45      |
| 55.8    | 27.9  |                                                 |                               | C <sub>2</sub> | 27.88            | .108             | 19.7                         | 9.2                              |                 | Ry.              | F. 1852, p. 226             | 46      |
| 55.8    | 27.9  | 3.0                                             |                               | C <sub>2</sub> | 27.88            | .108             | 19.7                         | 4.9                              |                 |                  | F. 1852, p. 292             | 47      |
| 55.8    | 27.9  | 3.1                                             |                               | C <sub>2</sub> | 27.9             | .111             | 19.7                         | 0.5                              |                 |                  | F. 1852, p. 292             | 48      |
| 55.8    | 27.9  |                                                 |                               | C <sub>2</sub> | 27.9             |                  |                              |                                  | Brick           | Ry.              | F. 1852, p. 222             | 49      |
| 37.9    | 18.9  |                                                 |                               | E              |                  |                  |                              |                                  | Stone           | Ry.              | F. 1852, p. 225             | 50      |
| *55.8   |       |                                                 |                               |                |                  |                  |                              |                                  |                 |                  |                             |         |
| *55.8   | 27.9  |                                                 |                               | C <sub>2</sub> | 27.9             |                  |                              | 15.8                             |                 | Ry.              | F. 1852, p. 228             | 51      |
| c-55.8  | c-9.8 | †1.0                                            | †2.0                          |                |                  |                  | 13.1                         |                                  |                 | H.W.             | G. 1891-1, p. 920           | 52      |
| 56.0    | 28.0  | 3.0                                             |                               | C <sub>2</sub> | 28.0             | .108             | 107.6                        |                                  |                 | Ry.              | D.                          | 53      |
| 56.0    | 14.0  | 2.0                                             | 2.0                           | E              | 55.8             | .036             | 80.8                         | †13.2                            |                 | H.W.             | Washington Bridge by Hutton | 54      |
| 56.7    | 28.4  | 5.3                                             |                               | C <sub>2</sub> | 28.4             | .187             | 77.4                         | 12.8                             |                 | H.W.             | F. 1852, p. 274             | 55      |
| to 31.2 | 15.6  |                                                 |                               |                |                  |                  |                              |                                  |                 |                  |                             |         |
| 57.5    | 19.2  | 4.0                                             |                               | E              | 41.5             | .096             | 35.2                         | 14.4                             |                 | H.W.             | F. 1852, p. 280             | 56      |
| 51.1    | 17.1  | 3.7                                             |                               | E              |                  |                  | 35.2                         |                                  |                 |                  |                             |         |
| 57.5    | 21.3  | 3.8                                             |                               | E              | 42.6             | .089             |                              | 12.8                             |                 |                  | F. 1852, p. 282             | 57      |
| 57.5 to |       |                                                 |                               | E              | 45.8             | .093             |                              | 12.8                             |                 |                  | F. 1852, p. 280             | 58      |
| *53.9   | 21.3  | 4.3                                             |                               |                |                  |                  |                              |                                  |                 |                  |                             |         |
| *58.0   | 29.0  |                                                 |                               | C <sub>2</sub> | 29.0             |                  | 4.7                          |                                  |                 | Aqued't          | B. July 7, 1888, p. 2       | 59      |
| 58.0    | 22.5  | 3.0                                             |                               | E              |                  |                  | 24.0                         | 11.0                             |                 |                  | F. 1852, p. 284             | 60      |
| 50.0    | 20.0  |                                                 |                               |                |                  |                  |                              |                                  |                 |                  |                             |         |
| 58.0    | 14.5  | 2.7                                             | 2.7                           | C              | 36.2             | .075             | 48.0                         | 6.0                              | Sandstone       | Ry.              | B. July 20, 1889            | 61      |
| 40.0    | 14.5  | 2.7                                             | 2.7                           | C              | 21.0             | .135             | 48.0                         |                                  |                 |                  |                             |         |
| *58.0   | 17.0  | 1.5                                             |                               | P              |                  |                  | 14.0                         | 10.0                             |                 | H.W.             | E. L.                       | 62      |
| 58.3    | 29.2  | 2.5                                             | 2.5                           | C <sub>2</sub> | 29.2             | .085             |                              | 10.0                             | Granite         | Ry.              | I. , F. 1852, p. 237        | 63      |
| 58.5    | 10.8  | 2.5                                             |                               | C              | 43.7             | .057             | 40.0                         |                                  |                 | H.W.             | F. 1852, p. 290             | 64      |
| 57.8    | 10.5  |                                                 |                               |                |                  |                  |                              |                                  |                 |                  |                             |         |
| 55.5    | 9.7   |                                                 |                               |                |                  |                  |                              |                                  |                 |                  |                             |         |
| 52.0    | 8.3   |                                                 |                               |                |                  |                  |                              |                                  |                 |                  |                             |         |
| *58.6   | 26.6  | 3.2                                             |                               | E              | 45.8             | .069             | 20.1                         | 11.7                             |                 |                  | F. 1852, p. 280             | 65      |
| 58.7 to | 29.8  | 5.4                                             |                               | C <sub>2</sub> | 29.8             | .281             | 53.3                         | 12.8                             |                 | H.W.             | F. 1852, p. 276             | 66      |
| 44.8    | 22.4  |                                                 |                               |                |                  |                  |                              |                                  |                 |                  |                             |         |
| 58.7 to | 29.8  | 4.3                                             |                               | C <sub>2</sub> | 29.3             | .147             | 77.7                         | 11.7                             |                 | H.W.             | F. 1852, p. 276             | 67      |
| 44.8    | 22.4  |                                                 |                               |                |                  |                  |                              |                                  |                 |                  |                             |         |
| 59.0    | 29.5  |                                                 |                               | C <sub>2</sub> | 29.5             |                  | †33.5                        | †23.0                            | Travert'e       | H.W.             | M. Feb. 18, 1899            | 68      |
| 59.0    | 29.5  | 3.3                                             |                               | C <sub>2</sub> | 29.5             | .112             | 42.6                         | 12.1                             |                 | H.W.             | A. April, 1847              | 69      |
| 16.4    | 8.2   |                                                 |                               |                |                  |                  |                              |                                  |                 |                  | F. 1852, p. 288             |         |
| 59.2    | 9.5   | 3.2                                             | 3.2                           | C              | 50.9             | .063             | 62.2                         |                                  |                 | Ry.              | D.                          | 70      |
| 59.7    | 29.8  | 2.0                                             | 4.5                           | C <sub>2</sub> | 29.8             | .067             | 80.8                         |                                  |                 | H.W.             | See No. 54                  | 71      |
| 59.7 to | 11.7  | 3.2                                             |                               | E              |                  |                  |                              | 8.5                              |                 |                  | F. 1852, p. 282             | 72      |
| 59.1    |       |                                                 |                               |                |                  |                  |                              |                                  |                 |                  |                             |         |
| 60.0    | 12.0  | 3.3                                             | 3.3                           | C              | 43.5             | .076             | 52.0                         | 8.0                              |                 |                  | B. Jan. 30, 1902            | 73      |
| 60.0    | 30.0  | 2.0                                             |                               | C <sub>2</sub> | 30.0             | .067             | 22.2                         |                                  | Berea sandstone | Ry.              | N. June 8, 1899             | 74      |

† About.

62. On 41° curve. Arches Cyl. H.=66'. 64. 1st Bridge by Mylne, 1768-72, called New Jamaica St. Bridge. Old Bromielaw Bridge. 68. Originally 8 arches, 5 now buried. To give access to St. Ange Castle. 69. Paris-Versailles. 70. Five tracks. Ribbed arch Skew 45° 44'. Pa. Ry. N. Y. Div. 73. Two abut. piers, 22'. Skew 71° 30'. 74. Twin arches 4' 2" apart. Two tracks. L. S. & M. S. Ry.



TABLE II.—DATA FOR ABOUT 500 ARCH BRIDGES

MASONRY

| Number. | Name.                   | Place.                                | Over.            | Date.      | Engineer.        | No. of Spans. |
|---------|-------------------------|---------------------------------------|------------------|------------|------------------|---------------|
| 75      |                         | Minneapolis, Minn., U.S.A.            | Mississippi R.   |            |                  | 1             |
| 76      | Dee                     | Val. Llangallen, Wales                | Dee R.           |            |                  | 2             |
| 77      | S. approach Voyne Viad. | Drogheda Ireland                      | Boyne R.         | 1851-67    | Macneill         | 10            |
| 78      | Muddy Crk.              | Addystone, O., U.S.A.                 | Muddy Crk.       | †1895      | Kittredge        | 13            |
| 79      | W. Jersey St.           | Elizabeth N. J. U.S.A.                | W. Jersey St.    | 189 -      | Brown            | 1             |
| 80      | Kenet                   | Near Caversham, England               | Kenet R.         | †1840      | Brunel           | 1             |
| 81      | Staines                 | Staines, England                      | Thames R.        | 1796       | Sanby            | 2             |
| 82      | Ballater                | Ballater, Scotland                    | Dee R.           | 1809       | Telford          | 5             |
| 83      | Stirling                | Stirling, Scotland                    | Forth R.         | 1829-32    | Stevenson        | 1             |
|         |                         |                                       |                  |            |                  | 2             |
| 84      | Richmond                | Richmond, England                     | Thames R.        | 1774-77    | Payne & Couse    | 5             |
| 85      | Alness                  | Alness, Scotland                      |                  | 1816       | Telford          | 1             |
| 86      |                         | Warfield, England                     |                  | 1846       | Grainger         | 21            |
| 87      | Anker                   | England                               |                  |            |                  | 1             |
| 88      | Dutton Viad.            | England                               | Weaver Val.      |            | Stephenson       | 19            |
| 89      | Holy Cross (old)        | Feldkirch, Austria                    | Ill. R.          | 13th cen.? |                  | 20            |
| 90      | Kingston                | Kingston England                      | Thames R.        | 1825-28    | Lapidge          | 1             |
|         |                         |                                       |                  |            |                  | 5             |
|         |                         |                                       |                  |            |                  | 2             |
|         |                         |                                       |                  |            |                  | 2             |
| 91      |                         | Saumur, France                        | Arm of Loire R.  | 1756-64    | Voglio & Cessart | 12            |
| 92      | Conemaugh               | W. of Ben's Crk, Penn., U.S.A.        | Conemaugh R.     | 1896       | Brown            | 1             |
| 93      | Ben's Creek             | Lilly-Portage, Pa., U.S.A.            | Conemaugh R.     | 1896       | Brown            | 1             |
| 94      | L. Juniata No. 7        | 1 m. E. Schoenberger's, Penn., U.S.A. | L. Juniata R.    | 1889       | Brown            | 3             |
| 95      | Big Chiques             | Penn., U.S.A.                         | Big Chiques Crk. | 1884       | Brown            | 2             |
| 96      | Big Viaduct             | Viad. Sta., Penn., U.S.A.             | L. Conemaugh R.  | 1889       | Brown            | 2             |
| 97      | L. Conemaugh No. 6      | E. of L. Conemaugh, Pa., U.S.A.       | L. Conemaugh R.  | 1889-90    | Brown            | 3             |
| 98      | Chestnut St.            | Philadelphia, Pa., U.S.A.             | Schuylkill R.    | 1861-66    | Kneass           | 1             |
| 99      |                         | Bewdley England                       | Severn R.        | 1797-9     | Telford          | 2             |
| 100     | Congleton Viad.         | Congleton England                     | Dane R. & H.W.   | 1830-      | Stevenson        | 42            |
| 101     |                         | Ratisbon, Germany                     | Danube R.        | 1135       |                  | 15            |
| 102     | Tweed                   | Berwick, England                      | Tweed R.         | 1847-50    | Stevenson        | 28            |
| 103     |                         | Charmes, France                       | Moselle R.       | 1740       |                  | 10            |
| 104     |                         | Kew England                           | Thames R.        |            |                  | 5             |
| 105     | Görlitz                 | Near Görlitz, Silesia                 | Neisse R. & Val. | 1844-47    |                  | 6             |
|         |                         |                                       |                  |            |                  | 6             |
|         |                         |                                       |                  |            |                  | 18            |
|         |                         |                                       |                  |            |                  | 1             |
| 106     |                         | Dôle France                           | Doubs R.         | 1760-64    | Gueret           | 7             |
| 107     | W. Grand St.            | Elizabeth, N. J. U.S.A.               | W. Grand St.     | 189-       | Brown            | 1             |

\* Maximum.

REMARKS.—76. H.=147'6. Intrados to rail=6'1. Shrewsbury-Chester, stone facing. 78. Approach to metal spans Av. H.=90'. 78. Big 4 Ry. 79. Pa. Ry., N. Y. Div. Skew 60°. Ribbed 80. Great Western Ry. Co. 81. Closed 1797 on account of poor foundation. 84. Clear headway above L.W.=25'0. 86. Leeds-Thirsk. H.=00'0. 87. 816' long. 88. Grand Junc. Ry. H.=73'0. 89. Replaced. 90. Found. upon blue clay. Kingston-Hampton. wick. 91. A. 1856, p. 376. Found. upon piles. 92. Pa. Ry. Pitts. Div. Four tracks. 93. Pa-

## ARRANGED ACCORDING TO SPAN—(Continued).

## ARCHES.

| Span.        | Rise. | Thickness of Arch Ring at Crown $t_0$ . | At Springing, $t_s$ . | Curve.         | Radius at Crown. | $t_0/R$ | Width, Face to Face at Crown. | Thickness of Piers at Springing. | Material.                    | Class of Bridge. | Reference.           | Number. |
|--------------|-------|-----------------------------------------|-----------------------|----------------|------------------|---------|-------------------------------|----------------------------------|------------------------------|------------------|----------------------|---------|
| 60.0         | 15.0  |                                         |                       | C              |                  |         | 40.0                          | 8.0                              |                              | H.W.             | N. Nov. 23, 1895     | 75      |
| 57.0         | 14.3  |                                         |                       |                |                  |         |                               |                                  |                              |                  |                      |         |
| 54.0         | 13.5  |                                         |                       |                |                  |         |                               |                                  |                              |                  |                      |         |
| 60.0         | 30.0  |                                         |                       | C <sub>2</sub> | 30.0             |         | 27.8                          | 13.1                             | Brick                        | Ry.              | F. 1852, p. 156      | 76      |
| 60.0         | 30.0  |                                         |                       | C <sub>2</sub> | 30.0             |         |                               |                                  |                              | Ry.              | A. July 1851, p. 384 | 77      |
| 60.0         | 12.0  | 3.0                                     | 3.0                   | C              | 43.5             | .069    | 30.0                          | 10.0                             | Berea sandstone              | Ry.              | Blue                 | 78      |
| 55.0         | 12.0  | 3.0                                     | 3.0                   | C              | 37.5             | .080    |                               |                                  |                              |                  |                      |         |
| 60.0         | 9.5   | 3.5                                     | 3.5                   | C              | 52.1             | .067    | 51.0                          |                                  |                              | Ry.              | D.                   | 79      |
| 60.0         |       | 3.0                                     | 4.5                   |                |                  |         | 136.0                         |                                  | Brick                        |                  |                      | 80      |
| 18.0         |       | 1.5                                     | 1.5                   |                |                  |         |                               |                                  | Brick                        | Ry.              | K. Dec. 20, 1895     |         |
| 60.0         |       |                                         |                       |                |                  |         |                               | 8.0                              |                              | H.W.             | K. Sept. 13, 1895    | 81      |
| 52.0         |       |                                         |                       |                |                  |         |                               |                                  |                              |                  |                      |         |
| 60.0         |       |                                         |                       |                |                  |         |                               |                                  |                              |                  |                      |         |
| 60.0         | 13.5  | 2.8                                     | 3.5                   | C              | 40.0             | .070    | 32.8                          | 9.0                              | Granite                      | H.W.             | C. 1855-56           | 82      |
| 58.0         | 12.5  | 2.8                                     | 3.5                   |                | 38.6             | .073    |                               |                                  | Greenst'e from near Stirling | H.W.             | C. 1855-56           | 83      |
| 53.5         | 10.3  | 2.8                                     | 3.5                   |                | 39.8             | .073    |                               |                                  |                              |                  |                      |         |
| *60.0        |       |                                         |                       |                |                  |         |                               |                                  |                              |                  |                      |         |
| 60.0         | 20.0  |                                         |                       | C              |                  |         | 25.0                          |                                  |                              | H.W.             | K. July 12, 1895     | 84      |
| 60.0         | 30.0  |                                         |                       |                |                  |         |                               |                                  |                              |                  | F. 1852, p. 288      | 85      |
| 60.0         |       |                                         |                       |                |                  |         |                               |                                  | Brick                        | Ry.              | F. 1852, p. 169      | 86      |
| 30.0         |       |                                         |                       |                |                  |         | 31.0                          |                                  | BrickRing                    |                  | F. 1852, p. 188      | 87      |
| 60.0         |       |                                         |                       |                |                  |         | 30.0                          |                                  |                              |                  |                      |         |
| 60.0         | 30.0  |                                         |                       | C <sub>2</sub> | 30.0             |         | 11.0                          | 10.0                             |                              | Ry.              | A. 1837, 8, p. 125   | 88      |
| 60.0         | 19.0  |                                         |                       | E              |                  |         | 25.0                          |                                  |                              | H.W.             | O. June 1898         | 89      |
| 56.0         | 18.3  |                                         |                       |                |                  |         |                               | 9.3                              | Brick                        | H.W.             | A. Dec. 1842         | 90      |
| 52.0         | 16.5  |                                         |                       |                |                  |         |                               | 8.7                              | faced with stone             |                  |                      |         |
| *60.0        | 21.0  | 4.8                                     |                       | E              | 55.4             | .087    | 44.7                          | 12.8                             |                              |                  | K. July 26, 1895     |         |
| 60.0         | 20.0  | 3.0                                     | 3.0                   | C              | 32.5             | .092    | 83.5                          |                                  |                              | H.W.             | L. F. 1852, p. 276   | 91      |
|              |       |                                         |                       |                |                  |         |                               |                                  |                              | Ry.              | D.                   | 92      |
| 60.0         | 20.0  | 3.0                                     | 3.0                   | C              | 32.5             | .092    |                               |                                  |                              |                  |                      |         |
| 60.0         | 15.0  | 3.0                                     | 3.0                   | C              | 37.5             | .080    | 36.6                          | 7.0                              |                              | Ry.              | E.                   | 93      |
|              |       |                                         |                       |                |                  |         |                               |                                  |                              | Ry.              | E.                   | 94      |
| 60.0         | 15.0  | 2.8                                     | 2.8                   | C              | 37.5             | .075    | 11.0                          | 12.0                             | BrickRing                    | Ry.              | E.                   | 95      |
| 60.0         | 30.0  | 2.7                                     | 2.7                   | C <sub>2</sub> | 30.0             | .090    | 36.5                          | 8.0                              |                              | Ry.              | E.                   | 96      |
| 60.0         | 20.0  | 3.0                                     | 3.0                   | C              | 32.5             | .092    | 41.0                          | 7.0                              |                              | Ry.              | E.                   | 97      |
|              |       |                                         |                       |                |                  |         |                               |                                  |                              |                  |                      |         |
| 60.0         | 18.0  | 2.5                                     |                       | C              | 34.0             | .074    |                               |                                  | Brick                        | H.W.             | I.                   | 98      |
| 60.0         | 18.0  | 2.0                                     |                       | C              | 34.0             | .059    | 28.0                          | 8.0                              |                              |                  | I. L. F. 1852 p. 284 | 99      |
| 52.0         | 16.0  |                                         |                       |                |                  |         |                               |                                  |                              |                  |                      |         |
| 60.8         | 20.0  |                                         |                       | C              |                  |         | 31.0                          |                                  | Brick                        | Ry.              | A. 1839, p. 444      | 100     |
| 60.8         | 30.4  | 3.2                                     |                       | C <sub>2</sub> | 30.4             | .105    | 25.6                          | 20.3                             |                              | H.W.             | F. 1852, p. 274      | 101     |
| 61.5         | 30.8  |                                         |                       | C <sub>2</sub> | 30.8             |         |                               | 5.2                              | Brick                        | Ry.              | F. 1852, p. 155      | 102     |
| 61.8 to 34.1 | 30.9  | 4.3                                     |                       | C <sub>2</sub> | 30.9             | .139    |                               | 17.1                             |                              | H.W.             | F. 1852, p. 276      | 103     |
| 61.8 to 38.4 | 30.9  |                                         |                       |                |                  |         |                               |                                  |                              |                  |                      |         |
| 61.8 to 38.4 | 30.9  | 2.7                                     |                       | C <sub>2</sub> | 30.9             | .087    |                               | 8.5                              |                              |                  | F. 1852, p. 282      | 104     |
| 61.8 to 41.2 | 30.9  |                                         |                       | C <sub>2</sub> | 30.9             |         |                               |                                  |                              |                  |                      |         |
| 61.8 to 41.2 | 30.9  |                                         |                       |                |                  |         |                               |                                  | Red Gran.                    | Ry.              | F. 1852, p. 215      | 105     |
| 30.0         | 15.5  |                                         |                       |                | 20.6             |         |                               |                                  |                              |                  |                      |         |
| 24.8         | 12.4  |                                         |                       |                | 15.5             |         |                               |                                  |                              |                  |                      |         |
| 61.8 to 51.2 | 19.2  | 4.3                                     |                       |                | 12.4             |         |                               |                                  |                              |                  |                      |         |
| 62.0         | 9.0   | 3.6                                     | 3.6                   | E              | 44.7             | .096    |                               |                                  | 11.5 to 10.7                 |                  | F. 1852, p. 280      | 106     |
|              |       |                                         |                       | C              | 57.9             | .062    | 51.0                          |                                  |                              | Ry.              | D.                   | 107     |

† About.

Ry. Pitts. Div. 04. Ribbed. Skew 45°. Three tracks. Pa. Ry. 95. Pa. Ry. Phila. Div. 96. On 2° curve. Replaced 80' arch, destroyed May 31, 1880, Johnstown Flood. 97. On 5° 33' curve. Skew 57° 54'. Ribbed. Pa. Ry. Three tracks. 100. Manchester-Birmingham Ry. 101. J. 1896, p. 126, gives span=53-33 and C<sub>2</sub>. 102. Stone facing. H.=124'.6. 105. H.=115'.3. 328'.0 on curve. Berlin-Breslau. 107. Pa. Ry., N. Y. Div. Skew 57° 41'. Ribbed. Four tracks

TABLE II.—DATA FOR ABOUT 500 ARCH BRIDGES

MASONRY

| Number. | Name.              | Place.                       | Over.            | Date.     | Engineer.          | No. of Spans. |
|---------|--------------------|------------------------------|------------------|-----------|--------------------|---------------|
| 108     | Holy Cross (new)   | Feldkirch, Austria           | Ill R.           | 1898      |                    | 1             |
| 109     | St. Angelo         | Rome, Italy                  | Tiber R.         | 135       | Hadrian            |               |
| 110     | Barton Aq.         | Worsley, England             | Irwell R.        | 1760—     | Brinkley           | 3             |
| 111     |                    | Athlone, Ireland             | Shannon R.       | 1844      | Rhodes             | 3             |
| 112     |                    | Mirepoix, France             | Lers R.          | 1776-90   | Garipuy            | 7             |
| 113     |                    | Frouard, France              | Moselle R.       | 1788      | Le Creux           | 7             |
| 114     |                    | Ferté, France                | Marne R.         |           | Pitrou             | 1             |
| 115     |                    | Montélimar, France           | Roubion R.       | 1806      | Voglie             | 3             |
| 116     | Actius             | Rome, Italy                  |                  | 138       | M. Rusticus        | 7             |
| 117     | Neuf               | Paris, France                | Seine R.         | 1578-1604 | Cerceau & Marchand | 12            |
| 118     | Rock River         | Watertown, Wis., U.S.A.      | Rock R.          | 1902-03   | Loweth             | 4             |
| 119     | Coldstream         | Coldstream, Scotland         | Tweed R.         | 1771—     | Smeaton            | 5             |
| 120     | L. Conemaugh No. 3 | Summerhill, Penn., U.S.A.    | L. Con. R. H. W. | 1887      | Brown              | 3             |
| 121     | L. Conemaugh No. 2 | Penn., U.S.A.                | L. Conemaugh R.  |           | Brown              | 2             |
| 122     | Stockport Viad.    | Stockport, England           |                  |           |                    | 22            |
| 123     |                    | Carlisle, Scotland           | Eden R.          |           | Smirke             | 5             |
| 124     | Rivanna Aq.        | U.S.A.                       |                  |           | Ellet              | 5             |
| 125     | Teviot-Tweed       | Near Kelso, Scotland         | Teviot R.        | 1794-95   | Ellet              | 3             |
| 126     | Houghton River     |                              |                  |           | Haskoll            |               |
| 127     | Conon              | England                      | Conon R.         | 1809      | Telford            | 1             |
|         |                    |                              |                  |           |                    | 2             |
|         |                    |                              |                  |           |                    | 2             |
| 128     | Boberthal          | Near Bunzlau, Silesia        | Bober R. & Val.  |           |                    | 5             |
|         |                    |                              |                  |           |                    | 30            |
| 129     | Cher               | France                       | Cher R.          |           | Beaudemoulin       | 6             |
| 130     | Scrvia             | Italy                        | Scrvia R.        | 1850      | Ferraris           | 3             |
| 131     | Cinq-Mars          | France                       | Loire R.         | 1845-46   | Bailloud           | 10            |
| 132     |                    | Val-Benoist, Belgium         |                  | 1832      |                    | 5             |
| 133     |                    | Furand                       | Furand, R.       | 1834      | Montluisant        | 1             |
| 134     |                    | Auzon, France                | Vienne R.        | 1846-47   | Beaudemoulin       | 5             |
| 135     | Chante-Perdrix     | France                       |                  |           | Lamothe            | 9             |
| 136     | Landwasser V.      |                              | Landwasser R.    | 1901      |                    | 5             |
| 137     | Raritan River      | New Brunswick, N. J., U.S.A. | Raritan R.       | 1902-3    | Brown              | 10            |
|         |                    |                              |                  |           |                    | 2             |
|         |                    |                              |                  |           |                    | 8             |
| 138     | Kew                | Kew, England                 | Thames R.        | 1789      | Paine              | 1             |
|         |                    |                              |                  |           |                    | 2             |
|         |                    |                              |                  |           |                    | 4             |
| 139     | Bow                | Stratford, England           | Lea R.           | 1835-39   | Walker & Burgess   | 1             |
| 140     |                    | Near York, England           | Ouse R.          |           | J. & B. Greene     | 3             |
| 141     | Montignac          | France                       | Vézère R.        | 1766-72   | Tardif             | 3             |
| 142     |                    | Lancaster, England           |                  |           |                    | 5             |
| 143     | Brig o' Balgownie  | Old Aberdeen, Scotland       | Don R.           | 1281      | Bishop Cheyne      | 1             |
| 144     |                    | Horbury, England             | Aire R.          | 1775      | Clinchamp          | 5             |
| 145     | Bellecour          | Lyon, France                 |                  |           |                    | 5             |
| 146     | Viad. d'Arles      | Near d'Arles, France         | Valley           | 1780-1810 |                    | 31            |

\* Maximum

REMARKS.—108. Replaced No. 89. 110. Removed for Manch. Ship. Canal. 111. Gravel foundation. Coffers built under the circular. See No. 12. 118. C. M. & St. P. Ry. 120. Pitts. Div. Pa. Ry. Skew 60°. Ribbed. On piles. 121. Pitts. Div. Pa. Ry. Stone parapet. 122. Manchester-Birmingham. H.=105'9. 123. Intrados has five centres. 124. James River and Kanawha Canal. 128. Berlin-Breslau. Intrados of each at same elevation; 75 ft. high.



## ARRANGED ACCORDING TO SPAN—(Continued).

## ARCHES.

| Span.        | Rise. | Thickness of Arch Ring at Crown $h_c$ . | At Springing, $h_s$ . | Curve. | Radius at Crown. | $\frac{h_c}{R}$ . | Width, Face to Face at Crown. | Thickness of Piers at Springing. | Material.       | Class of Bridge. | Reference.            | Number. |
|--------------|-------|-----------------------------------------|-----------------------|--------|------------------|-------------------|-------------------------------|----------------------------------|-----------------|------------------|-----------------------|---------|
| 62.3         | 15.6  |                                         |                       |        |                  |                   |                               |                                  |                 | H.W.             | O. June 1898          | 108     |
| *62.3        |       |                                         |                       |        |                  |                   | 50.8                          |                                  |                 | H.W.             | L. Q.                 | 109     |
| *63.0        | 31.5  |                                         |                       | $C_2$  | 31.5             |                   |                               |                                  | Limestone       | H.W.             | R. Dec. 1888          | 110     |
| 63.0         |       |                                         |                       | $C_2$  |                  |                   | 42.0                          |                                  |                 |                  | A. 1844, p. 444       | 111     |
| 63.9         | 10.7  | 5.3                                     |                       | $E$    | 55.4             | .096              | 125.6                         | 11.7                             |                 |                  | F. 1852, p. 282       | 112     |
| 63.9         | 19.2  | 5.3                                     |                       | $E$    | 55.7             | .092              |                               | 12.8                             |                 |                  | F. 1852, p. 284       | 113     |
| 63.9         | 23.5  | 3.7                                     |                       | $E$    | 59.7             | .062              | 22.4                          |                                  |                 |                  | F. 1852, p. 276       | 114     |
| 64.0         | 21.3  | 4.2                                     |                       | $E$    | 50.6             | .084              |                               | 19.4                             |                 |                  | F. 1852, p. 286       | 115     |
| 64.0 to 25.4 | 32.0  | 4.6                                     |                       | $C_2$  | 32.0             | .150              | 50.8                          | 24.5                             |                 | H.W.             | F. 1852, p. 274       | 116     |
| 64.0 to 45.3 | 32.0  | 4.5                                     |                       | $C_2$  | 32.0             | .141              | 72.5                          | 13.8                             |                 | H.W.             | F. 1852, p. 276       | 117     |
| 64.0 to 45.3 | 22.7  |                                         |                       |        | 22.7             |                   |                               |                                  |                 |                  |                       |         |
| 64.0 to 64.0 | 16.5  | 3.0                                     | 3.0                   |        | 39.3             | .076              | 28.3                          | 8.0                              | Ring sand stone | Ry.              | B. Mar. 26, 1903      | 118     |
| 64.0 to 64.0 | 16.0  | 2.7                                     | 2.7                   | $C$    | 40.0             | .067              | 23.0                          |                                  |                 |                  | L.                    | 119     |
| 64.0 to 65.0 | 32.5  | 2.8                                     |                       | $C$    | 40.0             |                   |                               |                                  | Brick           | Ry.              | E.                    | 120     |
| 65.0         | 21.0  | 3.8                                     | 7.3                   | $E$    | 32.5             | .086              | 32.0                          |                                  |                 | Ry.              | F. 1852, p. 158. J    | 122     |
| 65.0         | 15.0  | 2.8                                     | 2.8                   | $C$    | 42.7             | .066              | 136.0                         | 7.0                              |                 | H.W.             | C. 1855-6             | 123     |
| *65.0        | 17.0  |                                         |                       | $C$    | 32.5             | .086              | 23.0                          |                                  | Canal           | Ry.              | Pub. Wks, U. S., '41  | 124     |
| 65.0         | 32.5  | 2.8                                     | 2.8                   | $C_2$  | 32.5             | .086              |                               |                                  |                 | H.W.             | L.                    | 125     |
| 65.0         | 21.8  | 3.0                                     |                       | $C$    | 36.4             | .082              | 20.0                          | 8.0 to 6.5                       | Freestone       | H.W.             | I.                    | 126     |
| 55.0         |       |                                         |                       |        |                  |                   |                               |                                  |                 |                  | L. J. F. 1852, p. 286 | 127     |
| 45.0         |       |                                         |                       |        |                  |                   |                               |                                  |                 |                  |                       |         |
| 65.6 less    |       |                                         |                       | $E$    |                  |                   |                               |                                  |                 | Ry.              | F. 1852, p. 212       | 128     |
| 65.6         | 21.9  | 3.3                                     |                       | $E$    | 47.6             | .069              | 8.5                           |                                  |                 |                  |                       |         |
| 65.6         | 13.1  | 3.9                                     |                       | $E$    | 47.5             | .082              | 29.5                          | 13.1                             | Brick ring      | Ry.              | F. 1852, p. 294       | 129     |
| 65.6         | 21.6  | 3.9                                     |                       | $E$    | 47.7             | .082              | 28.9                          | 11.5                             |                 | Ry.              | F. 1852, p. 296       | 130     |
| 65.6         | 8.8   | 3.3                                     |                       | $C$    | 65.5             | .054              |                               |                                  |                 | Ry.              | F. 1852, p. 294       | 131     |
| 65.6         | 32.8  | 3.3                                     |                       | $C_2$  | 32.8             | .101              | 26.2                          |                                  | Freestone       | Ry.              | J. F. 1852, p. 290    | 132     |
| 65.6         | 21.9  | 3.3                                     |                       | $E$    | 47.5             | .069              | 8.5                           |                                  |                 | H.W.             | J. F. 1852, p. 294    | 133     |
| 65.6         | 32.8  |                                         |                       |        |                  |                   | 26.2                          |                                  |                 | Ry.              | G. 1st Tri., 1901     | 134     |
| 65.6         | 32.8  | 3.0                                     | 4.4                   | $C_2$  | 32.8             | .091              | 8.5                           | 11.5                             | Limestone       | Ry.              | Engineer, April, '04  | 135     |
| 66.0         | 33.0  |                                         |                       | $C_2$  | 33.0             |                   | 55.0                          | 8.0                              |                 | Ry.              | N. Oct. 10, 1903      | 137     |
| 56.0         | 28.0  | 3.2                                     |                       |        | 28.0             | .114              |                               | to 11.0                          |                 |                  |                       |         |
| 51.0         | 25.5  |                                         |                       |        | 25.5             |                   |                               |                                  |                 |                  |                       |         |
| 72.0         | 24.0  | 3.3                                     |                       |        | 39.0             | .084              | 24.0                          |                                  |                 | H.W.             | K. June 14, 1895      | 138     |
| 66.0         |       |                                         |                       |        |                  |                   |                               |                                  |                 |                  |                       |         |
| 66.0         | 13.8  | 2.5                                     | 4.0                   | $E_2$  | 81.0             | .031              | 42.5                          |                                  | Granite ring    | H.W.             | A. Oct. 1837, p. 14   | 139     |
| 66.0         | 19.3  | 3.5                                     |                       | $E$    |                  |                   | 29.6                          | 10.0                             | Brick interior  | Ry.              | A. April 1839. S      | 140     |
| 66.1 to 42.6 | 21.3  |                                         |                       | $C$    |                  |                   | 17.1                          |                                  |                 |                  | F. 1852, p. 280       | 141     |
| 66.9         | 33.5  |                                         |                       | $C_2$  | 33.5             |                   | 13.1                          |                                  |                 | Canal            | F. 1852, p. 286       | 142     |
| 67.0         |       |                                         |                       | $P$    |                  |                   |                               |                                  |                 | H.W.             | L.                    | 143     |
| 67.9 to 55.4 | 12.6  | 6.4                                     |                       | $E$    | 57.5             | .111              | 11.7                          |                                  |                 |                  | F. 1852, p. 282       | 144     |
| 68.2         | 24.4  | 2.7                                     |                       | $E$    | 53.3             | .050              | 18.6                          |                                  |                 |                  | F. 1852, p. 284       | 145     |
| 68.9         | 23.0  |                                         |                       | $E$    |                  |                   |                               |                                  |                 | Ry.              | F. 1852, p. 129       | 146     |

† About.

120. Tours-Bordeaux. Skew  $34^\circ 30'$ . 130. Turin-Genoa. 131. Tours-Nantes. 134. Tours-Bordeaux. 135. Cost 1,017,300 f. 136. Thuis-Engadine. 137. Penn. Ry. 139. Replaced old bridge of 1100-1118. Slight skew. Foundation on gravel. 140. Great North of England Ry. On piles. 142. Bottom of canal to intrados = 8'.5. 146. Avignon-Marseilles. H. = 27'.9. Pile foundation.



TABLE II.—DATA FOR ABOUT 500 ARCH BRIDGES

MASONRY

| Number. | Name.             | Place.                            | Over.             | Date.      | Engineer.      | No. of Spans. |
|---------|-------------------|-----------------------------------|-------------------|------------|----------------|---------------|
| 147     | Central Ave.      | Indianapolis, Ind., U.S.A.        | Fall Crk.         | 1899       |                | 2             |
| 148     | Bord              |                                   | Oeil R. (?)       | 1764       | Leclerc        | 1             |
| 149     | Karlsbùrcke       | Prague, Austria                   | Moldau            | 1357-1507  |                | 16            |
| 150     | Potarch           | Wales(?)                          | Dee R.            | 1813       | Telford        | 1             |
| 151     | Lockwood          | Nr. Huddersfield, England         | Sheffield R. Val. | 1846-49    | Hawkshaw       | 2             |
| 152     | Wellesley         | Limerick, Ireland                 | Shannon R.        | 1827       | Nimmo          | 32            |
| 153     | Wharncliffe Viad. | Brent-Knoll, England              | Brent R.          | 1836-37    | Brunel         | 5             |
| 154     | Crum Elbow        | Hyde-Pk.-on-Hudson, N. Y., U.S.A. | Crum Elbow Crk.   | 1898       | Morris         | 1             |
| 155     | Wissahickon       | Pa., U.S.A.                       | Lea R.            |            | Braithwaite    | 1             |
| 156     | Shock's Mills     | Shock's Mills, Pa., U.S.A.        | Susquehanna R.    | 1881-2     | Buchholz       | 1             |
| 157     | Rockville         | Rockville, Penn., U.S.A.          | Susquehanna R.    | 1903-      | Brown          | 28            |
| 158     |                   |                                   |                   | 1901       | Brown          | 48            |
| 159     | Mazares           | France(?)                         | Lère R.           | 1787       | Pertichamp     | 3             |
| 160     |                   | Pavia, Italy                      | Ticino R.         | 14th Cent. | Under Visconti | 7             |
| 161     |                   | Swatara, Penn., U.S.A.            |                   |            | Osborn         | 6             |
| 162     | Homps             |                                   | Aude R.           | 1785       | Ducros         | 3             |
| 163     | Spoletto Aq.      | Spoletto, Italy                   | Valley(?)         | 741        | Theodelapius   | 10            |
| 164     | Colorado St.      | St. Paul, Minn., U.S.A.           |                   | 1889       | Rundlett       | 1             |
| 165     |                   | Helmsdale, Scotland(?)            | Helmsdale R. (?)  | 1816       | Telford        | 2             |
| 166     |                   | Luxemburg, Germany                | Petrusse R.       | 1899-1903  | Sejourne       | 2             |
| 167     | North             | Edinburgh, Scotland               | Waverley Ry. Sta. | 1763†      |                | 3             |
| 168     | North Loch        | Nr. Edinburgh, Scotland           | N. Loch Valley    |            | Mylne          | 3             |
| 169     | Schuyllkill       | Philadelphia, Pa., U.S.A.         | Schuyllkill R.    |            |                | 4             |
| 170     | Black Rock Tunnel | Penn., U.S.A.                     |                   | 1836       | Robinson       | 4             |
| 171     | London (old)      | London, England                   | Thames R.         | 1176-1209  | Peter of Cole- | 19            |
| 172     | Brunswick         | N. Brunswick, N. J., U.S.A.       | Raritan R.        | 1758       | [church        | 1             |
|         |                   |                                   |                   | 1902       | Brown          | 1             |
| 173     | Aulne             | France                            |                   |            | Arnoux         | 1             |
| 174     | Boston Ave.       | Medford, Mass., U.S.A.            | Mystic R.         | 1900       | Bailey         | 12            |
| 175     | Zeniec            | Austria                           |                   |            |                | 1             |
| 176     | Schmiedtobel      | Near Klösterle, Austria           | Schmiedtobel      | 1882?      | Huss           | 3             |
| 177     | Po                | Near Valenza, Italy               | Po R.             | 1850       | Rovere         | 2             |
| 178     |                   | Dresden, Saxony                   | Elbe R.           | 1170-1260  | Fotius         | 21            |
| 179     | Teviot-Tweed      | Kelso, Scotland                   | Tweed R.          | 1799-1803  | Rennie         | 18            |
| 180     | Conon Viad.       | Conon, Scotland                   | Conon R.          |            | Mitchell       | 5             |
| 181     | Staines           | Staines, England                  | Thames R.         | 1832       | Rennie         | 5             |
| 182     |                   | Fucecchio, Italy                  | Arno R.           | 1869       |                | 1             |
|         |                   |                                   |                   |            |                | 2             |
|         |                   |                                   |                   |            |                | 5             |

\* Maximum.

REMARKS.—147. Cost about \$39,000. Pile foundation 3' center to center. 149. Partially destroyed, flood 1890. 151. Huddersfield & Sheffield Ry. 70' and 45' arches on skew and ribbed. H.=122'. 152. "Bell-mouthed" type. 153. Great Western Ry. of England. 156. Cost \$375,000. 157. Penn. Ry., two tracks. 158. Penn. Ry., four tracks, 6° curve at one end. 160. Replaced by another bridge. Covered. Roof supported by marble columns 161. Lebanon Valley Ry. 163. Now in use. Piers of stone. H.=292'. at springing. 164

## ARRANGED ACCORDING TO SPAN—(Continued).

## ARCHES.

| Span.   | Rise. | Thickness of Arch King at Crown ft. | At Springing, ft. | Curve.         | Radius at Crown. | $\frac{L}{R}$ . | Width, Face to Face at Crown. | Thickness of Piers at Springing. | Material.          | Class of Bridge. | Reference.                        | Number. |
|---------|-------|-------------------------------------|-------------------|----------------|------------------|-----------------|-------------------------------|----------------------------------|--------------------|------------------|-----------------------------------|---------|
| 69.0    | 14.0  | 2.0                                 | 2.0               |                |                  |                 | 50.0                          | 6.0                              | Limestone          | H.W. and E. Ry.  | H. W. Klausman                    | 147     |
| 69.0    | 15.0  | 2.0                                 | 2.0               |                |                  |                 | 50.0                          | 6.0                              | Ring Oolitic       |                  | F. 1852, p. 280                   | 148     |
| 69.2    | 25.6  |                                     |                   | E              |                  |                 |                               |                                  |                    |                  | P. 1896, p. 126                   | 149     |
| 69.5    |       |                                     |                   |                |                  |                 |                               |                                  |                    |                  | F. 1852, p. 288                   | 150     |
| 70.0    | 25.8  | 2.5                                 |                   | C              | 36.6             | .068            | 20.0                          | 10.0                             |                    |                  |                                   |         |
| 60.0    |       |                                     |                   |                |                  |                 |                               |                                  |                    |                  |                                   |         |
| 70.0    | 8.0   | 2.6                                 | 2.6               |                |                  |                 | 28.0                          | 4.5                              | Sandstone          | Ry.              | C. 1850                           | 151     |
| 45.0    |       |                                     |                   |                |                  |                 |                               |                                  |                    |                  | A. April, 1851, p. 215            |         |
| 30.0    | 15.0  | 1.5                                 | 1.5               | E              | 15.0             | .100            | 43.0                          | 10.0                             |                    | H.W. Ry.         | C. 1855-6. S. Q.                  | 152     |
| 70.0    | 9.0   | 12.0                                | 13.0              | E              |                  |                 | 35.0                          |                                  | Brick, stone fac'g |                  | A. 1837-8, p. 126                 | 153     |
| 70.0    | 17.5  | 3.0                                 |                   |                |                  |                 |                               |                                  |                    | H.W.             | F. 1852, p. 181                   |         |
|         |       |                                     |                   |                |                  |                 |                               |                                  |                    |                  | B. Feb. 16, 1899                  | 154     |
| 70.0    | 7.0   | 2.5                                 | 2.5               |                |                  |                 |                               |                                  |                    |                  |                                   |         |
| 70.0    | 17.5  | 3.8                                 |                   | E              |                  |                 | 30.0                          |                                  | Brick ring         | Ry.              | S.                                | 155     |
| 70.0    | 23.0  | 3.0                                 |                   |                |                  |                 | 28.0                          | 9.5                              | Talcose sl.        | Ry.              | B. May 2, 1902                    | 156     |
| 70.0    | 20.0  | 3.5                                 | 3.5               | C              | 40.6             | .086            | 28.0                          | 8.0                              |                    | Ry.              | T. March 11, 1904                 | 157     |
| 70.0    | 20.0  | 3.5                                 | 3.5               | C              | 40.6             | .086            | 50.0                          | 8.0                              | Stone ring         | Ry.              | B. March 10, 1900                 | 158     |
|         |       |                                     |                   |                |                  |                 |                               |                                  |                    |                  | T. Oct. 25, 1901                  |         |
| 70.3    | 35.2  |                                     |                   | C <sub>2</sub> | 35.2             |                 | 16.0                          |                                  |                    |                  | F. 1852, p. 284                   | 159     |
| 70.4    | 64.0  | 3.9                                 |                   | P              | 85.3             | .046            | 18.7                          |                                  | Brick              | H.W. Ry.         | F. 1852, p. 276                   | 160     |
| 70.0    | 25.0  | 3.5                                 |                   | C              | 37.0             | .093            | 22.5                          | 8.5                              | Brick              |                  | I.                                | 161     |
| 70.2    | 9.4   | 4.3                                 |                   | C              | 68.2             | .063            | 11.7                          |                                  | Freestone          |                  | J. F. 1852, p. 284                | 162     |
| 70.3    | 35.2  |                                     |                   | C <sub>2</sub> | 35.2             |                 | 11.7                          |                                  | Brick              |                  | J. Q.                             | 163     |
| 70.5    | 11.0  | 3.5                                 | 4.6               | C              | 62.0             | .056            | 150.0                         |                                  | Limestone          | Aqued't H.W.     | N. Nov. 23, 1889                  | 164     |
| 70.7    | 25.0  |                                     |                   | C              |                  |                 | 21.0                          | 14.0                             |                    |                  | F. 1852, p. 288                   | 165     |
| 70.8    | 35.4  | 2.9                                 |                   |                |                  |                 |                               |                                  |                    | H.W.             | N. Oct. 12, 1901                  | 166     |
|         |       |                                     |                   |                |                  |                 |                               |                                  |                    |                  | N. Mar. 1, 1902                   |         |
| 71.0    | 35.5  |                                     |                   | C <sub>2</sub> | 35.5             |                 | 54.5                          | 13.5                             | Craigleith stone   | H.W.             | K. Oct. 1899, pp. 423, 491        | 167     |
| *72.0   | 36.0  | 2.8                                 |                   | C <sub>2</sub> | 36.0             | .078            | 42.3                          |                                  |                    | H.W. Ry.         | I.                                | 168     |
| 72.0    | 16.5  | 2.0                                 |                   | C              | 47.5             | .042            |                               |                                  |                    | Ry.              | F. 1852, p. 202                   | 169     |
| 72.0    | 16.5  | 2.8                                 | 2.8               | C              | 47.5             | .059            | 18.3                          | 8.0                              |                    | Ry.              | I.                                | 170     |
| 9 to 20 |       |                                     |                   |                |                  |                 | 40†                           | 25' to 34'                       |                    | H.W.             | L.                                | 171     |
| 72.0    |       |                                     |                   |                |                  |                 |                               |                                  |                    |                  |                                   |         |
| 72.0    | 36.0  | 3.3                                 | 3.3               | C              | 36.0             | .092            | 55.0                          |                                  |                    | Penn. Ry.        | B. Jan. 30, 1902, p. 86           | 172     |
| 66.0    | 33.0  | 3.3                                 | 3.3               | C              | 33.0             | .100            | 55.0                          | 9.0                              |                    | 4 tracks         |                                   |         |
| 56.0    | 28.0  | 3.2                                 | 3.2               | C              | 28.0             | .114            | 55.0                          |                                  |                    |                  |                                   |         |
| 51.0    | 25.5  | 3.0                                 | 3.0               | C              | 25.5             | .118            | 55.0                          | 8.0                              |                    |                  |                                   |         |
| 72.2    | 31.1  |                                     |                   | C <sub>2</sub> | 31.1             |                 | 26.6                          |                                  |                    |                  |                                   |         |
| 72.2    | 15.5  | 2.5                                 | 2.5               |                | 49.8             | .050            | 56.0                          |                                  | Granite            | Ry.              | G. 1st Tri. 1901                  | 173     |
| 72.2    |       |                                     |                   |                |                  |                 |                               |                                  |                    | H.W. Ry.         | Wm. G. Taylor                     | 174     |
| 72.2    |       | 2.6                                 | 4.3               |                |                  |                 |                               |                                  |                    | Ry.              | B. Dec. 7, 1893                   | 175     |
| 72.2    | 36.1  | 4.1                                 | 7.5               | C <sub>2</sub> | 36.1             | .114            |                               |                                  |                    |                  | G. 1888, XVI, p. 575              | 176     |
| 39.4    | 19.7  |                                     |                   |                | 19.7             |                 | 117.1                         |                                  |                    |                  |                                   |         |
| 72.2    | 11.2  | 3.8                                 |                   | C              | 63.0             | .050            | 32.8                          | 0.84&                            | Brick              | Ry.              | F. 1852, p. 296                   | 177     |
| 72.5    | 36.3  | 6.4                                 |                   | C <sub>2</sub> | 36.3             | .180            | 37.3                          | 29.9                             |                    | H.W.             | F. 1852, p. 274                   | 178     |
| 73.0    | 21.0  |                                     |                   | E              |                  |                 | 26.0                          |                                  |                    | H.W. Ry.         | L.                                | 179     |
| 73.0    |       | 3.0                                 | 4.0               |                |                  |                 |                               |                                  |                    |                  | K. Sept. 20, 1867, pp. 206, 257   | 180     |
| 74.0    | 9.3   | 3.0                                 | 6.0               |                | 78.0             | .038            | 34.0                          | 8.3                              | Granite            | H.W.             | C. S. K. Sept. 13, 1881           | 181     |
| 66.0    | 8.3   |                                     |                   |                |                  |                 |                               |                                  |                    |                  | 1895, p. 336                      |         |
| 74.0    |       |                                     |                   |                |                  |                 |                               |                                  |                    |                  | Jour. F. Inst., Feb. 1870, p. 101 | 182     |

† About.

Skew 60°. Pile foundation. 166. Four lateral arches in each spandrel. 167. Replaced by steel, 1808-9. 168. H.=65' to top of parapet. 169. To Mt. Carbon, W. Va. 170. Phila. & Reading Ry. 171. Replaced in 1824-31. 172. 72' span. Stepped Skew 63° 10'. Backing of masonry 24' to 28' above springing, then earth fill. 173. Cost 2,165,000 f. 174. Cost \$12,300. Skew 16°. 175. Austrian State Ry. 176. Austrian State Ry. 177. Alessandria to Lake Maggiore. 179. Below Elliot's Bridge. 180. Highland Ry. one track.

TABLE II.—DATA FOR ABOUT 500 ARCH BRIDGES

MASONRY

| Number. | Name.                   | Place.                      | Over.          | Date.                | Engineer.              | No. of Spans. |
|---------|-------------------------|-----------------------------|----------------|----------------------|------------------------|---------------|
| 183     | Albany St.              | Scotland                    | Earn R.        | 1781-1821            | Rennie                 | 3             |
| 184     |                         | N. Brunswick, N. J., U.S.A. | Raritan R.     | 1892                 | Dean and Westbrook     | 7             |
| 185     | Whitadder               | Allantown England(?)        | Whitadder R.   | 1842                 | Stevenson              | 2             |
| 186     | Westminster (old)       | Westminster, England        | Thames R.      | 1738-50              | Labelye                | 15            |
| 187     |                         | St. Maxence, France         | Oise R.        | 1774-85              | Perronet               | 3             |
| 188     |                         | Navilly, France             | Doubs R.       | 1780                 | Gauthey                | 5             |
| 189     |                         | Roanne, France              | Loire R.       | 1789-1809            | Vareigne and Vimar     | 7             |
| 190     |                         | Compiègne, France           | Oise R.        | 1783                 | Lahite                 | 3             |
| 191     | Pont Royal              | Semur, France               | Armançon R.    | 1780                 | Dumorey                | 1             |
| 192     |                         | Paris, France               | Seine R.       | 1685                 | Mansard                | 5             |
| 193     | Cestius                 | Rome, Italy                 | Tiber R.       | 1st c. B.C.          | Under Cestius          | 1             |
| 194     | Hyde Park               | Perth, Scotland             | Tay R.         | 1760-71              | Smeaton                | 9             |
| 195     |                         | Readville, Mass., U.S.A.    | Hyde Park Ave. | 1897-98              | Curtis                 | 1             |
| 196     |                         | Italy                       | Taro R.        | 1816-20              | Cocconcelli            | 20            |
| 197     |                         | Orleans, France             | Loire R.       |                      |                        |               |
| 198     | Crown St. or Hutchinson | Glasgow, Scotland           | Clyde R.       | 1829-33              | Stevenson              | 1             |
| 199     | Molle                   | Near Rome, Italy            | Tiber R.       | †100 B.C.            | Scaurus                | 2             |
| 200     | Fabricius               | Rome, Italy                 | Tiber R.       | †62 B.C.             | Fabricius              | 1             |
| 201     |                         | Scotland                    | Avon R.        | 1820                 | Telford                | 1             |
| 202     | Annan                   | Near Johnstown, Scotland    | Annan R.       | 1820                 | Telford                | 1             |
| 203     | High Bridge             | New York, N. Y., U.S.A.     | Harlem R.      | 1837-42              | Jervis                 | 8             |
| 204     | Conewago                | W. of Conew'g, Pa., U.S.A.  | Conewago Crk.  | 1891-92              | Brown                  | 7             |
| 205     | Schuylkill Falls        | Philadelphia, Pa., U.S.A.   | E. Pk. Drive   | 1890                 | Nichols                | 3             |
| 206     | Conemaugh               | Viad. Station, Pa., U.S.A.  | Conemaugh R.   | 1833                 | Penn. Ry.              | 1             |
| 207     | Posen Viad.             | Posen, Germany              |                |                      |                        |               |
| 208     | Vittorio                | Turin, Italy                | Po R.          | 1810                 | Pertinchamp            | 5             |
| 209     | Painsville Viad.        | Near Painsville, O., U.S.A. |                |                      |                        | 4             |
| 210     |                         | Trilport, France            | Marne R.       | 1758-64              | Peronnet               | 3             |
| 211     | Pont du Gard            | 14 m. from Nismes, France   | Gardon R.      | Bet. 27 B.C.-14 A.D. | Under Agrippa          | 1             |
| 212     |                         | Prague Austria              | Moldau R.      | 1st tier             |                        | 3             |
| 213     |                         | York, England               | Ouse R.        | 2d tier              |                        | 12            |
| 214     |                         | Near Montlouis, France      | Loire R.       | top tier             |                        | 36            |
| 215     |                         | Tablonica Austria           |                |                      |                        | 8             |
| 216     |                         | Baiersbann, Germany         | Forbach R.     | 1845                 | Morandière             | 3             |
| 217     | Oise                    | Near Pontoise, France       | Oise R.        | 1890                 | Leibbrand              | 1             |
|         |                         |                             |                | 1843                 | De Breville and Couche | 3             |

\* Maximum.

REMARKS.—183. On piles. 184. Skew. 186. First use of modern caisson. Replaced by cast-iron bridge. 187. Radial joints in spandrels. 193. Continuation of Fabricius Bridge. 195. Skew  $61^{\circ}$  and  $77^{\circ} 52'$ . N. Y., N. H. & H. Ry. 200. 13' arch in pier. 201. Glasgow-Carlisle. 203. H.=100'0. Parapet 116' above water. 204. Phila. Div. Pa. Ry.  $6^{\circ}$  curve. Two tracks. 205. Phila. & Reading Ry. 206. Pitts. Div. Pa. Ry. Destroyed by Johnstown Flood, 1889. 208. Commenced by French 1810. Completed by King Victor



## ARRANGED ACCORDING TO SPAN—(Continued).

## ARCHES.

| Span.        | Rise. | Thickness of Arch Ring at Crown $h_c$ . | At Springing, $h_s$ . | Curve. | Radius at Crown. | $\frac{h_c}{R}$ . | Width, Face to Face at Crown. | Thickness of Piers at Springing. | Material.                                                          | Class of Bridge. | Reference.                   | Number. |
|--------------|-------|-----------------------------------------|-----------------------|--------|------------------|-------------------|-------------------------------|----------------------------------|--------------------------------------------------------------------|------------------|------------------------------|---------|
| 75.0         | 18.5  | 2.7                                     |                       | $E$    |                  |                   | 40.0                          | 18.6                             | Brick<br>stone fac'g<br>Soft red<br>sandstone<br>Portland<br>stone | H.W.             | S. 1839                      | 183     |
| *75.0        | 15.0  | 2.4                                     | 2.4                   | $C$    | 54.4             | .044              | 35.0                          | 10.0                             |                                                                    | H.W.             | B. April 16, 1802,<br>p. 373 | 184     |
| 75.0         | 11.5  | 2.5                                     | 3.0                   |        |                  |                   | 30.0                          | 10.0                             |                                                                    | H.W.             | A. March, 1844, p. 128       | 185     |
| *76.0        | 38.0  | 7.6                                     | 14.0                  | $C_2$  | 38.0             | .200              | 46.0                          | 18.1                             |                                                                    | H.W.             | K. 1895, p. 306; L. F.       | 186     |
| 76.0         | 6.4   | 4.8                                     |                       | $C$    | 118.2            | .041              | 76.7                          | 9.6                              | H.W.                                                               | H.W.             | S. F. 1852, p. 282           | 187     |
| 76.7         | 25.6  | 4.3                                     |                       | $E$    | 53.3             | .080              |                               | 16.0                             |                                                                    | H.W.             | F. 1852, p. 284              | 188     |
| 76.7         | 20.6  | 3.2                                     |                       | $E$    | 63.0             | .050              | 35.1                          | 13.3                             |                                                                    | H.W.             | F. 1852, p. 284              | 189     |
| 76.7 to 70.3 | 25.6  | 4.3                                     |                       | $E$    | 53.3             | .080              | 32.0                          | 12.8                             |                                                                    | H.W.             | F. 1852, p. 278              | 190     |
| 76.8         | 19.3  | 3.2                                     |                       | $C_2$  | 38.4             | .084              |                               |                                  | H.W.                                                               | H.W.             | F. 1852, p. 282              | 191     |
| 76.8 to 68.2 | 38.4  | 3.2                                     |                       | $E$    |                  |                   | 55.4                          | 14.9                             |                                                                    | H.W.             | F. 1852, p. 276              | 192     |
| 76.8         | 25.7  | 4.8                                     |                       |        |                  |                   |                               |                                  |                                                                    | H.W.             | Q. F. 1852, p. 274           | 193     |
| *76.8        | 24.6  | 4.8                                     |                       | $C_2$  | 38.4             | .125              | 30.0                          |                                  |                                                                    | H.W.             | L. Aug. 12, 1808             | 194     |
| *77.0        | 38.4  | 4.8                                     |                       |        |                  |                   | 20.0                          |                                  | Brick                                                              | H.W.             | T. Aug. 12, 1808             | 195     |
| 78.0         | 14.3  | 3.0                                     | 3.0                   | $C$    | 60.3             | .040              | 165.0                         | 15.1                             |                                                                    | Ry.              | F. 1852, p. 288              | 196     |
| 78.7         | 21.6  | 1.2                                     |                       | $C$    | 40.6             | .026              |                               |                                  |                                                                    | Ry.              | I. C. 1855-6                 | 197     |
| 79.0         | 26.3  | 4.0                                     | 4.5                   | $E$    | 64.0             | .054              | 38.0                          |                                  |                                                                    | H.W.             | C. 1855-6                    | 198     |
| 79.3 to 51.0 | 13.4  | 3.5                                     |                       | $C$    | 64.7             | .054              |                               |                                  | Sandstone                                                          | H.W.             | Q. Cresy's Enc. C. E.        | 199     |
| 80.0         | 11.8  | 3.5                                     |                       |        | 65.0             | .054              | 29.0                          |                                  |                                                                    | H.W.             | F. 1852, p. 274              | 200     |
| 80.0         | 8.7   | 3.5                                     |                       |        |                  |                   |                               |                                  |                                                                    | H.W.             | M. No. 1207                  | 201     |
| 80.0         | 40.0  | 6.0                                     |                       | $C_2$  | 40.0             | .150              | 51.2                          | 32.0                             |                                                                    | H.W.             | F. 1852, p. 288              | 202     |
| 80.0         | 39.8  | 6.0                                     |                       | $C$    | 39.8             | .151              | 27.0                          |                                  | Peperino,<br>tufa and<br>travertine                                | H.W.             | F. 1852, p. 288              | 203     |
| 80.0         | 20.0  |                                         |                       |        | 50.0             |                   |                               |                                  |                                                                    | H.W.             | F. 1852, p. 288              | 204     |
| 80.0         | 20.0  | 3.0                                     |                       | $C$    | 50.0             | .060              | 20.0                          |                                  |                                                                    | H.W.             | F. 1852, p. 288              | 205     |
| 80.0         | 40.0  | 2.5                                     | 2.5                   | $C_2$  | 40.0             | .062              | 21.0                          |                                  |                                                                    | H.W.             | F. 1852, p. 288              | 206     |
| 80.0         | 25.0  |                                         |                       |        | 25.0             |                   |                               |                                  | Aqued't                                                            | H.W.             | Johnson's Ency.              | 207     |
| 80.0         | 40.0  | 3.5                                     | 3.5                   | $C_2$  | 40.0             | .088              | 25.0                          | 12.0                             |                                                                    | H.W.             | Johnson's Ency.              | 208     |
| 80.0         | 20.0  | 3.0                                     | 3.0                   | $C$    | 43.8             | .069              | 30.0                          |                                  |                                                                    | H.W.             | Johnson's Ency.              | 209     |
| 80.0         | 40.0  | 3.0                                     |                       | $C_2$  | 40.0             | .075              |                               |                                  |                                                                    | H.W.             | Johnson's Ency.              | 210     |
| 80.0         | 16.0  | 4.7                                     |                       | $C$    | 58.0             | .081              |                               |                                  | Sandstone<br>Brick<br>Granite                                      | H.W.             | Johnson's Ency.              | 211     |
| *80.0        |       |                                         |                       | $E$    |                  |                   |                               |                                  |                                                                    | H.W.             | Johnson's Ency.              | 212     |
| 80.0         | 40.0  | 3.0                                     | 3.0                   | $C_2$  | 40.0             | .075              | 90.0                          | 10.0                             |                                                                    | H.W.             | Johnson's Ency.              | 213     |
| 80.4 to 76.7 | 28.8  | 4.8                                     |                       | $C$    | 42.0             | .114              | 32.0                          | 16.0                             |                                                                    | H.W.             | Johnson's Ency.              | 214     |
| 80.5         | 27.7  |                                         |                       |        |                  |                   |                               |                                  | Freestone                                                          | H.W.             | Johnson's Ency.              | 215     |
| 80.5         | 40.3  | 5.3                                     |                       | $C_2$  | 40.3             | .124              | 20.8                          |                                  |                                                                    | H.W.             | Johnson's Ency.              | 216     |
| 83.0         | 31.5  | 5.0                                     |                       |        | 31.5             | .150              |                               |                                  |                                                                    | H.W.             | Johnson's Ency.              | 217     |
| 81.0         | 25.5  | 5.0                                     |                       |        | 25.5             | .100              |                               |                                  |                                                                    | H.W.             | Johnson's Ency.              | 218     |
| 15.8         | 7.9   | 2.6                                     |                       |        | 7.9              |                   | 15.0                          |                                  | Granite                                                            | H.W.             | Johnson's Ency.              | 219     |
| *80.0        |       |                                         |                       | $P$    |                  |                   | 11.8                          |                                  |                                                                    | H.W.             | Johnson's Ency.              | 220     |
| *81.0        | 26.3  |                                         |                       | $E$    | 78.5             | .056              | 28.2                          | 10.7                             |                                                                    | H.W.             | Johnson's Ency.              | 221     |
| 81.2         | 23.3  | 4.4                                     |                       |        |                  |                   |                               |                                  |                                                                    | H.W.             | Johnson's Ency.              | 222     |
| 82.0         |       | 3.6                                     | 5.2                   |        |                  |                   |                               |                                  | Granite                                                            | H.W.             | Johnson's Ency.              | 223     |
| 82.0         | 9.8   | 2.0                                     | 2.6                   |        |                  |                   | 21.7                          |                                  |                                                                    | H.W.             | Johnson's Ency.              | 224     |
| 82.0         | 11.7  | 4.6                                     |                       | $C$    | 77.6             | .059              | 25.4                          | 10.1                             |                                                                    | H.W.             | Johnson's Ency.              | 225     |

† About.

Emmanuel. 209. Lake Shore & M. S. R.R. 210. First bridge entirely designed by Peronnet.  
 211. Fifth century, ends destroyed. Repaired 1743 and piers prolonged for new bridge. H. =  
 160'0. 212. Between Karlin and Bubua. Viaduct has 87 arches. 214. Orleans-Tours.  
 Damaged in War 1870-71. 215. Austrian State Ry. 216. Three-lead hinges. Cost 18,260 f.  
 217. Skew 76°. Ch. de fer du Nord.



TABLE II.—DATA FOR ABOUT 500 ARCH BRIDGES

MASONRY

| Number. | Name.          | Place.                                   | Over.            | Date.                | Engineer.                        | No. of Spans. |
|---------|----------------|------------------------------------------|------------------|----------------------|----------------------------------|---------------|
| 218     | Crueize Viad.  | Near Marvejois, France                   | Crueize R.       |                      |                                  | 6             |
| 219     | Stulz Viad.    |                                          | Stulz Gorge      |                      |                                  |               |
| 220     | Mussy Viad.    | Mussy, France                            | Mussy R.         | 1892-6               | Geoffroy, Morris<br>and Pouthier | 18            |
| 221     | Pont Royal     | Paris, France                            | Seine R.         | 1685                 | Mansard                          | 5             |
| 222     |                | Moret, France                            | Loing R.         | 1771                 | Peronnet                         | 3             |
| 223     |                | Elkader, Iowa, U.S.A.                    | Turkey R.        | 1888                 | Tschirgi                         | 2             |
| 224     | Cart           | Paisley, Scotland                        | Cart R.          | 1839                 | Locke                            |               |
| 225     | Big Walnut     | U.S.A.                                   |                  | 1902                 | Graham                           | 3             |
| 226     |                | Sisteron, France                         | Durance R.       | 1500                 |                                  | 1             |
| 227     | Cognet         | Hautes Alpes, France                     | Drac R.          | 1605                 |                                  | 2             |
| 228     |                | Maligny                                  | Serin R.         |                      | Werbruge                         | 1             |
| 229     |                | Darlaston, England                       |                  |                      |                                  |               |
| 230     |                | Coatsville, Pa., U.S.A.                  | W. B. Brandywine | 1902                 | Brown                            | 2             |
| 231     |                | Blois, France                            | Loire R.         | 1723                 | Gabriel                          | 7             |
| 232     |                | Bordeaux, France                         | Garonne R.       | 1813-22              | Deschamps                        | 11            |
| 233     | Lea Cut        | Lea Cut, England                         | Lea Cut R.       |                      |                                  | 17            |
| 234     | Salarius       | Narses, Italy                            | Teverone R.      | Rebuilt<br>6th cent. |                                  | 1             |
| 235     | Fouchards      | Samur, France                            | Thouet R.        |                      | Trudaine or<br>Voglie<br>Picot   | 3             |
| 236     | Pont de Pierre | Grenoble, France                         | Isère            | 1839                 |                                  | 1             |
| 237     |                | La Voulte, France                        | Allier R.        |                      |                                  | 2             |
| 238     |                | Albois(?), France                        | Aveyron R.       | 1770                 | Boesnier                         | 3             |
| 239     | Dee Viad.      | Bet. Rhos-y-Medre and<br>Chirk, Wales(?) | Dee R.           | †1849                |                                  | 3             |
| 240     | Dunkeld        | Dunkeld, Scotland                        | Tay R.           | 1809                 | Telford                          | 19            |
| 241     | Dean           | Near Edinburgh, Scotland                 |                  |                      | Telford                          | 7             |
| 242     | Licking Aq.    |                                          | Licking R.       |                      | Fisk                             |               |
| 243     |                | Castellane, France                       | Verdon R.        | 1404                 |                                  | 1             |
| 244     |                | Romans, France                           | Isère            |                      |                                  | 4             |
| 245     | Enz            | Near Hofen, Germany                      | Enz R.           | 1885                 | Leibbrand                        | 1             |
| 246     | Jena           | Paris, France                            | Seine R.         | 1806-12              | Lamandé                          | 5             |
| 247     |                | Stonleigh, England                       | Avon R(?)        | 1781-1821            | Rennie                           | 1             |
| 248     | Alcantara      | Toledo, Spain                            | Tagus R.         | 097                  | Romans(?)                        | 3             |
| 249     | Louis XVI(?)   | France                                   |                  | 1791                 | Perronet                         |               |
| 250     | Spey           | Fochabers, Scotland                      | Spey R.          |                      | Burn                             | 4             |
| 251     | Trinity        | Florence, Italy                          | Arno R.          | 1569                 | Ammanati                         | 3             |
| 252     |                | Pontoise, France                         | Oise R.(?)       | 1772                 | Peronnet                         | 3             |
| 253     | St. Edme       | Nogent-on-Seine, France                  | Seine R.         | 1766-69              | Peronnet                         | 1             |
| 254     | Vecchio        | Florence, Italy                          | Arno R.          | 1177                 | Gaddi                            | 3             |
| 255     |                | Neuville, France                         | Ain R.           | 1775                 | Aubry                            | 2             |

\* Maximum.

REMARKS.—218. H.=207'.6. Midland Ry. 219. Thusis-Engadine. 220. B. Nov. 8, 1804, p. 388. Paris, Lyon. 224. Glasgow & Paisley Joint Ry. 225. B. & O. Ry., Newark Div., two tracks. 230. Over Wilmington & Northern Ry. and deep ravine. 234. Blown up in

## ARRANGED ACCORDING TO SPAN—(Continued).

## ARCHES.

| Span.                                                           | Rise.                                | Thickness of Arch Ring at Crown $\frac{1}{2}$ . | At Springing, $\frac{1}{8}$ . | Curve.                                | Radius at Crown.                     | $\frac{1}{2} R$ .                    | Width, Face to Face at Crown. | Thickness of Piers at Springing. | Material.                               | Class of Bridge.             | Reference.                                                                                        | Number                                 |
|-----------------------------------------------------------------|--------------------------------------|-------------------------------------------------|-------------------------------|---------------------------------------|--------------------------------------|--------------------------------------|-------------------------------|----------------------------------|-----------------------------------------|------------------------------|---------------------------------------------------------------------------------------------------|----------------------------------------|
| *82.0<br>82.0                                                   | †41.0<br>41.0                        | †4.2<br>3.3                                     | †8.2<br>4.9                   | 3C                                    | 41.0                                 | .102                                 | 26.2<br>8.5                   | 16.4                             |                                         | Ry.<br>Ry.                   | R. March, 1801<br>Engineer, April 8, 1804, pp. 357, 366                                           | 218<br>219                             |
| 82.2                                                            | 41.1                                 | 4.6                                             |                               | C <sub>2</sub>                        | 41.1                                 | .112                                 | 26.5                          | 16.4-23.0                        | Granite                                 | Ry.                          | K. April 30, 1807, p. 575                                                                         | 220                                    |
| *82.3<br>83.1<br>84.0<br>85.0<br>85.0<br>85.2                   |                                      | 4.3<br>3.0<br>18.0<br>3.3<br>2.7                |                               | C<br>C<br>C                           | 153.4<br>45.5                        | .028<br>.066                         | †60.0<br>41.6<br>30.0         | 8.0                              | Limestone                               | H.W.<br>H.W.<br>Ry.<br>Ry.   | L. F. 1852, p. 280<br>B. April 11, 1801<br>A. 1839, p. 313<br>T. Aug. 22, 1902<br>F. 1852, p. 282 | 221<br>222<br>223<br>224<br>225<br>226 |
| *85.3<br>85.3<br>86.0<br>86.0<br>86.0<br>78.0                   |                                      | 4.6<br>3.0<br>3.5<br>43.0<br>39.0               |                               | C <sub>2</sub><br>C <sub>2</sub><br>C | 42.6<br>42.6<br>75.2                 | .108<br>.070<br>.047                 | 11.8<br>21.3<br>26.5          |                                  | Freestone                               | H.W.<br>H.W.<br>H.W.<br>Ry.  | F. 1852, p. 276<br>J. F. 1852, p. 282<br>C. 1855-56<br>T. Nov. 21, 1902, p. 808                   | 227<br>228<br>229<br>230               |
| 86.3 to 54.3<br>86.9 to 65.9<br>87.0                            | 30.9<br>28.0<br>16.0                 | 6.9<br>3.9<br>3.8                               |                               | E<br>E                                | 57.5                                 | .120                                 | 22.4-15.9<br>13.8             |                                  | H.W.                                    | H.W.                         | F. 1852, p. 276<br>P. 1886, p. 134<br>J. F. 1852, p. 288                                          | 231<br>232                             |
| 87.8                                                            | 43.9                                 |                                                 |                               | C <sub>2</sub>                        | 43.9                                 |                                      | 27.8                          |                                  | Brick and stone<br>Brick,<br>stone trim | H.W.                         | C. 1855-56                                                                                        | 233                                    |
| 88.4                                                            | 7.4                                  |                                                 |                               | C                                     | 84.2                                 |                                      | 10.1                          |                                  |                                         | H.W.                         | M. Feb. 18, 1809, p. 19346<br>F. 1852, p. 282                                                     | 234<br>235                             |
| 88.6<br>75.4<br>*89.5<br>*89.5<br>*90.0                         | 22.1<br>20.7<br>28.8<br>33.0         | 3.9<br>3.5<br>28.8<br>3.7                       |                               | E<br>E<br>E                           | 114.8<br>52.2                        | .034<br>.070                         | 32.8<br>38.4                  | 16.4<br>12.8<br>13.9             |                                         |                              | F. 1852, p. 292<br>F. 1852, p. 282<br>F. 1852, p. 280<br>A. Oct. 48, p. 317                       | 236<br>237<br>238<br>239               |
| 90.0 to 22.0<br>90.0<br>90.0<br>90.3<br>91.1 to 70.2<br>c-c91.9 | 30.0<br>30.0<br>15.0<br>20.9<br>23.9 | 3.0<br>3.0<br>2.8<br>4.3<br>4.3                 |                               |                                       | 48.7<br>48.8<br>76.0<br>73.5<br>55.1 | .062<br>.061<br>.037<br>.058<br>.077 | 27.0<br>16.0-14.0             |                                  | Fine stone                              | Ry.                          | L. F. 1852, p. 286                                                                                | 240                                    |
| 90.0 to 22.0<br>90.0<br>90.0<br>90.3<br>91.1 to 70.2<br>c-c91.9 | 30.0<br>30.0<br>15.0<br>20.9<br>23.9 | 3.0<br>3.0<br>2.8<br>4.3<br>4.3                 |                               | C<br>C<br>C                           | 48.8<br>76.0<br>73.5<br>55.1         | .061<br>.037<br>.058<br>.077         | 27.0<br>16.0-14.0             |                                  | Aqued't                                 | H.W.                         | L.<br>L.<br>F. 1852, p. 274<br>F. 1852, p. 282                                                    | 241<br>242<br>243<br>244               |
| c-c91.9                                                         | c-c9.2                               | 3.3                                             | 4.9                           | C                                     | 119.4                                | .041                                 | 10.7                          |                                  | Sandstone                               | H.W.                         | K. 1802, p. 560;<br>G. 1801, p. 920                                                               | 245                                    |
| *91.8                                                           | 10.8                                 | 4.7                                             | †8.0                          | C                                     | 102.0                                | .046                                 | 46.4                          | 9.8                              | Freestone                               | H.W.                         | C. 1855-56; H. F. 1852, p. 286                                                                    | 246                                    |
| 92.0<br>*93.0<br>94.0<br>*95.0                                  | 13.0<br>46.5<br>9.8                  | †4.6<br>3.7                                     |                               | C                                     | 87.0                                 | .052                                 | 16.0                          |                                  |                                         | H.W.<br>H.W.<br>H.W.<br>H.W. | S.<br>P. 1806, p. 130<br>Woodbury, 1858<br>L.                                                     | 247<br>248<br>249<br>250               |
| 95.8 to 87.6<br>87.6<br>95.9<br>95.9                            | 16.0<br>14.8<br>7.1<br>28.8          | 3.2<br>3.2<br>5.3<br>5.3                        | 3.2                           | E<br>E                                | 165.6<br>79.9                        | .032<br>.066                         | 41.5<br>32.0                  | 9.8                              | White marble                            | H.W.                         | A. April, 1847, p. 104<br>F. 1852, p. 280<br>F. 1852, p. 280; V. July 17, 1807                    | 251<br>252<br>253                      |
| *95.9                                                           | 19.2                                 | 5.3                                             |                               | P                                     | 85.2                                 | .062                                 | 105.0                         | 23.5                             | Freestone                               | H.W.                         | J. F. 1852, p. 276;<br>P. 1806, p. 129<br>F. 1852, p. 282                                         | 254<br>255                             |
| 96.0                                                            | 26.6                                 | 4.3                                             |                               | E                                     | 71.1                                 | .060                                 |                               | 19.2                             |                                         |                              |                                                                                                   |                                        |

† About.

1867. 239. Shrewsbury & Chester Ry. 242. Chesapeake & Ohio Canal. 245. Three lead  
 "hinges." 247. On piles. 254. Covered. Rebuilt about 1350.

TABLE II.—DATA FOR ABOUT 500 ARCH BRIDGES

## MASONRY

| Number. | Name.             | Place.                                | Over.                        | Date.   | Engineer.              | No. of Spans.                                       |
|---------|-------------------|---------------------------------------|------------------------------|---------|------------------------|-----------------------------------------------------|
| 256     | Dean              | Edinburgh, Scotland                   |                              | 1831    | Telford                | 4                                                   |
| 257     | Fleischbrücke     | Drôme, France                         |                              | 1774    | Bouchet                | 3                                                   |
| 258     |                   | Nuremberg, Bavaria                    |                              | 1599    | Carln                  | 1                                                   |
| 259     | Imnau             | Near Imnau                            | Eyach R.                     | 1896    | Liebbrand              | 1                                                   |
| 260     |                   | Charrey, France                       | Saône R.                     | 1888    | Mocquery               | 5                                                   |
| 261     | Rialto            | Near Chalonnès, France                | Loire R.                     | 1864-5  | Morandiere             | 17                                                  |
| 262     |                   | Venice, Italy                         | Grand Canal                  | 1588-91 | Ant. da Ponte          | 1                                                   |
| 263     | Margherita        | Rome, Italy                           | Tiber R.                     | 1891    | Vescovali              | 3                                                   |
| 264     | Pont du Jour      | Carbonne, France                      | Garonne R.                   | 1770    | Saget                  | 3                                                   |
| 265     |                   | Paris, France                         | Seine R.                     | 1864    | Bassompierre           | 5<br>31                                             |
| 266     | Alcantara Aq.     | Near Lisbon, Portugal                 |                              | 1731-75 |                        | 35                                                  |
| 267     | Bishop Auckland   | England                               | Wear R.                      | 1388    |                        |                                                     |
| 268     | Etherow River     |                                       | Etherow R.                   |         | Hoskoll                | 4                                                   |
| 269     | Blackfriars (old) | London, England                       | Thames R.                    | 1760-70 | Robt. Mylne            | 9                                                   |
| 270     | Alcantara         | Alcantara, Portugal                   | Tagus R.                     | 100†    | Trajan                 | 6                                                   |
| 271     | Wellington        | Leeds, England                        | Aire R.                      | 1816-19 | Jno. Rennie            | 1                                                   |
| 272     | Rutherglen        | Bet. Glasgow and Rutherglen, Scotland | Clyde R.                     | 1895    | Crouch & Hogg          | 1<br>2                                              |
| 273     |                   | Minneapolis, Minn., U.S.A.            | Mississippi R.               | 1882-93 | Smith                  | 4<br>15<br>1<br>1<br>18                             |
| 274     | Elster Viad.      | Bet. Reichenbach and Plauen, Saxony   | Elster R. & V. (two tiers)   | 1846-50 | Wilke                  | 18                                                  |
| 275     | Göltzsh           | Bet. Reichenbach and Plauen, Saxony   | Göltzsh R. & V. (four tiers) | 1846-5- | Wilke(?)               | 1<br>1<br>20<br>23<br>16<br>10<br>10<br>1<br>5<br>1 |
| 276     | Lempde            |                                       | Alagnon                      | 1785    | Mauriset               |                                                     |
| 277     | Montlyon          | Rouen, France                         | Seine R.                     |         | Lamandé                | 1                                                   |
| 278     |                   | France                                | Durance R.                   | 1805    | Delbergue-Cormon       | 5                                                   |
| 279     | Pont au Double    | Paris, France                         | Seine R.                     | 1847    | De Lagalisserie        | 1                                                   |
| 280     | Guillotiére       | Lyons, France                         | Rhone R.                     | 1265    | Ass. des frères Dupont | 18                                                  |
| 281     | P. de la Concorde | Paris, France                         | Seine R.                     | 1787-92 | Perronet               | 1<br>2                                              |
| 282     |                   | Munich, Bavaria                       | Isar R.(?)                   | 1814    | Wiebeking              | 3                                                   |
| 283     | Gère              | Vienna, Austria                       |                              | 1781    | Vimar                  | 1                                                   |
| 284     | Avignon           | Avignon, France                       | Rhone R.                     | 1177-87 | Benezet                | 21                                                  |

\* Maximum.

REMARKS.—256. 96'0 arches are under sidewalks. 257. 90'0 arches are under roadway.  
 259. Three granite "hinges." 261. Granite piers on concrete foundation. Two tracks. 265.  
 Parapets, etc., Jura marble. 266. H.=230'0. Highest single tier of stone arches in the world.



## ARRANGED ACCORDING TO SPAN—(Continued).

## ARCHES.

| Span.   | Rise.  | Thickness of Arch Ring at Crown $\frac{1}{2}$ . | At Springing, $\frac{1}{2}$ . | Curve.         | Radius at Crown. | $\frac{1}{2} R$ . | Width, Face to Face at Crown. | Thickness of Piers at Springing. | Material.                  | Class of Bridge. | Reference.                                 | Number. |
|---------|--------|-------------------------------------------------|-------------------------------|----------------|------------------|-------------------|-------------------------------|----------------------------------|----------------------------|------------------|--------------------------------------------|---------|
| 96.0    | 16.7   | 3.0                                             |                               |                | 77.5             | .030              | 41.0                          |                                  |                            | H.W.             | F. 1852, p. 192                            | 256     |
| 99.0    | 30.0   | 2.3                                             |                               |                | 48.7             | .062              |                               |                                  |                            |                  |                                            |         |
| *96.0   | 27.7   | 6.4                                             |                               | E              | 74.5             | .086              |                               | 17.1                             |                            | H.W.             | F. 1852, p. 282                            | 257     |
| 97.0    | 13.0   | 4.0                                             |                               | P              | 80.0             | .050              | 53.3                          | —                                |                            |                  | P. 1896, p. 132; F. 1852, p. 276           | 258     |
| c-c98.4 | c-c9.8 | 1.5                                             | 1.6                           | C              | 128.4            | .012              | ?13.0                         |                                  | Beton                      | H.W.             | G. '98, 2d Tri.                            | 259     |
| *98.4   | 12.3   | 3.8                                             | 4.9                           | C              | 104.5            | .036              | 18.2                          | 11.5                             | Limestone                  | H.W.             | G. VI, 1896, p. 737                        | 260     |
| 98.4    | †24.6  |                                                 |                               | E              |                  |                   | ?26.2                         |                                  | Marble                     | Ry.              | K. Oct. 18, 1867                           | 261     |
| 98.5    | 23.0   |                                                 |                               | P              |                  |                   | 64.0                          |                                  |                            | H.W.             | Q. P. 1896, p. 122; F. 1852, p. 276        | 262     |
| *99.0   | 16.5   | †5.0                                            | †6.0                          | 5C             |                  |                   | ?67.5                         |                                  | Rezzato & travertine stone | H.W.             | R. June, 1892, p. 260                      | 263     |
| 99.1    | 40.5   | 3.7                                             |                               | E              | 73.5             | .050              | 25.6                          | 22.6                             |                            |                  | F. 1852, p. 280                            | 264     |
| 99.2    | 31.2   | 5.3                                             |                               | E              |                  |                   | 101.7                         |                                  | Stone from                 | H.W.             | K. Feb. 8 & Jan. 25, 1867                  | 265     |
| 15.8    | 7.9    |                                                 |                               | C              | 7.9              |                   | 29.5                          |                                  | Château Haudon             | Ry.              |                                            |         |
| *100.0  | 88.0   |                                                 |                               | P              |                  |                   |                               |                                  |                            | Aqued't          | P. 1896, p. 137                            | 266     |
| 100.0   | 22.0   | 1.8                                             | 1.8                           |                |                  |                   |                               |                                  |                            | H.W.             | I.                                         | 267     |
| *100.0  | 25.0   | 4.0                                             | 4.0                           | C              | 62.5             | .064              | ?45.0                         |                                  |                            | Ry.              | I.                                         | 268     |
| *100.0  | 43.0   | 5.0                                             |                               | E              |                  |                   |                               |                                  |                            | H.W.             | L. P. 1896, p. 136; F. 1852, p. 280        | 269     |
| *100.0  | 50.0   |                                                 |                               | C <sub>2</sub> | 50.0             |                   |                               |                                  | Granite                    | H.W.             |                                            | 270     |
| 100.0   | 15.0   | 4.0                                             | 7.0                           | C              | 90.8             | .043              |                               |                                  | Brown sandstone            | H.W.             | A. 1844, p. 128 and 246                    | 271     |
| 100.0   | 12.6   | 4.0                                             | 4.0                           | C              | 97.6             | .041              |                               |                                  |                            |                  |                                            | 272     |
| 90.0    | 11.7   | 4.0                                             | 4.0                           |                | 91.4             | .044              | †50.0                         | 13.5                             | Granite                    | H.W.             | Engineer, Aug. 23, 1895, p. 182            |         |
| 100.0   | 39.7   | 3.0                                             |                               |                |                  |                   | 28.0                          | 7.0                              | Limestone                  | Ry.              | Jour. West. Soc. Eng. Vol. 8, 1903, p. 421 | 273     |
| 80.0    | 40.0   | 2.7                                             |                               |                | 40.0             | .067              |                               | 14.0                             |                            |                  |                                            |         |
| 71.4    | 15.0   | 2.7                                             |                               |                |                  |                   |                               |                                  |                            |                  |                                            |         |
| 43.9    | 13.0   | 2.5                                             |                               |                |                  |                   |                               |                                  |                            |                  |                                            |         |
| 40.0    | 5.3    | 2.7                                             |                               |                |                  |                   |                               |                                  |                            |                  |                                            |         |
| 100.3   |        |                                                 |                               | C <sub>2</sub> |                  |                   | 26.1                          |                                  | Brick mostly               | Ry.              | F. 1852, p. 209                            | 274     |
| 23.2    |        |                                                 |                               |                |                  |                   |                               |                                  | Brick mostly               |                  |                                            |         |
| 100.3   |        | 3.7                                             |                               | C              |                  |                   | ?26.1                         |                                  | Rings of brick             | Ry.              | F. 1852, p. 199                            | 275     |
| 92.9    | 46.4   | 3.7                                             |                               | C <sub>2</sub> | 46.8             | .080              |                               |                                  |                            |                  | Q. Am. Sup.                                |         |
| 46.8    | 23.4   | 1.5                                             |                               | C <sub>2</sub> | 23.4             | .064              |                               |                                  |                            |                  |                                            |         |
| 44.6    |        | 1.5                                             |                               | C              | 23.4             | .064              |                               |                                  |                            |                  |                                            |         |
| 41.8    |        | 1.5                                             |                               | C              | 23.4             | .064              |                               |                                  |                            |                  |                                            |         |
| 39.0    |        | 1.5                                             |                               | C              | 23.4             | .064              |                               |                                  |                            |                  |                                            |         |
| 101.2   | 32.0   |                                                 |                               | E              |                  |                   |                               |                                  |                            |                  | F. 1852, p. 284                            | 276     |
| *101.7  | 13.7   | 4.5                                             |                               | E              | 95.7             | .047              |                               |                                  |                            | H.W.             | H.                                         | 277     |
| 101.7   | 32.0   |                                                 |                               | E              | 77.8             |                   |                               |                                  |                            |                  | F. 1852, p. 286                            | 278     |
| 101.8   | 9.8    | 5.3                                             |                               | C              | 130.2            | .039              | 52.2                          |                                  | Millstone grit             | H.W.             | J. F. 1852, p. 296                         | 279     |
| 102.3   | 38.4   | 2.1                                             |                               |                | 62.9             | .034              |                               | 34.1                             |                            | H.W.             | F. 1852, p. 274                            | 280     |
| 26.2    |        |                                                 |                               |                |                  |                   |                               |                                  |                            |                  |                                            |         |
| 102.3   | 9.8    | 3.7                                             |                               | C              | 148.0            | .025              | 51.1                          | 9.6                              | Freestone                  | H.W.             | J. F. 1852, p. 284                         | 281     |
| 92.7    | 8.7    | 3.4                                             |                               |                | 127.5            | .027              |                               |                                  |                            |                  |                                            |         |
| 83.1    | 6.4    | 3.2                                             |                               |                | 138.2            | .023              |                               |                                  |                            |                  |                                            |         |
| 102.3   | 17.1   | 4.3                                             |                               | C              | 82.5             | .050              | 42.6                          | 9.6                              | Freestone                  | H.W.             | J. F. 1852, p. 288                         | 282     |
| 102.7   | 28.2   | 5.2                                             |                               | C              | 57.4             | .090              | 35.6                          |                                  |                            |                  | F. 1852, p. 284                            | 283     |
| *102.9  | 51.5   | 2.4                                             |                               | C <sub>2</sub> | 51.5             | .047              | 15.4                          | 22.8                             | Freestone                  | H.W.             | J. L. P. 1896, F. 1852, p. 274             | 284     |

† About.

260. Replaced by cast iron, 1865. 270. H. = 210'.0. 271. Coffor-dams employed. 273. Minneapolis Union Ry. Two tracks. 274. Saxony-Bavaria. 275. Saxony-Bavaria. H. = 264'.0. 284. In ruins.



TABLE II.—DATA FOR ABOUT 500 ARCH BRIDGES

MASONRY

| Number. | Name.          | Place.                                          | Over.            | Date.     | Engineer.              | No. of Spans. |
|---------|----------------|-------------------------------------------------|------------------|-----------|------------------------|---------------|
| 285     |                | Port de Piles, France                           | Creuse R.        | 1846-47   | Bayeux                 | 1             |
| 286     | Herault        | Route of Nice, France                           | Herault R.(?)    |           | Grangent               | 2             |
| 287     |                | Prague, Bohemia                                 | Moldau R.        | 1878      | Reiter                 | 1             |
|         |                |                                                 |                  |           |                        | 2             |
|         |                |                                                 |                  |           |                        | 2             |
| 288     |                | Marbach, Germany                                | Murr R.          | 1887      | Leibbrand              | 1             |
| 289     | Wissahickon    | Philadelphia, Pa., U.S.A.                       | Wissahickon Crk. | 1897      | Gen. Thayer            | 1             |
| 290     | Potomac Aq.    | Washington, D. C., U.S.A.                       | Potomac R.       |           |                        | 7             |
| 291     | Ponthaut       | Germany                                         | Bonne R.         | 1793      |                        | 1             |
| 292     |                | Orleans France                                  | Loire R.         | 1750-60   | Hupear                 | 9             |
| 293     |                | Hartford, Conn., U. S. A.                       | Connecticut R.   | 1903      | Graves                 | 1             |
| 294     |                | Baiersbronn, Germany                            | Murg. R.         | 1889      | Leibbrand              | 1             |
| 295     |                | Wurttemberg, Germany                            | Nagold R.        | 1882      |                        | 1             |
| 296     |                | Winstone, England                               | Tees R.          | 1762      | Robinson               |               |
| 297     |                | Sault, France                                   | Rhone R. Br.     | 1825-27   | Montluisant            | 1             |
| 298     | Lodi St.       | Elyria, Ohio, U.S.A.                            | W. Br. Black R.  | 1894      | Jackson and            | 2             |
|         |                |                                                 |                  |           | Bunce                  | 1             |
| 299     |                | Toulouse, France                                | Garonne R.       | 1543-1632 | Souffron               | 7             |
| 300     | 2d Worochta    | Worochta, Austria                               | Pruth R.         | 1892-93   | Huss                   | 12            |
| 301     |                | St. Esprit, France                              | Rhone R.         | 1265-1309 | Ass. des freres Dupont | 19            |
| 302     |                | Nantes, France                                  | Loire R.         |           |                        |               |
| 303     |                | Mantes, France                                  | Seine R.         | 1757-65   | Hupear                 | 3             |
| 304     | Grand-Maitre   | Fontainebleau, France                           | Fontainebleau V. | 1869      | Belgrand               | 21            |
| 305     | Cresheim       | Fairmont Park, Philadel-<br>phia, Penn., U.S.A. | Cresheim Crk.    | 1892      | Webster                | 1             |
| 306     | Napoleon       | Paris, France                                   |                  |           |                        | Many          |
| 307     | Tongueland     | Near Kirkendbright, Scot-<br>land               | Dee R.           | 1806      | Telford                | 1             |
| 308     |                | Hartford, Conn., U.S.A.                         | Connecticut, R.  | 1904-     | Graves                 | 6             |
|         |                |                                                 |                  |           |                        | 1             |
|         |                |                                                 |                  |           |                        | 1             |
|         |                |                                                 |                  |           |                        | 2             |
|         |                |                                                 |                  |           |                        | 1             |
| 309     | Waterloo (new) | London, England                                 | Thames R.        | 1817      | Rennie                 | 2             |
| 310     | Devil's Br.    | Near Lucca, Italy                               | Serchio R.       | 1000†     |                        | 9             |
| 311     | Têtes          | France                                          | Durance R.       | 1732      | Hanriana               | 1             |
| 312     |                | Bourbonnais, France                             |                  |           | Vaudray                |               |
| 313     | Vingeanne Val. | Near Oisilly, France                            | Vingeanne Val.   |           |                        |               |
| 314     |                | Rumilly, France                                 | Cheran R.        | 1785      | Garella                | 7             |
| 315     | Maidenhead     | Maidenhead, England                             | Thames R.        | 1832-38   | Brunel                 | 1             |
|         |                |                                                 |                  |           |                        | 6             |

\* Maximum.

REMARKS.—285. Tours-Bordeaux. Two tracks. 288. Three lead "hinges." 289. Skew  
 69° 26'. Ten 4' ribs. 293. See No. 308. 294. Three lead "hinges." Cost 23,800 f.  
 299. Stone trimmings. 300. Austrian State Ry. 301. Small arches in piers. 304. Paris

## ARRANGED ACCORDING TO SPAN—(Continued).

## ARCHES.

| Span.  | Rise. | Thickness of Arch Ring at Crown $t_0$ . | At Springing $t_s$ . | Curve.         | Radius at Crown. | $\frac{t_0}{R}$ . | Width, Face to Face at Crown. | Thickness of Piers at Springing. | Material.           | Class of Bridge. | Reference.                            | Number. |
|--------|-------|-----------------------------------------|----------------------|----------------|------------------|-------------------|-------------------------------|----------------------------------|---------------------|------------------|---------------------------------------|---------|
| 103.8  | 40.5  | 4.3                                     | 4.3                  | E              | 70.8             | .060              | 30.0                          | 19.2                             | Freestone           | Ry.              | C. 1851-52; J. F. 1852 p. 276         | 285     |
| 98.4   |       |                                         |                      |                |                  |                   |                               |                                  |                     |                  | F. 1852, p. 294                       |         |
| 104.4  | 15.4  | 2.7                                     |                      | C              | 90.5             | .029              | 22.4                          |                                  | Granite             | H.W.             | F. 1852, p. 280                       | 286     |
| 105.0  | 16.2  | 4.0                                     | 5.3                  |                |                  |                   | 39.8                          | 13.1                             |                     |                  | K. May 10, 1878, p. 359               | 287     |
| 99.8   | 15.3  |                                         |                      |                |                  |                   |                               |                                  |                     |                  |                                       |         |
| 94.5   | 14.5  |                                         |                      |                |                  |                   |                               |                                  |                     |                  |                                       |         |
| 89.3   | 13.7  |                                         |                      |                |                  |                   |                               |                                  |                     |                  |                                       |         |
| 105.0  | 10.2  | 3.9                                     | 4.9                  | C              | 140.2            | .028              | 18.4                          |                                  | Coushohocken stone  | H.W.             | G. 1891, 1, p. 922                    | 288     |
| 105.0  | 11.0  | 3.0                                     | 4.5                  | C(?)           | 118.1            | .025              | 235.0                         |                                  |                     | H.W.             | B. Sept. 9, 1897, p. 162              | 289     |
| *105.0 |       |                                         |                      |                |                  |                   |                               |                                  |                     |                  |                                       |         |
| 106.3  | 53.1  | 5.7                                     |                      | C <sub>2</sub> | 53.1             | .108              | 20.5                          |                                  |                     | Aqued't          | A. 1837, 8, p. 148                    | 290     |
| 106.5  | 28.8  | 6.9                                     |                      |                | 83.9             | .083              | 49.0                          | 19.2                             |                     | H.W.             | F. 1852, p. 284                       | 291     |
| 98.0   | 16.0  | 5.8                                     |                      |                |                  |                   |                               | 18.1                             | Granite             |                  | L. F. 1852, p. 276                    | 292     |
| 108.0  | 27.0  |                                         |                      | E              |                  |                   |                               | 18.0                             |                     |                  |                                       |         |
| 108.2  | 10.8  | 2.0                                     | 2.6                  |                |                  |                   | 21.7?                         |                                  |                     | H.W.             | N. Dec. 26, 1903                      | 293     |
| 108.8  | 10.8  | 3.3                                     | 5.3                  |                |                  |                   | 16.4                          |                                  |                     | H.W.             | G. 1st Tri., 1901                     | 294     |
| 108.8  |       |                                         |                      |                |                  |                   |                               |                                  |                     | H.W.             | G. 1891, 1, p. 903                    | 295     |
| 111.5  | 31.9  | 4.6                                     |                      | E              | 114.2            | .057              | 23.0                          | 22.2                             |                     |                  | L.                                    | 296     |
| 91.8   | 29.5  | 4.3                                     |                      | E              | 81.0             | .037              | 38.0                          |                                  | Elyria sandstone    | H.W.             | F. 1852, p. 288                       | 297     |
| 112.0  | 19.5  | 3.5                                     | 4.3                  |                |                  |                   |                               |                                  | Brick               | H.W.             | C. H. Snow, City Engineer, Elyria, O. | 298     |
| 113.0  | 38.4  | 3.7                                     |                      |                | 76.8             | .049              | 64.0                          | 26.6                             |                     |                  | F. 1852, p. 276                       | 299     |
| 44.8   |       |                                         |                      |                |                  |                   |                               |                                  |                     |                  |                                       |         |
| 113.5  | 56.8  | 4.3                                     | 6.7                  | C <sub>2</sub> | 56.8             | .076              | 14.7                          |                                  |                     | Ry.              | B. Dec. 7, 1893, p. 448               | 300     |
| 32.8   |       |                                         |                      |                |                  |                   |                               |                                  |                     |                  |                                       |         |
| 114.1  | 44.8  | 5.9                                     |                      | C              | 70.4             | .084              | 17.6                          | 27.8                             |                     | H.W.             | F. 1852, p. 274                       | 301     |
| 81.0   |       |                                         |                      |                |                  |                   |                               |                                  |                     |                  |                                       |         |
| 115.2  | 34.4  | 6.4                                     |                      |                |                  |                   |                               |                                  |                     |                  |                                       |         |
| *115.4 | 34.0  | 6.4                                     |                      | E              |                  |                   |                               |                                  |                     | H.W.             | Q. I.                                 | 302     |
|        |       |                                         |                      |                |                  |                   |                               |                                  |                     |                  |                                       | 303     |
| 115.8  | 19.3  |                                         |                      |                |                  |                   |                               |                                  | Beton               | Aqued't          | K. Oct. 1869, p. 275                  | 304     |
| 142.5  |       |                                         |                      |                |                  |                   |                               |                                  |                     |                  |                                       |         |
| 116.0  | 21.2  | 3.5                                     | 4.5                  |                |                  |                   | 10.0                          |                                  | Buff sandstone      | Sewer            | B. Aug. 31, 1893, p. 170              | 305     |
| 116.0  |       |                                         |                      |                |                  |                   |                               |                                  |                     |                  |                                       |         |
| 116.0  | 14.8  | 4.0                                     |                      |                |                  |                   |                               |                                  |                     |                  |                                       |         |
| Small  | 38.0  | 3.6                                     |                      | C              | 64.8             | .056              | 24.0                          |                                  |                     | Ry.              | I.                                    | 306     |
| 68.0   | 21.1  |                                         |                      |                |                  |                   |                               |                                  |                     | H.W.             | L. F. 1852, p. 286                    | 307     |
|        |       |                                         |                      |                |                  |                   |                               |                                  |                     |                  |                                       |         |
| 74.0   | 22.9  |                                         |                      |                |                  |                   |                               |                                  |                     |                  |                                       |         |
| 81.0   | 25.1  |                                         |                      |                |                  |                   |                               |                                  |                     |                  |                                       |         |
| 108.0  | 27.0  |                                         |                      |                |                  |                   |                               |                                  |                     |                  |                                       |         |
| 115.0  | 28.8  |                                         |                      |                |                  |                   |                               |                                  |                     |                  |                                       |         |
| 119.0  | 29.8  |                                         |                      |                |                  |                   |                               |                                  |                     |                  |                                       |         |
| 120.0  | 34.6  | 4.5                                     | 10.0                 | E              |                  |                   | 44.0                          | 20.0                             | Granite             | H.W.             | S. K. Feb. 22, 1895, p. 236; F. 1852  | 309     |
| 120.5  | 60.3  | 4.5                                     |                      | C <sub>2</sub> | 60.3             | .074              | 12.0                          |                                  | Limestone sandstone | H.W.             | C.                                    | 310     |
| 123.6  | 61.8  | 4.7                                     |                      | C <sub>2</sub> | 61.8             | .065              | 15.9                          |                                  |                     |                  |                                       |         |
| 124.0  | 6.9   | 2.7                                     | 3.6                  | C              | 255.7            | .010              |                               |                                  | Granite             | H.W.             | F. 1852, p. 278                       | 311     |
| 127.0  | 46.0  |                                         |                      | E              |                  |                   | 14.0                          |                                  |                     | Ry.              | I.                                    | 312     |
| 127.6  | 63.8  | 5.3                                     |                      | C <sub>2</sub> | 63.8             | .086              | 23.5                          |                                  |                     | Ry.              | B. Dec. 7, 1893                       | 313     |
| 128.0  | 24.3  | 5.3                                     | 7.5                  | E              | 169.0            | .031              | 236.0                         |                                  | Freestone           | H.W.             | J. F. 1852, p. 284                    | 314     |
| 21.0   |       |                                         |                      |                |                  |                   |                               |                                  | Brick               | Ry.              | P. Wks. G. B., 46; K. Oct. 25, 1895   | 315     |

† About.

water-supply. 308. Pneumatic foundations. Cost (est.) \$1,600,000. 310. Four small side  
 arches. 313. E. Ry. of France. 315. Great W. Ry.

TABLE II.—DATA FOR ABOUT 500 ARCH BRIDGES

MASONRY

| Number. | Name.            | Place.                     | Over.             | Date.                 | Engineer.                | No. of Spans. |
|---------|------------------|----------------------------|-------------------|-----------------------|--------------------------|---------------|
| 316     |                  | Neuilly, France            | Seine R.          | 1768-74               | Perronet                 | 5             |
| 317     |                  | Mantes, France             | Seine R.          | 1757-65               | Hupear under<br>Perronet | 1             |
| 318     | Echo Br.         | Newton Upper Falls, Mass.  | Charles R.        | 1876                  | Fitzgerald               | 2             |
|         |                  | U.S.A.                     |                   |                       |                          | 1             |
| 319     |                  | Elyria, Ohio               | Black R.          |                       | Telford                  | 4             |
| 320     |                  | Aberdeen, Scotland         | Den Burn Rill     | 1801+                 |                          | 1             |
| 321     |                  | Wan Hsien, China           |                   |                       |                          | 1             |
| 322     | North Ave.       | Baltimore, Md., U.S.A.     | Gorge-Jones'F'lls | 1893-95               | Smith                    | 3             |
| 323     | 1st Worochta     | Worochta, Austria          | Pruth R.          | 1892-93               | Huss                     | 1             |
| 324     | Boucicault       | Verjux, France             | Saône R.          | 1888-90               | Jozon                    | 6             |
| 325     | Moret Viad.      | Moret, France              | Loing Val.        | 1847-49               |                          | 1             |
| 326     |                  | Scrvia, Italy              | Scrvia R.         | 1850+                 | Ranco                    | 5             |
| 327     | St. Martin       | Toledo, Spain              | Tagus R.          | 1203                  |                          | 2             |
| 328     |                  | Villeneuve, France         | Lot R.            | 1732                  |                          | 30            |
| 329     | Vizille          | Near Grenoble, France      | Romanche R.       | 1766                  | Bouchet                  | 1             |
| 330     | Waldi-Tobel      | Near Bludenz, Austria      | Gorge             | 1884                  | Huss                     | 1             |
| 331     |                  | Verdun, France             | Doubs R.          | 1895-97               | Jozon                    | 1             |
| 332     | Castalet         |                            |                   |                       | Sejourné                 | 2             |
| 333     | Albula R. Viad.  | Sales                      | Al. R. Gorge      | 1903                  |                          |               |
| 334     | Br. C33          | Bellows Falls, Vt., U.S.A. | Connecticut R.    | 1899                  | Cheever                  | 2             |
| 335     | St. Sauveur      | France                     |                   |                       |                          | 1             |
| 336     | Pont-y-tu-prydd  | Nr. Newbridge, S. Wales.   | Taff R.           | 1755                  | Edwards                  | 1             |
| 337     | Alma             | Paris, France              | Seine R.          | 1855                  | Darcel                   | 1             |
| 338     |                  | Near Narni, Italy          |                   | Bet. 27<br>B.C.—14A.D |                          | 2             |
| 339     | Putney Road      | Putney, England            | Thames R.         | †1882                 | Bazalgette               | 1             |
| 340     | Outer Maximilian | Munich, Bavaria            | Isar R.           | 1904                  |                          | 2             |
| 341     | Verone           | Near Vieux-Château, Italy  | Adige R.          | 1354                  | Under Scala              | 2             |
| 342     |                  | Moulins, France            | Allier R.         | 1705-1710             | Mansard                  | 1             |
| 343     | Pont-du-Cèret    | Near Perpignan, France     | Tech R.           | 1336                  |                          | 1             |
| 344     |                  | Turin, Italy               | Dora Riparia R.   | 1834                  | Mosca                    | 1             |

\* Maximum.

REMARKS.—316. In design R.=160'.0. 317. Destroyed in War 1870. 318. Sudbury Aqueduct for Boston. H.=79'.0. 321. Slightly pointed. 322. Skew 55°. Ribbed. 323. Austrian. State Ry. 324. Radius at spr.=75'.5. Paris-Lyon. Two tracks. Approach to metal spans crossing river. Curve about 0° 52'. 330. H.=165'.0. Slight curve. 331. Extrados arc of circle 144'.3.



## ARRANGED ACCORDING TO SPAN—(Continued).

## ARCHES.

| Span.  | Rise. | Thickness of Arch Ring at Crown ft. | At Springing, ft. | Curve.         | Radius at Crown. | $\frac{t}{R}$ | Width, Face to Face at Crown. | Thickness of Piers at Springing. | Material.       | Class of Bridge.    | Reference.                                          | Number. |
|--------|-------|-------------------------------------|-------------------|----------------|------------------|---------------|-------------------------------|----------------------------------|-----------------|---------------------|-----------------------------------------------------|---------|
| *128.2 | 32.0  | 5.3                                 |                   | E              | 250.0            | .021          | 47.9                          | 14.0                             | Freestone       | H.W.                | L. J. P. 1896; F. 1852, p. 280                      | 316     |
| 128.2  | 38.5  | 6.4                                 |                   | C              | 89.5             | .071          | 35.6                          | 25.6                             |                 | H.W.                | L. V. July 17, 1897                                 | 317     |
| 115.4  | 34.0  |                                     | 6.0               |                | 67.5             | .074          | 18.0                          |                                  | Granite         | H.W.                | F. 1852, p. 276                                     | 318     |
| 129.0  | 42.3  | 5.0                                 |                   | C              |                  |               |                               |                                  | Aqued't         | H.W.                | Boston Water W'ks                                   |         |
| 37.0   | 18.5  |                                     |                   | C <sub>2</sub> | 18.5             |               |                               |                                  |                 | A. Fteley           | Fitzgerald                                          |         |
| 37.0   | 14.0  |                                     |                   |                |                  |               | 27.0                          |                                  |                 |                     | See No. 298.                                        | 319     |
| 129.0  | 29.0  |                                     |                   | C              |                  |               | 43.0                          |                                  | Granite         | H.W.                | L. p. 477                                           | 320     |
| 130.0  | 65.0  |                                     |                   | C <sub>2</sub> |                  |               |                               |                                  |                 | H.W.                | B. June 19, 1902                                    | 321     |
| 130.0  | 26.0  | 5.0                                 | 8.4               | E              |                  |               | 100.0                         | 16.0                             | Brick Ring      | H.W. & E. R.R.      | B. July 6, 1893, p. 7                               | 322     |
| 131.2  | 32.8  | 4.6                                 | 7.2               |                |                  |               | 14.7                          |                                  |                 | Ry.                 | B. Dec. 7, 1893, p. 448                             | 323     |
| 36.1   |       |                                     |                   |                |                  |               |                               |                                  |                 |                     |                                                     |         |
| 26.2   |       |                                     |                   |                |                  |               |                               |                                  |                 |                     |                                                     |         |
| 131.2  | 16.4  | 3.4                                 | †4.9              | E              | 177.0            | .019          | 26.0                          |                                  | Villebois stone | H.W.                | B. May 18, 1893                                     | 324     |
|        |       |                                     |                   |                |                  |               |                               |                                  |                 |                     | G. 1892, p. 445; P. 1892, p. 50                     |         |
| 131.2  | 16.4  | 2.6                                 |                   |                | 139.4            | .019          | 29.5                          | 8.2                              |                 | Ry.                 | F. 1852, p. 117                                     | 325     |
| 32.8   | 16.4  |                                     |                   |                | 16.4             |               |                               |                                  |                 |                     |                                                     |         |
| 131.2  | 43.7  | 5.9                                 |                   | E              | 86.9             | .068          |                               |                                  | Brick           | Ry., Turin to Genoa | F. 1852, p. 296                                     | 326     |
| *132.0 |       |                                     |                   | P              |                  |               |                               |                                  |                 | H.W.                | P. 1896, p. 130                                     | 327     |
| *132.6 | 66.3  | 5.3                                 |                   | C <sub>2</sub> | 66.3             | .080          | 19.6                          |                                  |                 | H.W.                | F. 1852, p. 276                                     | 328     |
| 133.8  | †38.2 | 7.7                                 |                   | E              | 115.0            | .067          | 32.3                          |                                  |                 | H.W.                | F. 1852, p. 280                                     | 329     |
| 134.5  | 42.6  | 5.6                                 | 10.2              |                |                  |               |                               |                                  |                 | Ry.                 | G. 1888, p. 575                                     | 330     |
| 134.5  | 30.1  | 3.9                                 |                   | E <sub>2</sub> |                  |               | 19.7                          | 13.1                             | Limestone       | H.W.                | G. 1897, 4 <sup>th</sup> , p. 179                   | 331     |
| 126.3  | 27.9  | 3.9                                 |                   |                |                  |               |                               |                                  |                 |                     |                                                     |         |
| 134.5  |       |                                     |                   |                |                  |               | 8.5                           |                                  |                 |                     | B. Feb. 27, 1902                                    | 332     |
| 137.8  |       |                                     |                   |                |                  |               |                               |                                  |                 | Ry.                 | Engineer, April 8 and Mar. 4, 1904, pp. 228 and 355 | 333     |
| 98.4   |       |                                     |                   |                |                  |               |                               |                                  |                 |                     | B. June 21, 1900, p. 402, and Blues P. 1896, p. 140 |         |
| 140.0  | 20.0  | 4.0                                 | 4.0               | C              | 132.6            | .030          | 27.0                          |                                  |                 | Ry.                 | Q. L. Q. J.                                         | 334     |
| 140.0  | 35.0  | 1.5                                 |                   |                |                  |               | 15.8                          |                                  | Freestone       | H.W.                |                                                     | 335     |
| 141.4  | 28.2  | 4.9                                 |                   | C              |                  |               |                               | 16.4                             | Millstone grit  | H.W.                |                                                     | 336     |
| 126.0  | 25.2  | 4.9                                 |                   |                |                  |               |                               |                                  |                 |                     | A. 1856, p. 376                                     | 338     |
| 142.0  |       |                                     |                   |                |                  |               |                               |                                  |                 | H.W.                | L.                                                  |         |
| 135.0  |       |                                     |                   |                |                  |               |                               |                                  |                 |                     |                                                     |         |
| 114.0  |       |                                     |                   |                |                  |               |                               |                                  |                 |                     |                                                     |         |
| 75.0   |       |                                     |                   |                |                  |               |                               |                                  |                 |                     |                                                     |         |
| 144.0  | 19.3  | 4.5                                 | 5.5               |                | 144.0            | .031          | 47.0                          | 18.0                             | Granite ring    | H.W.                | K. May 17, 1895                                     | 339     |
| 129.0  | 16.3  | 4.3                                 | 5.3               |                | 136.0            | .032          |                               | and 10.0                         |                 |                     | K. July 23, 1886, p. 85                             |         |
| 112.0  | 13.0  | 4.2                                 | 5.2               |                | 127.0            | .033          |                               | 20.9                             | Limestone       | H.W.                | B. Oct. 27, 1904                                    | 340     |
| 144.3  |       |                                     |                   |                |                  |               | 71.5                          | 36.2                             | Freestone       | H.W.                | J. F. 1852, p. 274                                  | 341     |
| 146.0  | 35.8  | 5.3                                 |                   | C              | 90.5             | .059          |                               | and 22.4                         |                 |                     |                                                     |         |
| 87.4   | 21.3  |                                     |                   |                |                  |               |                               | 36.2                             |                 |                     |                                                     |         |
| 33.0   | 17.1  |                                     |                   |                |                  |               |                               |                                  |                 |                     |                                                     |         |
| 147.1  |       |                                     |                   |                |                  |               |                               |                                  |                 |                     | F. 1852, p. 276                                     | 342     |
| 115.1  |       |                                     |                   |                |                  |               |                               |                                  |                 |                     |                                                     |         |
| 147.6  | 73.8  | 4.6                                 | 13.1              | C <sub>2</sub> | 73.8             | .062          | 12.8                          |                                  |                 | H.W.                | B. Dec. 7, 1893                                     | 343     |
|        |       |                                     |                   |                |                  |               |                               |                                  |                 |                     | F. 1852, p. 274                                     |         |
| 148.0  | 18.0  | 4.9                                 |                   | C              | 160.0            | .031          | 40.0                          |                                  | Granite         | H.W.                | I. U. 1846, p. 27; F. 1852, p. 290                  | 344     |

† About.

and 150'.9 radius. 333. Thusis-Engandine. H.=282'. 335. H.=215'.0. 337. Rubble, grouted. Foundation on piles. 338. Probably the most magnificent bridge built by the Romans in Italy. 340. Three metal hinges. Failed by hinges slipping, June 27, 1904. 342. Failed 1710. 343. Mostly brick. Stone ring.



TABLE II.—DATA FOR ABOUT 500 ARCH BRIDGES

MASONRY

| Number. | Name.                            | Place.                                            | Over.                                    | Date.          | Engineer.                 | No. of Spans. |
|---------|----------------------------------|---------------------------------------------------|------------------------------------------|----------------|---------------------------|---------------|
| 345     | Bellefield                       | Nr. Kleinwolmsdorff, Sax.                         | Roeder R.                                | 1806-          | Rust                      | 1             |
| 346     |                                  | Pittsburgh, Penn., U.S.A.                         | St. Pierre Hollow                        |                |                           | 1             |
| 347     | Claix                            | Near Grenoble, France                             | Drac R.                                  | 1611           |                           | 1             |
| 348     |                                  | Elyria, Ohio, U.S.A.                              | Black R.                                 | 1886           | Kinney                    | 1             |
| 349     | Gloucester                       | Gloucester, England                               | Severn R.                                | 1827           | Telford                   | 1             |
| 350     |                                  | Berne, Switzerland                                | Aar R.                                   | 11204          |                           |               |
| 351     | Vieille Brioude                  | Brioude, France                                   | Allier R.                                | 1454           | Greiner and Es-<br>tone   | 1             |
| 352     | London.                          | London, England                                   | Thames R.                                | 1821-30        | Rennie                    | 1             |
| 353     |                                  |                                                   |                                          |                |                           | 2             |
| 354     | Jamma                            | Near Tournon<br>Jamma, Austria                    | Doux R.<br>Pruth R.                      | 1545<br>1892-3 | Huss                      | 1             |
| 355     | Main St                          | Wheeling, W. Va., U.S.A.                          | Wheeling Crk.                            | 1892           | Hoge & White              | 5             |
| 356     | { Tyne<br>Wear Viad.<br>Victoria | Near Newcastle, Eng.<br>Sunderland<br>Low Lambton | { Tyne R.<br>Wear R.<br>Wear R. & Val. } |                |                           | 1             |
| 357     |                                  | Gignac, France                                    | Herauld R.                               | 1777-93        | Garipuy                   | 1             |
| 358     |                                  | Near Lavaur, France                               | Agout R.                                 | 1775           | Saget                     | 1             |
| 359     | Nydeck                           | Berne, Switzerland                                | Aar R.                                   | 1840-44        | Müller                    | 3             |
| 360     | Antoinette                       |                                                   |                                          |                | Sejourné                  |               |
| 361     | Ballochmoyle                     | Near Ballochmoyle, Scot.                          | Ayr R.                                   |                | Millar                    | 1             |
| 362     | Vieille Brioude                  | Brioude, France                                   | Allier R.                                |                | Romans                    | 1             |
| 363     |                                  | Near Coppel, Germany                              | Schwaendenholz<br>Ravine & Brook         | 1901           |                           | 1             |
| 364     | Gour Noir                        | 4 k. from Uzerche, France                         | Vézère R.                                | 1888-9         | Daigrement                | 1             |
| 365     |                                  | Turin, Italy                                      | Dora Riparia R.                          | 1833           | Hartley                   | 1             |
| 366     | Grosvenor                        | Chester, England                                  | Dee R.                                   | 1832-3         | Hartley                   | 1             |
| 367     | Lavaur                           | Near Lavaur, France                               | Agout R.                                 | 1888           | Sejourné                  |               |
| 368     |                                  | Bogenhausen, Bavaria                              | Isar R.                                  | 1901-02        | Fischer, Archt.           | 1             |
| 369     |                                  | Germany                                           | Gutach R.                                | 1901           |                           | 1             |
| 370     |                                  | Jaremce, Austria                                  | Pruth R.                                 | 1892-3         | Huss                      | 1             |
| 371     | Cabin John                       | Washington, D. C., U.S.A.                         | Cabin John Crk.                          | 1857-64        | Meigs                     | 1             |
| 372     |                                  | Italy                                             | Adda R.                                  | 1903           |                           | 1             |
| 373     | Trezzo                           | Near Trezzo, Italy                                | Adda R.                                  | 1380           | Under Barnabo<br>Visconti | 1             |
| 374     |                                  | Luxemburg, Germany                                | Petrusse R                               | 1899-03        | Sejourné                  | 1             |
| 375     | Plauen                           | Plauen, Saxony                                    | Valley                                   | 1905           | Leibold                   | 1             |

\* Maximum.

REMARKS.—345. Saxony-Silesia. Cut-stonering. 348. Rock foundation. 350. First stone bridge over Aar near Nydeck castle. 353. Rock foundation. 354. Austrian State Ry. 356. Durham Junc. Ry. H.=151' about. 358. H. says five arches. 361. Glasgow and S. W. Ry. 362. Fell 1822. See No. 351. 363. On curve  $R=2660'$ . Clear H.=124'5. 364. Limoges-Brive.

## ARRANGED ACCORDING TO SPAN—(Continued).

## ARCHES.

| Span.  | Rise. | Thickness of Arch Ring at Crown $t_c$ . | At Springing $t_s$ . | Curve.         | Radius at Crown. | $\frac{t_c}{R}$ . | Width, Face to Face at Crown. | Thickness of Piers at Springing. | Material.               | Class of Bridge. | Reference.                       | Number. |
|--------|-------|-----------------------------------------|----------------------|----------------|------------------|-------------------|-------------------------------|----------------------------------|-------------------------|------------------|----------------------------------|---------|
| 148.6  | 49.5  | 5.6                                     |                      | C              | 80.1             | .070              | 26.0                          |                                  | Gray sandstone          | Ry.              | F. 1852, p. 294                  | 345     |
| 150.0  | 36.6  | 4.0                                     | 6.0                  |                |                  |                   | 82.0                          |                                  |                         | H.W.             | B. June 22, 1890, p. 391         | 346     |
| 150.2  | 54.4  | 3.2                                     |                      | C              | 82.0             | .039              | 20.2                          |                                  | Sandstone               | H.W.             | H. F. 1852, p. 276               | 347     |
| 150.0  | 27.0  | 3.8                                     | 4.5                  | C              | 117.6            | .032              | 28.0                          |                                  |                         | H.W.             | B. May 31, 1890                  | 348     |
| 150.0  | 54.0  | 4.5                                     |                      | E              | 152.4            | .029              | 27.4                          |                                  |                         | H.W.             | H. F. 1852, p. 200               | 349     |
| 150.5  |       |                                         |                      |                |                  |                   |                               |                                  |                         | H.W.             | B. Dec. 10, 1805                 | 350     |
| 150.6  | 75.5  | 4.3                                     |                      | C <sub>2</sub> | 75.5             | .057              | 24.7                          |                                  | Granite                 | H.W.             | A. 1844, p. 247; F. 1852, p. 274 | 351     |
| 152.0  | 37.7  | 4.8                                     | 10.0                 | E <sub>2</sub> | 162.0            | .029              | 56.1                          | 24.0                             |                         | H.W.             | A. 1847, p. 106                  | 352     |
| 140.0  |       | 4.6                                     | 9.0                  |                |                  |                   |                               | 22.0                             |                         |                  | F. 1852, p. 290                  |         |
| 130.0  |       | 4.5                                     | 8.5                  |                |                  |                   |                               |                                  |                         |                  | P. 1896, p. 128                  |         |
| 156.7  | 65.0  | 2.8                                     |                      | C              | 78.9             | .035              | 16.0                          |                                  | H.W. Ry.                |                  | J. F. 1852, p. 276               | 353     |
| 157.4  |       | 5.6                                     | 8.5                  | C              |                  |                   | 14.7                          |                                  |                         |                  | B. Dec. 7, 1893, p. 448          | 354     |
| 29.5   |       |                                         |                      |                |                  |                   |                               |                                  | Soft Sandstone          | H.W. Ry.         | Blues                            | 355     |
| 150.0  | 28.4  | 4.5                                     | 6.0                  | C              | 125.4            | .036              | 48.0                          | 21.5                             |                         |                  | F. 1852, p. 178                  |         |
| 150.0  | 70.0  | 4.6                                     | 4.6                  | C <sub>2</sub> | 79.9             | .058              | 25.8                          | 23.8                             |                         |                  | A. 1837-38, p. 57                | 356     |
| 144.0  | 72.0  | 4.6                                     | 4.6                  | C <sub>2</sub> | 72.0             | .064              |                               | 21.5                             |                         |                  | C. 1855-56                       |         |
| 99.8   | 40.9  | 4.6                                     | 4.6                  | C <sub>2</sub> | 49.9             | .092              |                               |                                  | Freestone               |                  |                                  |         |
| 20.2   | 10.1  |                                         |                      |                |                  |                   |                               |                                  |                         |                  |                                  |         |
| 160.0  | 44.0  | 6.5                                     |                      | E              | 117.7            | .055              |                               | 25.6                             |                         | H.W.             | H. J. F. 1852, p. 284            | 357     |
| 83.1   | 41.6  | 6.5                                     |                      | C <sub>2</sub> | 41.6             | .156              |                               |                                  |                         | H.W.             |                                  |         |
| 160.5  | 65.0  | 6.3                                     |                      | E              | 103.4            | .095              | 38.4                          |                                  | H.W. Ry.                |                  | H. F. 1852, p. 282               | 358     |
| *160.7 |       |                                         |                      |                |                  |                   | 39.8                          |                                  |                         |                  | O. Nov. '97, p. 322              | 359     |
| 164.0  |       |                                         |                      |                |                  |                   |                               |                                  | Volcanic rock Sandstone |                  | B. Dec. 19, 1805                 |         |
| 180.0  | 90.0  | 6.0                                     | 6.0                  | C <sub>2</sub> | 90.0             | .067              | 28.0                          |                                  |                         | Ry.              | F. 1852, p. 204                  | 360     |
| 50.0   | 25.0  | 4.5                                     |                      |                | 25.0             | .180              |                               |                                  |                         |                  | C. 1851-52                       | 361     |
| 183.7  | 160.0 | 5.3                                     |                      | C              | 91.8             | .058              | 16.0                          |                                  |                         | H.W.             | A. June, 44, p. 247              | 362     |
| 187.0  | 55.8  | 5.9                                     | 8.5                  |                |                  |                   | 14.4                          |                                  | Granite ring            |                  |                                  |         |
| 196.8  | 52.8  | 5.6                                     | 13.8                 | C              | 118.1            | .047              |                               |                                  |                         | Ry.              | B. Dec. 26, 1901, p. 487         | 363     |
| 200.0  | 42.0  | 4.0                                     |                      | C              | 140.0            | .028              |                               |                                  | Sandstone               |                  | G. 1892, p. 545                  | 364     |
| 200.0  | 42.0  | 4.5                                     | 7.0                  | C              | 140.0            | .032              | 35.5                          |                                  |                         | H.W.             | H. p. 225                        | 365     |
| 201.7  | 60.2  |                                         |                      |                |                  |                   |                               |                                  | Limestone               | H.W.             | A. G. June, 1891;                | 366     |
| c.-c.  | c.-c. |                                         |                      |                |                  |                   | 60.6                          |                                  |                         |                  | B. Oct. '91, Dec. 7, '03         | 367     |
| 209.9  | 21.4  | 3.4                                     |                      |                |                  |                   |                               |                                  | Sandstone               | Ry.              | R. 1889, p. 584                  | 368     |
| 210.0  | 52.5  | 6.6                                     | 9.2                  |                |                  |                   | 13.7                          |                                  |                         | H.W.             | N. Oct. 4, 1902                  | 369     |
|        |       |                                         |                      |                |                  |                   |                               |                                  |                         |                  | B. Sept. 13, 1902                |         |
| 213.0  | 59.0  | 6.9                                     | 10.2                 |                |                  |                   | 14.7                          |                                  |                         | Ry.              | B. Jan. 18, 1902                 | 370     |
| 39.4   |       |                                         |                      |                |                  |                   |                               |                                  | Granite ring Granite    |                  | B. Dec. 26, 1901                 |         |
| 26.2   |       |                                         |                      |                |                  |                   |                               |                                  |                         |                  | p. 447                           |         |
| 220.0  | 57.3  | 4.2                                     | 6.2                  | C              | 134.3            | .031              | 20.3                          |                                  |                         | Aq. and H.W.     | K. April 19, 1867;               | 371     |
| †230.0 |       | 4.9                                     | 7.2                  | 3C             | 246.0            | .020              | 17.3                          |                                  |                         | Ry.              | N. July 29, 1899                 | 372     |
| 251.0  | 87.8  | 4.0                                     | 4.0                  | C?             | 133.6            | .030              |                               |                                  | Hard slate              | H.W.             | N. Oct. 17, 1903                 | 373     |
|        |       |                                         |                      |                |                  |                   |                               |                                  |                         |                  | B. Dec. 7, 1893;                 |         |
| 277.7  | 101.7 | 4.7                                     | 7.1                  |                |                  |                   | 18.1                          |                                  |                         |                  | C. 1855                          | 374     |
| 205.3  | 56.4  | 4.9                                     | 6.6                  |                | 344.5            | .044              | 52.5                          |                                  |                         | H.W.             | B. Feb. 27, 1902                 | 375     |
| 43.3   | 15.6  |                                         |                      |                |                  |                   |                               |                                  |                         | H.W.             | B. Aug. 17, 1905, p. 156         |         |

† About.

366. F. 1852, p. 200. Lead in ring joints  $\frac{1}{2}$  span from abutment. 367. Rough stone in cement.  
 368. Three metal hinges backed with granite. Five lateral arches in each spandrel. 369. Lateral arches. Max. H. = 111'5". 372. Three-hinged for D. L. Fixed for L.L. 373. Destroyed 1416.  
 374. Twin arches 19'4 apart. 375. Longest stone arch in the world.

TABLE II.—DATA FOR ABOUT 500 ARCH BRIDGES

PLAIN CON

| Number. | Name.             | Place.                             | Over.                        | Date.   | Engineer.          | No. of Spans. |
|---------|-------------------|------------------------------------|------------------------------|---------|--------------------|---------------|
| 1       | Fern St.          | W. Hartford, Conn., U.S.A.         | Trout Brook                  | 1902-3  | Crawford           | 1             |
| 2       | Casey R.          | Las Marias, Porto Rico             | Casey R.                     | 1899(?) | Buel               | 2             |
| 3       | Bridge No. 41     | Sharpsville, Penn., U.S.A.         | Pine Run                     | 1900    | Geer               | 1             |
| 4       |                   | Cheltenham, Mo., U.S.A.            | Des Peres R.                 | 1904(?) | Purdon             | 1             |
| 5       |                   | Bet. Manati and Aales, Porto Rico  | Quebrada R.(?)               | 1899(?) | Buel               | 3             |
| 6       |                   | Mansfield, Ohio, U.S.A.            |                              | 1904(?) | Keith              | 1             |
| 7       |                   | Bet. Santiago and El Caney, Cuba   | San Juan R.                  | 1902    | Rockenbach         | 2             |
| 8       | Cannington Viad.  | Cannington, England                |                              | 1900-02 | Pain               | 10            |
| 9       | Ewarton Br.       | Jamaica, W. I.                     | Ravine                       | 1881-82 | Bell               | 4             |
| 10      | Lochnanuamh Viad. | Scotland                           |                              | 1899†   | Simpson and Wilson | 8             |
| 11      |                   | Scotland                           | Arnaboll Burn                | 1899†   | Simpson and Wilson | 6             |
| 12      | Finnan Viad.      | Scotland                           | Finnan Valley                | 1899†   | Simpson and Wilson | 21            |
| 13      |                   | Washington, D.C., U.S.A.           | Broad Branch                 | 1901    | Douglas            | 1             |
| 14      |                   | Northampton, Pa., U.S.A.           | Hokendauqua Crk. and Highway | 1900    | Thompson           | 1             |
| 15      |                   | Adjuntas, Porto Rico               | Small stream                 | 1899(?) | Buel               | 1             |
| 16      |                   | Salt River, Ariz., U.S.A.          | Dam spillway                 | 1905-   |                    | 3             |
| 17      | Bridge No. 242    | W. of Cincinnati, O., U. S. A.     | Tanner's Crk.                | 1903-4  | Kittridge          | 1             |
| 18      |                   | Thebes, Ill., U. S. A.             | Bank of Mississippi R.       | 1902    | Noble and Mojeski  | 11            |
| 19      |                   | Concord, Mass., U.S.A.             | Assabet R.                   | 1901    | Worcester          | 1             |
| 20      | Bridge No. 163    | W. of Cincinnati, O., U.S.A.       | Tanner's Crk.                | 1903-04 | Kittridge          | 3             |
| 21      |                   | Ehingen, Wurtemberg                | Danube R.                    | 1893    |                    | 2             |
| 22      | Ashtabula Br.     | Ashtabula, Ohio, U.S.A.            | Ashtabula R.                 | 1904    | Beckwith           | 1             |
| 23      |                   | Near Rechtenstein, Wurtemberg      | Danube R.                    | 1893    | Braun              | 2             |
| 24      |                   | Plano, Ill., U.S.A.                | Big Rock Crk.                | 1903-4  | Breckenridge       | 1             |
| 25      |                   | Near San Leandro, Cal., U.S.A.     | S. Leandro Crk.              | 1901    | County Surveyor    | 1             |
| 26      | St. Ana Viad.     | Riverside, Cal., U.S.A.            | Santa Ana R.                 | 1902-04 | Hawgood            | 8             |
| 27      | Morar Viad.       | Scotland                           | Morar R. & H.W.              | 1898-9  | Simpson and Wilson | 2             |
| 28      |                   | Near Imnau, Germany                | Eyach R.                     | 1806    | Leibbrand          | 1             |
| 29      |                   | Pittsburg, Penn., U.S.A.           | Silver Lake                  | 1905    | Brown              | 2             |
| 30      |                   | Near Tarvis, Austria               | Schlitz R.                   | †1903   |                    | 5             |
| 31      |                   | Thebes, Ill., U. S. A.             | Bank of Mississippi R.       | 1902    | Noble and Mojeski  | 1             |
| 32      |                   | Near Mechanicsville, N. Y., U.S.A. | Anthony Kill                 |         |                    | 2             |

\* Maximum.

REMARKS.—1. Cost \$4050. 3. Skew, 15° 0'. Penn. Ry. 4. 1:6 "chats." St. L. & S. F. Ry.  
 6. Three cast-iron hinges. 7. Contract price, \$31,000. 9. On curve, 1980' R. Jamaica Govt. Rys.  
 12. On curve 1200' R. L=1248'; H.=100'. 13. Pebble-faced, Cost \$4150.17.  
 14. Three tracks. C. R.R. of N. J. Ex. metal used in radial planes. 15. 1600' above sea-level.  
 16. Very flat arches; about 12" fill over key. 17. "Big 4" Ry., Chicago Div. 18. Approach to



## ARRANGED ACCORDING TO SPAN—(Continued).

## CRETE ARCHES.

| Span. | Rise. | Thickness at Crown. | At Springing. | Curve.         | Radius at Crown. | Thickness at Crown = $t_0$ . | $\frac{t_0}{R}$ . | Width, Face to Face at Crown. | Thickness of Piers at Springing. | Class of Bridge. | Reference.                              | Number. |
|-------|-------|---------------------|---------------|----------------|------------------|------------------------------|-------------------|-------------------------------|----------------------------------|------------------|-----------------------------------------|---------|
| 26.0  | 5.0   | 2.0                 | 2.0           |                | 19.5             | 2.0                          | .103              |                               |                                  |                  |                                         | 1       |
| 10.0  | 3.0   | 1.3                 | 1.3           |                | 5.5              | 1.3                          | .236              | †30.0                         | 4.0                              | H.W.             | N. April 25, 1903                       |         |
| 30.0  | 15.0  | 2.5                 |               | C <sub>2</sub> | 15.0             | 2.5                          | .167              |                               |                                  | H.W.             | Cem., Jan. 1902                         | 2       |
| 30.0  | 15.0  | 2.5                 | 2.5           | C <sub>2</sub> | 15.0             | 2.5                          | .167              | 75.5                          |                                  | Ry.              | T. Nov. 16, 1900                        | 3       |
| 36.0  | 18.0  |                     |               | C <sub>2</sub> | 18.0             |                              |                   | 87.0                          | 5.2                              | Ry.              | B. Nov. 3, 1904                         | 4       |
| 39.4  | 9.8   | 1.6                 |               |                |                  | 1.6                          |                   |                               |                                  | H.W.             | Cem., Jan. 1902, p. 328                 | 5       |
| 40.0  | 7.5   | .7                  | .8            |                |                  | .7                           |                   | 40.0                          |                                  | H.W.             | N. Feb. 18, 1905                        | 6       |
| 40.0  | 11.5  | 1.5                 |               |                | 30.0             | 1.5                          | .050              | 20.0                          | 8.0                              | H.W.             | B. Jun. 13, 1903, p. 549                | 7       |
| 50.0  | 16.0  | 2.5                 | 2.5           | E              |                  | 2.5                          |                   | 16.0                          |                                  | Light Ry.        | N. Oct. 21, 1905                        | 8       |
| 50.0  | 22.2  | 2.0                 | 3.0           |                | 25.2             | 2.0                          | .079              | 16.0                          | 6.0                              | Ry.              | B. July 27, 1893                        | 9       |
| 50.0  |       |                     |               |                |                  |                              |                   |                               |                                  | Ry.              | B. Feb. 9, 1899, p. 85                  | 10      |
| 50.0  |       |                     |               |                |                  |                              |                   |                               |                                  | Ry.              | B. Feb. 9, 1899, p. 85                  | 11      |
| 50.0  | 25.0  | 2.5                 |               | C <sub>2</sub> | 25.0             | 2.5                          | .100              |                               | 6.0                              | Ry.              | B. Feb. 9, 1899, p. 85                  | 12      |
| 50.3  | †7.0  | †1.8                | †6.0          |                |                  | †1.8                         |                   | 26.0                          |                                  | H.W.             |                                         |         |
| 51.8  | 13.5  | 3.5                 |               |                | 31.5             | 3.5                          | .111              | 42.0                          |                                  | Ry.              | N. Jun. 8, 1901, p. 541                 | 13      |
| 34.0  | 11.3  | 2.8                 |               |                | 28.0             | 2.8                          | .100              |                               |                                  |                  |                                         |         |
| 55.0  | 11.0  | 1.5                 |               | E              |                  | 1.5                          |                   |                               |                                  | H.W.             | Cem., Jan. 1902                         | 15      |
| 59.0  |       | †1.5                |               |                |                  | 1.5                          |                   | 10.3                          | 6.5                              | H.W.             | N. Oct. 14, 1905                        | 16      |
| 60.0  | 26.0  | 2.7                 |               | C              |                  | 2.7                          |                   | †33.0                         | 9.3                              | Ry.              | N. Mar. 5, 1904, p. 292                 | 17      |
| 40.0  | 20.0  | 2.3                 |               | C <sub>2</sub> | 20.0             | 2.3                          | .115              |                               |                                  |                  |                                         |         |
| 65.0  | 32.5  | 3.3                 |               | C <sub>2</sub> | 32.5             | 3.3                          | .102              | 28.0                          | 12.0                             | Ry.              | T. Jan. 9, '03, p. 21; B. Nov. 20, 1902 | 18      |
| 66.0  | 11.0  |                     |               | E              |                  |                              |                   | †35.0                         |                                  | H.W.             | Municip. Engineering, March, 1902       | 19      |
| 68.0  | 17.0  | 3.5                 | 6.0           | 5C             | 64.0             | 3.5                          | .055              | 33.0                          | 12.5                             | Ry.              | N. Mar. 5, 1904                         | 20      |
| 69.0  |       | 2.3                 | 3.0           | C              |                  | 2.3                          |                   | 24.6                          | 6.6                              | H.W.             | B. Jan. 9, 1902, p. 35                  | 21      |
| 66.0  | 7.2   |                     |               |                |                  |                              |                   |                               |                                  |                  |                                         |         |
| 74.0  | 37.0  | †6.5                |               | C <sub>2</sub> | 37.0             | 6.5                          | .176              | 145.0                         |                                  | Ry.              | T. Jan. 27, 1905                        | 22      |
| 74.4  | 8.2   | 2.1                 |               |                |                  | 2.1                          |                   |                               |                                  |                  | Y. 1898                                 | 23      |
| 75.0  |       | 3.0                 |               |                |                  |                              |                   |                               |                                  | Ry.              | N. Jan. 2, '04, p. 18                   | 24      |
| 81.3  | 26.0  | 3.0                 | 15'-20'       | 5C             | 43.0             | 3.0                          | .070              | 44.0                          |                                  | H.W.             | B. Aug. 27, 1903, p. 174                | 25      |
|       |       |                     |               |                | 61.5             | 3.0                          | .048              | †50.0                         |                                  |                  |                                         |         |
| 86.0  | 43.0  | 3.5                 |               | C              | 43.5             | 3.5                          | .081              |                               |                                  | Ry.              | N. Sept. 9, 1905, p. 284                | 26      |
| 38.5  |       |                     |               |                |                  |                              |                   |                               |                                  |                  |                                         |         |
| 90.0  | 24.0  | 3.0                 |               |                |                  | 3.0                          |                   |                               |                                  | Ry.              | B. Feb. 9, 1899, p. 85                  | 27      |
| 50.0  |       |                     |               |                |                  |                              |                   |                               |                                  |                  |                                         |         |
| 20.0  |       |                     |               |                |                  |                              |                   |                               |                                  |                  |                                         |         |
| 98.4  | 9.8   | 1.5                 | 1.6           |                |                  | 1.5                          |                   | 8.2                           |                                  | H.W.             | G. 2 Tri., 1898                         | 28      |
| 100.0 | 50.0  | 4.0                 | 4.0           | C <sub>2</sub> | 50.0             | 4.0                          | .080              | 54.0                          | 12.0                             | Ry.              | N. May 6, 1905, p. 528                  | 29      |
| 80.0  | 40.0  | 3.6                 | 3.6           |                | 40.0             | 3.5                          | .088              |                               |                                  | H.W.             | Engineer. April 22, 1904, p. 424        | 30      |
| 100.0 | 10.0  | 2.3                 | 2.3           |                |                  |                              |                   |                               |                                  |                  |                                         |         |
| 100.0 | 50.0  | 4.5                 |               | C <sub>2</sub> | 50.0             | 4.5                          | .090              | 28.0                          |                                  | Ry.              | T. Jan. 9, '02, p. 21; B. Nov. 20, 1902 | 31      |
| 100.0 |       |                     |               |                |                  |                              |                   |                               |                                  | El. Ry.          | B. Nov. 5, 1903, p. 408                 | 32      |
| 50.0  |       |                     |               |                |                  |                              |                   |                               |                                  |                  |                                         |         |

† About.

Thebes Bridge. 20. "Big 4" Ry., Chicago Div. 21. Cost \$21,000. 22. L. S. & M. S. Ry.  
 Four tracks. 23. Three lead "hinges." 24. C. B. & Q. Ry. Two tracks. 25. Skew, 10°  
 Cost \$25,840. 26. One track. S. P., L. A. & S. L. Ry. 27. Mallaig Ex. of W. Highland Ry.  
 28. Three granite "hinges." 29. Penn. Ry. Four tracks, 5° curve. 30. Three steel "hinges."  
 31. Approach to Thebes Bridge.



TABLE II.—DATA FOR ABOUT 500 ARCH BRIDGES

PLAIN CON

| Number. | Name.          | Place.                              | Over.         | Date.     | Engineer.            | No. of Spans.    |
|---------|----------------|-------------------------------------|---------------|-----------|----------------------|------------------|
| 33      | Danville Arch  | 2 miles from Danville, Ill., U.S.A. | Vermillion R. | 1905      | "Big 4"              | 1                |
| 34      |                | Near Mittenberg, Germany            | Main R.       | 1898-99   | Fleischman and Bosch | 2<br>2<br>2<br>2 |
| 35      | Grand Maître   | Fontainebleu Forest, France         | Valley        | 1860      | Belgrand             |                  |
| 36      |                | Kirchheim, Wurtemberg               | Neckar R.     | 1808†     |                      | 4                |
| 37      | 16th St.       | Washington, D. C., U.S.A.           | Piney Branch  | 1905      | Douglas              |                  |
| 38      | Borrowdale     | Scotland                            | Bor'dale Burn | 1898-99   | Simpson and Wilson   | 1                |
| 39      | Coulouvrenière | Geneva, Switzerland                 | Rhone R.      | 1895      | Butticaz             | 2                |
| 40      | Big Muddy      | Near Grand Tower, Ill., U.S.A.      | Big Muddy R.  | 1901-03   | Parkhurst            | 2<br>3           |
| 41      | Inzigkofen     | Inzigkofen, Wurtemberg              | Danube R.     | 1896      | Leibbrand            | 1                |
| 42      | Vauxhall       | London, England                     | Thames R.     | 1899      | Binnie               | 1                |
| 43      | Conn. Ave. Br. | Washington, D. C., U.S.A.           | Rock Creek    | 1889-1906 | Morison & Biddle     | 2<br>5           |
| 44      |                | Munderkingen, Wurtemberg            | Danube R.     | 1893      | Douglas              | 2                |
| 45      |                | Near Oviédo, Spain                  | Nalon R.      | Proposed  | Leibbrand            | 1                |
| 46      |                | Neckarhausen, Germany               | Neckar R.     | 1903†     |                      |                  |
| 47      |                | Ulm, Germany                        | Ry. Yards     | 1905†     |                      | 1                |

\* Maximum.

REMARKS.—34. Three lead "hinges." 35. Paris water-supply from Vanne. 36. Three lead "hinges." 38. Mallaig Ex. of W. Highland Ry. 39. Three "hinges." 40. Two tracks, Ill. Cent. Ry. 41. Three cast-iron "hinges." 42. Three "hinges." 44. Three steel "hinges."

## ARRANGED ACCORDING TO SPAN—(Continued).

## CRETE ARCHES.

| Span.  | Rise. | Thickness at Crown. | At Springing. | Curve.         | Radius $R$ at Crown. | Thickness at Crown = 40. | $\frac{t_0}{R}$ . | Width, Face to Face at Crown. | Thickness of Piers at Springing. | Class of Bridge. | Reference.                      | Number. |
|--------|-------|---------------------|---------------|----------------|----------------------|--------------------------|-------------------|-------------------------------|----------------------------------|------------------|---------------------------------|---------|
| 100.0  | 40.0  | 4.0                 |               | C              | 51.3                 | 4.0                      | .088              | 42.0                          | 15.0                             | Ry.              | N. Mar. 3, 1906, p. 338         | 33      |
| 80.0   | 30.0  | 3.6                 |               |                | 41.7                 | 3.6                      | .086              |                               |                                  |                  |                                 |         |
| 112.0  | 16.4  | 2.5                 | 2.8           |                |                      | 2.5                      |                   | 23.0                          | 10.2                             | H.W.             | B. July 25, 1901, p. 61         | 34      |
| 107.3  | 17.5  |                     |               |                |                      |                          |                   |                               |                                  |                  |                                 |         |
| 107.3  | 14.8  | 2.5                 | 2.8           |                |                      | 2.5                      |                   |                               |                                  |                  |                                 |         |
| 102.3  | 18.3  |                     |               |                |                      |                          |                   |                               |                                  |                  |                                 |         |
| 102.3  | 13.8  | 2.3                 | 2.6           |                |                      | 2.3                      |                   |                               |                                  |                  |                                 |         |
|        | 19.0  |                     |               |                |                      |                          |                   |                               |                                  |                  |                                 |         |
| *115.8 | †19.3 | 1.3                 |               |                |                      | †1.3                     |                   |                               |                                  | Aqued't          | K. Oct. '60, p. 275             | 35      |
| 124.6  | 19.0  | 2.6                 | 3.0           | Par            |                      | 2.6                      |                   | †18.0                         |                                  | H.W.             | B. Mar. 9, 1900                 | 36      |
| 125.0  | 39.0  | 5.0                 |               |                |                      | 5.0                      |                   | 25.0                          |                                  | H.W.             | B. Nov. 16, 1905                | 37      |
| 127.5  | 22.5  | 4.0                 |               |                |                      | 4.0                      |                   |                               |                                  | Ry.              | B. Feb. 9, 1899, p. 85          | 38      |
| 20.0   |       |                     |               |                |                      |                          |                   |                               |                                  |                  |                                 |         |
| 131.2  | 18.2  | 3.0                 | 3.0           | C              | 127.3                | 3.0                      | .024              |                               |                                  | H.W.             | Y. 1898                         | 39      |
| 140.0  | 30.0  | 7.0                 | †20.0         | E              | 167.0                | 7.0                      | .042              | 50.6                          |                                  | Ry.              | B. Nov. 12, 1903, p. 423        | 40      |
| 141.0  | 14.4  | 2.3                 | 2.6           |                |                      | 2.3                      |                   | †12.5                         |                                  |                  | B. April 22, 1897               | 41      |
| 144.6  | †18.6 | 3.9                 | 3.9           |                |                      | 3.9                      |                   |                               |                                  |                  |                                 | 42      |
| 130.6  | †20.0 | 3.9                 | 3.9           |                |                      | 3.9                      |                   | †84.0                         |                                  | H.W.             | N. Feb. 25, 1899                |         |
| 150.0  | 75.0  | 5.0                 |               | C <sub>2</sub> | 75.0                 | 5.0                      | .067              | 52.0                          | 20.0                             | H.W.             | N. July 8, 1905, p. 30          | 43      |
| 82.0   | 41.0  | 3.3                 |               |                | 41.0                 | 3.3                      | .080              |                               |                                  |                  | B. June 1, 1905                 |         |
| 164.0  | 16.4  | 3.3                 | 3.6           |                |                      | 3.3                      |                   | †26.2                         |                                  | H.W.             | G. 3 Tri., 1897, p. 356         | 44      |
| 165.0  | 18.8  | 3.7                 | 3.7           |                |                      | 3.7                      |                   | †17.0                         |                                  | H.W.             | B. Sept. 26, 1901               | 45      |
| 165.0  | 13.5  | 2.8                 | 3.7           |                |                      | 2.8                      |                   | 15.8                          |                                  | H.W.             | Engineer, Dec. 30, 1904, p. 650 | 46      |
| 215.0  |       |                     |               |                |                      |                          |                   | †46.0                         |                                  | H.W.             | B. March 15, 1906               | 47      |

† About.

45. Three "hinges." 46. Three cast-iron and steel "hinges." 47. Three "hinges;" centre to centre of hinges 187'.0; rise centre to centre of hinges 18'.7; cost \$45,000.

TABLE II.—DATA FOR ABOUT 500 ARCH BRIDGES

REINFORCED CON

| Number. | Name.                             | Place.                            | Over.             | Date.   | Engineer.                                                                                                                              | No. of Spans. |
|---------|-----------------------------------|-----------------------------------|-------------------|---------|----------------------------------------------------------------------------------------------------------------------------------------|---------------|
| 1       | Ridgewood Ave.                    | Elkhart, Ind., U.S.A.             | La Rue H. W.      | 1903    | Osgood<br>M. A. C. Co.<br>M. A. C. Co.<br>M. A. C. Co.<br>M. A. C. Co.<br>M. A. C. Co.<br>M. A. C. Co.<br>M. A. C. Co.<br>M. A. C. Co. | 1             |
| 2       |                                   | Vulcanite, N. J., U.S.A.          | Highway           | 1905-6  |                                                                                                                                        | 1             |
| 3       |                                   | Rock Rapids, Ia., U.S.A.          | Ravine            | 1894    |                                                                                                                                        | 1             |
| 4       |                                   | Ridgewood, N. J., U.S.A.          | Br. of Saddle C.  | 1897    |                                                                                                                                        | 1             |
| 5       |                                   | Marion Co., Ind., U.S.A.          |                   | 1890    |                                                                                                                                        | 1             |
| 6       |                                   | Waldwick, N. J., U.S.A.           | Stream            | 1898?   |                                                                                                                                        | 1             |
| 7       |                                   | Mahwah, N. J., U.S.A.             | Stream            | 1898    |                                                                                                                                        | 1             |
| 8       |                                   | Crystal Lake, N. J., U.S.A.       | Stream            | 1898    |                                                                                                                                        | 1             |
| 9       |                                   | Delaware Co., Penn., U.S.A.       | Stream            | 1905?   |                                                                                                                                        | 1             |
| 10      |                                   | Wayne Township, N. J., U.S.A.     | Stream            | 1896    |                                                                                                                                        | 1             |
| 11      | Linwood Ave.                      | Ridgewood, N. J., U.S.A.          | Saddle R.         | 1895    | M. A. C. Co.<br>Walter                                                                                                                 | 1             |
| 12      |                                   | W. Edwardsville, Kan., U.S.A.     | Mission Creek     | 1904?   |                                                                                                                                        | 1             |
| 13      | S. Jefferson St.<br>McKinley Arch | Indian Creek, Ill., U.S.A.        | Indian Creek      | 1903    | Smith<br>Hall<br>Reiseger<br>Phillips                                                                                                  | 3             |
| 14      |                                   | Oconomowoc, Wis., U.S.A.          | Lake              | 1890    |                                                                                                                                        | 2             |
| 15      |                                   | Battle Creek, Mich., U.S.A.       | Kalamazoo R.      | 1899    |                                                                                                                                        | 2             |
| 16      |                                   | St. Louis, Mo., U.S.A.            | Des Peres R.      | 1902    |                                                                                                                                        | 1             |
| 17      | San Miguel                        | Sorsogon, Philippines             | Stream            | 1905    | Stevens<br>White & Co.<br>Keepers & Thacher                                                                                            | 1             |
| 18      |                                   | Manila, Philippines               | Estero S. Miguel  | 1905    |                                                                                                                                        | 1             |
| 19      |                                   | Albion, Mich., U.S.A.             |                   | 1898?   |                                                                                                                                        | 3             |
| 20      | Como Park                         | St. Paul, Minn., U.S.A.           | Rapid Transit Ry. | 1904    | Wilson<br>Wilson<br>M. A. C. Co.                                                                                                       | 1             |
| 21      | Como Park                         | St. Paul, Minn., U.S.A.           | Rapid Transit Ry. | 1904    |                                                                                                                                        | 1             |
| 22      | Mount St.                         | Atlantic Highlands, N. J., U.S.A. | Grand Ave.        | 1895-96 |                                                                                                                                        | 1             |
| 23      | Salem St.                         | Carbondale, Penn., U.S.A.         | Lackawanna R.     | 1896    | M. A. C. Co.<br>Brady                                                                                                                  | 1             |
| 24      | Florida Keys Viad.                | Florida Keys, U.S.A.              | Salt Water        | 1905-   |                                                                                                                                        |               |
| 25      | Lamington Br.                     | Marysborough, Queensland          | Mary R.           | 1896    |                                                                                                                                        | 11            |
| 26      |                                   | Louisville, Ky., U.S.A.           | Beargrass Creek   | 1897    | Keepers & Thacher<br>Nelson                                                                                                            | 1             |
| 27      |                                   | Decatur Township, Ind., U.S.A.    | Goose Creek       |         |                                                                                                                                        | 1             |
| 28      | Mich. Cent. Ry.                   | Detroit, Mich., U.S.A.            | Southern Bvd.     | 1895-6  | Keepers & Thacher<br>M. A. C. Co.                                                                                                      | 1             |
| 29      |                                   | Hyde Park, N. Y., U.S.A.          | Crum Elbow C.     | 1897    |                                                                                                                                        | 1             |
| 30      |                                   | Plainwell, Mich., U.S.A.          | Kalamazoo R.      | 1903    | Courtwright<br>Schwiers<br>Osgood                                                                                                      | 7             |
| 31      | Arch St.                          | Paterson, N. J., U.S.A.           | Passaic R.        | 1903    |                                                                                                                                        | 3             |
| 32      | Jackson St.                       | Newark, N. J., U.S.A.             | Jackson St.       | 1904    |                                                                                                                                        | 1             |
| 33      | Sixth Ave.                        | Carbondale, Penn., U.S.A.         | Lackawanna R.     | 1896    | M. A. C. Co.<br>Buck (State)<br>Waldo (Con.)                                                                                           | 1             |
| 34      | Goat Island                       | Niagara Falls, N. Y., U.S.A.      | Niagara R.        | 1900-1  |                                                                                                                                        |               |
| 35      |                                   | Clifton, N. J., U.S.A.            | Passaic R.        | 1903    | Schwiers                                                                                                                               | 2             |
| 36      |                                   | Route Neutra, Hungary             |                   | 1892    |                                                                                                                                        | 6             |
| 37      |                                   | Route Nymphenburg, Wurtemberg     |                   |         |                                                                                                                                        | 1             |
| 38      | Castle Eichorn                    | Mähren, Austria                   | Ravine            | 1898    | Venier<br>M. A. C. Co.<br>Dean<br>M. A. C. Co.                                                                                         |               |
| 39      | Eighth Ave.                       | Carbondale, Penn., U.S.A.         | Lackawanna R.     | 1896    |                                                                                                                                        |               |
| 40      | Franklin Br.                      | St. Louis, Mo., U.S.A.            | Des Peres R.      | 1897-98 |                                                                                                                                        |               |
| 41      | Montgomery St.                    | Jersey City, N. J., U.S.A.        | Street            | 1895-96 |                                                                                                                                        |               |

\* Maximum.

REMARKS.—1. L. S. & M. S. Ry. 2. C. R. R. of N. J.; two tracks. 3. Six 4" 7.5-lb I beams, 36" centre to centre. 4. Nine 5" 9.75-lb. I beams; 36" centre to centre. 5. Melan type. 6. Nine 5" 9.75-lb. I beams; 34" centre to centre. 7. Nine 5" 9.75-lb. I beams; 31" centre to centre. 8. Nine 5" 9.75-lb. I beams; 36" centre to centre. 9. Skew. Phila. R. T. Co. 10. Nine 5" 9.75-lb. I beams; 35" centre to centre. 11. Seven 5" 9.75-lb. I beams. 36" centre to centre. 12. U. P. Ry.; two tracks. 13. C. I. & St. L. Ry. Short Line; two tracks. 14. Flat bars and expanded metal. 15. Melan type; twenty-one 6" I beams. 17. Various sizes and shapes of bars. 19. Thacher type. 20. 5" 9.75-lb. I beams; 38" centre to centre (?). 21. Four angles, 2" X 2" X 1/4"; 38" centre to centre (?). 22. Eight 6" 12.25-lb. I beams; 36" cen-



## ARRANGED ACCORDING TO SPAN—(Continued).

## CRETE ARCHES.

| Span. | Rise. | Thickness at Crown. | At Springing. | Curve.         | Radius R at Crown. | Kind of Steel.                 | Per Cent Steel at the Crown. | Width, Face to Face at Crown | Thickness of Piers at Springing. | Class of Bridge. | Reference.          | Number. |
|-------|-------|---------------------|---------------|----------------|--------------------|--------------------------------|------------------------------|------------------------------|----------------------------------|------------------|---------------------|---------|
| 30.0  | 9.0   | 3.3                 | 6.4           |                |                    | $\frac{3}{4}$ " J <sup>†</sup> | .55                          |                              |                                  | Ry.              | B. July 14 1904     | 1       |
| 30.0  | †9.0  |                     |               |                |                    | $\frac{3}{4}$ " O              |                              | 32.5                         |                                  | Ry.              | N. Sept. 9, 1905    | 2       |
| 30.0  | 6.6   | 0.5                 | 2.5           | 3C             | 39.0               |                                | 1.02                         | 17.5                         |                                  | H.W.             | X.                  | 3       |
| 30.0  | 3.0   | 0.9                 | 1.8           |                |                    |                                | .80                          | 26.0                         |                                  | H.W.             | X.                  | 4       |
| 32.0  | 3.2   |                     |               |                |                    |                                |                              |                              |                                  | H.W.             | Cement, Sept. 1900  | 5       |
| 32.0  | 3.2   | 0.9                 | 2.0           |                |                    |                                | .85                          | 24.0                         |                                  | H.W.             | X.                  | 6       |
| 32.0  | 3.2   | 0.9                 | 2.0           |                |                    |                                | .93                          | 22.0                         |                                  | H.W.             | X.                  | 7       |
| 32.0  | 6.4   | 0.9                 | 3.3           |                |                    |                                | .80                          | 26.0                         |                                  | H.W.             | X.                  | 8       |
| 33.0  | 16.0  | 0.5                 | 0.9           | E              |                    | O                              |                              |                              |                                  | Int'r.R.R.       | N. Dec. 2, 1905     | 9       |
| 35.0  | 3.5   | 0.8                 | 2.0           |                |                    |                                | .82                          | 25.0                         |                                  | H.W.             | X.                  | 10      |
| 40.0  | 8.0   | 1.0                 |               |                |                    |                                | .66                          | 20.0                         |                                  | H.W.             | X.                  | 11      |
| 40.0  | †20.0 | 2.2                 | 10.0          |                | 21.0               | $\frac{1}{4}$ " J              | .70                          | 38.0                         |                                  | Ry.              | T. Dec. 8, 1905     | 12      |
| 40.0  | 20.0  | 2.5                 |               | C <sub>2</sub> | 20.0               | 1 $\frac{1}{4}$ " J            |                              |                              |                                  | Ry.              | T. March 11, 1904   | 13      |
| 42.0  | 6.7   | 0.5                 |               |                |                    |                                |                              | 32.3                         |                                  | H.W.             | B. Oct. 10, 1899    | 14      |
| 42.0  |       |                     |               |                |                    |                                |                              | 42.0                         |                                  | H.W.             | T. Sept. 24, 1900   | 15      |
| 45.0  |       |                     |               | E              |                    | $\frac{3}{4}$ " J              |                              | 66.0                         |                                  | H.W.             | B. June 11, 1903    | 16      |
| 45.0  | 6.0   |                     |               | E              |                    | Var.                           |                              | 45.0                         |                                  | H.W.             | N. Oct. 21, 1905    | 17      |
| 45.9  |       |                     |               | 5C             | 43.0               | +                              |                              | †35.0                        |                                  | H.W.             | N. July 8, 1905     | 18      |
| 46.7  | 6.3   |                     |               |                |                    |                                |                              | 34.0                         |                                  | H.W.             | B. Sept. 21, 1899   | 19      |
|       |       |                     |               |                |                    |                                |                              | †28.0                        |                                  |                  |                     |         |
| 50.0  | 12.5  | 0.8                 | 2.5           |                |                    |                                | †.75                         | †17.0                        |                                  | Foot-b'rie       | N. Dec. 3, 1904     | 20      |
| 50.0  | 12.5  | 0.8                 | 2.5           |                |                    |                                | †.99                         | †17.0                        |                                  | Foot-b'rie       | B. April 6, 1905    | 21      |
| 50.0  | 11.0  | 0.8                 | 3.0           | C              | 33.9               |                                | 1.11                         | 25.0                         |                                  | H.W.             | N. August 22, 1896  | 22      |
| 50.0  | 8.3   | 0.8                 | 2.8           |                |                    |                                | 1.25                         | 55.0                         |                                  | H.W.             | X.                  | 23      |
| 50.0  | 25.0  | 2.0                 |               | C <sub>2</sub> | 25.0               | $\frac{3}{4}$ " J              | .76                          | 15.0                         |                                  | Ry.              | B. Oct. 19, 1905    | 24      |
| 50.0  | 4.0   | 1.7                 | 5.7           |                |                    |                                | .71                          | 22.7                         |                                  | H.W.             | N. Nov. 17, 1900    | 25      |
| 50.0  | 11.2  | 1.0                 | 5.0           | 3C             | 41.7               | 3"× $\frac{3}{4}$ "            |                              | †60.0                        |                                  | H.W.             | B. Feb. 14, 1901    | 26      |
| 50.0  |       |                     |               |                |                    |                                |                              |                              |                                  |                  | K. & T. Blues       |         |
| 50.0  |       |                     |               |                |                    |                                |                              |                              |                                  | H.W.             | Cement, Nov. 1901   | 27      |
| 50.3  | 9.5   | 1.5                 | 7.5           | C              | 38.2               |                                | 3.75                         | 109.9                        |                                  | Ry.              | N. Sept. 28, 1895   | 28      |
| 53.0  | 7.5   | 0.8                 | 2.5           |                |                    |                                | 1.23                         | 17.0                         |                                  | H.W.             | T. March 3, 1899    | 29      |
| 26.0  | 7.5   | 0.7                 | 1.7           |                |                    |                                | 1.00                         |                              |                                  |                  | B. Nov. 10, '98; X. |         |
| 54.0  | 8.0   |                     |               |                |                    |                                |                              | †24.0                        | 6.0                              | H.W.             | B. May 12, 1904     | 30      |
| 54.2  | 2.4   | 1.7                 | 4.2           |                |                    |                                | .60                          | †45.5                        |                                  | H.W.             | N. Sept. 10, 1904   | 31      |
| 54.3  | 10.5  | 2.5                 |               | C              | 48.4               | 1 $\frac{1}{4}$ " O            | 1.37                         | †32.0                        |                                  | Ry.              | N. August 6, 1904   | 32      |
| 54.6  | 5.5   | 0.9                 | 3.0           |                |                    |                                | 1.12                         | 48.0                         |                                  | H.W.             | X.                  | 33      |
| 55.0  | 10.0  |                     |               |                |                    |                                | .63                          | †40.0                        | 8.0                              | H.W.             | B. Dec. 6, 1900     | 34      |
| 50.5  | 9.0   |                     |               |                |                    |                                | .66                          |                              |                                  |                  |                     |         |
| †55.0 | 3.0   | 2.0                 |               |                |                    |                                | 2.40                         | †30.0                        |                                  | H.W.             | N. Sept. 10, 1904   | 35      |
| 55.8  | 3.7   | 0.8                 |               |                |                    |                                |                              | †10.7                        |                                  | H.W.             | G. 1st Tri., 1904   | 36      |
| 56.7  | 5.9   | 1.0                 |               |                |                    |                                |                              | †32.8                        |                                  | H.W.             | G. 1st Tri., 1904   | 37      |
| 57.6  | 19.7  | 1.9                 | 2.3           | 3C?            | 32.8               |                                |                              | 21.3                         |                                  | H.W.             | Öst. Monat.Baud'st  | 38      |
| 58.7  | 6.0   | 1.0                 |               |                |                    |                                | 1.02                         | 49.0                         |                                  | H.W.             | X.                  | 39      |
| 60.0  | 15.5  | 0.8                 |               | 3C             | 48.0               |                                | 1.40                         | 32.0                         |                                  | H.W.             | X.                  | 40      |
| 61.2  | 12.0  |                     |               |                |                    |                                | 1.02                         | 83.3                         |                                  | H.W.             | X.                  | 41      |

† About.

‡ J=Johnson bars.

O=Round bars.

tre to centre. 23. Nineteen 7" 15-lb. I beams; 36" centre to centre. 24. Arches vary in span but all same type. 25. 41.25-lb. rails; 2' centre to centre. 27. Melan type. 28. Fou, 4"×4"× $\frac{1}{2}$ " angles; 22 $\frac{1}{2}$ " centre to centre. 29. Five 7" 15-lb. I beams; 36" centre to centre; five 5" 6.75-lb. I beams; 36" centre to centre. 30. 4"-6" channels; 1.0' centre to centre. 32. Ce R. R. of N. J. two tracks. 33. Sixteen 7" 15-lb. I beams; 36" centre to centre. 34. Thacher type. 35. See No. 31. 36. Wülich type. 37. Monier type. 38. Melan type; four I beams. 39. Sixteen 7" 15-lb. I beams; 36" centre to centre. 40. Eleven 8" 18-lb. I beams; 36" centre to centre. 41. Twenty-one 7" 15-lb. I beams 36" centre to centre; two elevated tracks.



TABLE II.—DATA FOR ABOUT 500 ARCH BRIDGES

REINFORCED CON

| Number. | Name.           | Place.                                     | Over.             | Date.   | Engineer.     | No. of Spans. |
|---------|-----------------|--------------------------------------------|-------------------|---------|---------------|---------------|
| 42      | Herkimer Viad.  | Troy, N. Y., U.S.A.                        | Wynant's Kill     | 1897    | Kenney        | 1             |
| 43      |                 | Route Ebhausen, Wurtemberg                 |                   | 1891    |               | 1             |
| 44      |                 | Vigneux, France                            | Dora R.           | 1900    |               | 1             |
| 45      |                 | Italy                                      | W. Canada Crk.    | 1902    |               | 2             |
| 46      |                 | Herkimer, N. Y., U.S.A.                    |                   | 1902-3  | Osborn E. Co. | 3             |
| 47      |                 | Jacksonville, Fla., U.S.A.                 | McCoy's C. & R.R. | 1903-4  |               | 7             |
| 48      |                 | Auch, France                               | Gers R.           | 1899    |               | 11            |
| 49      |                 | Military Road, San Juan, Ponce, Porto Rico | Guayo R.          | 1900-01 | Thacher       | 1             |
| 50      | Bloomfield Ave. | Newark, N. J., U.S.A.                      | Park drive        | 1904    | Reynolds      | 3             |
| 51      |                 | Cincinnati, Ohio, U.S.A.                   | Park drive        | 1894-95 | M. A. C. Co.  | 1             |
| 52      |                 | Trinidad, Col., U.S.A.                     | Purgatorie R.     | 1905    | Hibbard       | 2             |
| 53      |                 | Copenhagen, Denmark                        | Railway           | 1879    |               | 1             |
| 54      |                 | Route Painpardu, Belgium                   |                   | 1899    |               | 1             |
| 55      |                 | La Salle, Ill., U.S.A.                     | Gorge             | 1905    | Strauss       | 1             |
| 56      | Cedar R.        | Waterloo, Ia., U.S.A.                      | Cedar R.          | 1902-3  | Z             | 1             |
| 57      | Meridian St.    | Indianapolis, Ind., U.S.A.                 | Fall Creek        | 1900    | Jeup          | 7             |
| 58      | Illinois St.    | Indianapolis, Ind., U.S.A.                 | Fall Creek        | 1900    | Jeup          | 3             |
| 59      | Wealthy Ave.    | Grand Rapids, Mich., U.S.A.                | Grand R.          | 190     | Anderson      | 3             |
| 60      |                 | Wabash, Ind., U.S.A.                       | Creek             | 1905    |               | 1             |
| 61      | Hamilton St.    | Hartford, Conn., U.S.A.                    | Park R.           | 1898    | M. A. C. Co.  | 2             |
| 62      |                 | Hyde Park, N. Y., U.S.A.                   | Crum Elbow C.     | 1897    | M. A. C. Co.  | 1             |
| 63      |                 | Polasky, Cal., U.S.A.                      | S. Joaquin R.     | 1905    | Leonard       | 10            |
| 64      |                 | Route Bade, Austria                        |                   | 1900    |               | 1             |
| 65      | Rock Creek      | Washington, D. C., U.S.A.                  | Rock Creek        | 1901-   | Beach         | 1             |
| 66      | Soissons        | Soissons, France                           | L'Aisne           | 1902    | Riboud        | 1             |
| 67      |                 | Halder                                     | Lenne R.          | 1904    |               | 1             |
| 68      | De l'Empereur   | Sarajero, Bosnie                           |                   | 1897    |               | 1             |
| 69      | Fabrizio Viad.  | Italy                                      |                   | 1905    |               | 2             |
| 70      |                 | Rt. Payerbach, Austria                     |                   | 1900    |               | 1             |
| 71      | Seeley St       | Brooklyn, N. Y., U.S.A.                    | Prospect Ave.     | 1903-4  | Foot          | 1             |
| 72      |                 | Austria                                    | Bialka R.?        | 1894    |               | 1             |
| 73      |                 | Gr'd Rapids, Mich., U.S.A.                 | Grand R.          | 1903-4  | Anderson      | 1             |
| 74      | Main St.        | Dayton, Ohio, U.S.A.                       | Great Miami R.    | 1902-3  | Turner        | 2             |
| 75      | West St.        | Paterson, N. J., U.S.A.                    | Passaic R.        | 1897-8  | M. A. C. Co.  | 7             |
| 76      |                 | Yorktown, Ind., U.S.A.                     | Stream            | 1905?   | Luten         | 1             |
| 77      |                 | Papiguis, Italy                            | Nera R.           |         |               | 2             |
| 78      | N. Sixth Ave.   | Des Moines, Ia., U.S.A.                    | Des Moines R.     | 1901-2? | Z             | 3             |

\* Maximum.

REMARKS.—42. Nine 8" 18-lb. I beams; 36" centre to centre. 43. Monier type. 44. Piketty type. 45. Hennebique type. 46. U. & M. V. Ry. Two tracks. 47. Melan ribs and Thacher bars. 48. Bonna type. 49. Thacher type. 50. Melan type. Two E. Ry. tracks. 51. Eleven 9" 21-lb. I beams; 36" centre to centre. 52. Five 18.8 lb. (per foot) rails. 53. Hennebique type. 54. Two ribs. In Deer Park. 55. Thacher type. 56. 10" 25-lb. I beams; 36" centre to centre. 57. 10" 25-lb. I beams; 36" centre to centre. 58. 10" 25-lb. I beams; 36" centre to centre. 59. Kahn

## ARRANGED ACCORDING TO SPAN—(Continued).

## CRETE ARCHES.

| Span. | Rise. | Thickness at Crown. | At Springing. | Curve. | Radius R at Crown. | Kind of Steel. | Per Cent Steel at the Crown. | Width, Face to Face at Crown. | Thickness of Piers at Springing. | Class of Bridge. | Reference.                    | Number. |
|-------|-------|---------------------|---------------|--------|--------------------|----------------|------------------------------|-------------------------------|----------------------------------|------------------|-------------------------------|---------|
| 65.0  | 8.5   | 1.0                 |               |        |                    |                | 1.23                         | 27.0                          |                                  | H.W.             | X.                            | 42      |
| 65.6  | 8.2   | 0.7                 |               |        |                    |                |                              | †13.1                         |                                  | H.W.             | G. 1st Tri., 1904             | 43      |
| 65.6  | 14.8  | 1.6                 |               |        |                    |                |                              | †13.1                         |                                  | H.W.             | G. 1st Tri., 1904             | 44      |
| 65.6  | 6.6   |                     |               |        |                    |                |                              | †46.6                         |                                  | H.W.             | G. 1st Tri., 1904             | 45      |
| 66.0  | 14.0  | 1.8                 | 4.5           |        | 46.5               | 1½" T†         | .96                          |                               | 8.0                              | E. Ry.           | N. Feb. 22, 1903, p. 240      | 46      |
| 62.0  | 12.0  | 1.8                 | 4.5           |        | 46.0               |                | .96                          |                               |                                  |                  |                               | 47      |
| *68.0 | 7.0   | 1.5                 |               |        |                    |                |                              | 52.0                          | †7.0                             | H.W. & E. Ry.    | T. July 3, 1903, p. 428       | 48      |
| 68.9  | 6.6   | 1.0                 |               |        |                    |                |                              |                               |                                  | H.W.             | G. 1st Tri., 1904             | 48      |
| 70.0  | 2-7.0 |                     |               |        |                    |                |                              | 20.0                          |                                  | H.W.             | N. Aug. 3, 1901, p. 98        | 49      |
| 70.0  | 1-7.5 |                     |               |        |                    |                |                              |                               |                                  |                  |                               |         |
| 70.0  | 8.5   |                     |               |        |                    |                |                              | †65.0                         |                                  | H.W.             | N. Aug. 12, 1905, p. 32.5     | 50      |
| 70.0  | 10.0  | 1.3                 | 4.0           | C      | 106.3              |                | †.80                         | 32.5                          |                                  | H.W.             | M. A. C. Co., B. Oct. 3, 1895 | 51      |
| 70.0  | 7.0   | 1.2                 | 3.0           | 3C     |                    | 1½" T          | 2.67                         | †65.0                         | 7.0                              | H.W.             | N. Feb. 10, 1906              | 52      |
| 71.7  | 8.5   | 0.9                 | 1.2           | 3C     | 77.2               |                |                              |                               |                                  | Foot-brie        | B. July 21, 1898              | 53      |
| 71.8  | 9.2   | 1.3                 |               |        |                    |                |                              | †39.4                         |                                  | H.W.             | G. 1st Tri., 1904             | 54      |
| 72.0  | 7.5   | †2.0                | †2.0          |        |                    | 1×1" Ts        |                              | †5.0                          |                                  | Foot-brie        | B. Sept. 21, 1905             | 55      |
| 72.0  | 7.2   | 1.2                 | 2.7           |        |                    | 2½"×½"         |                              | †46.0                         |                                  | H.W.             | N. Feb. 13, 1904              | 56      |
| 74.0  | 9.5   | 1.3                 | 1.8           | 3C     |                    |                | 1.3                          | †70.0                         | 8.0                              | H.W.             | B. April 11, 1901             | 57      |
| 74.0  | 9.5   | 1.3                 | 1.8           | 3C     |                    |                | 1.3                          | †60.0                         | 8.0                              | H.W.             | B. April 11, 1901             | 58      |
| 75.0  | 14.0  |                     |               | 3C     |                    |                |                              | †39.0                         |                                  | H.W.             | B. Mar. 22, 1906, p. 321      | 59      |
| 75.0  | 18.0  | 1.5                 | 3.3           | Par    |                    | 1½"×1½" 1"×3"  |                              | †32.0                         |                                  | H.W.             | B. Mar. 15, 1906              | 60      |
| 75.0  | 7.5   | 1.3                 | 4.5           |        |                    |                | 1.17                         | 49.3                          |                                  | H.W.             | N. Dec. 2, 1905               | 61      |
| 75.0  | 14.7  | 1.3                 | †1.9          | 5C     | 97.5               |                | .60                          | 20.0                          |                                  | H.W.             | X.                            | 62      |
| 75.0  | †11.0 | 1.5                 |               |        | 62.5               | ½" J           | 1.12                         | 19.5                          | 5.0                              | H.W.             | N. Feb. 24, 1906              | 63      |
| 77.4  | 7.7   |                     |               |        |                    |                |                              | †39.4                         |                                  | H.W.             | G. 1st Tri., 1904             | 64      |
| 80.0  | 15.0  | 1.5                 |               |        |                    |                | .90                          | 27.0                          |                                  | H.W.             | B. Aug. 14, 1902              | 65      |
| 80.7  | 7.9   | 1.0                 | 9.0           |        |                    | O              |                              | 45.9                          | 4.9                              | H.W. & Ry.       | G. 1st Tri., 1904             | 66      |
|       |       |                     |               |        |                    |                |                              |                               |                                  |                  | G. 3d Tri., 1903, p. 47       |         |
| 80.3  | 8.2   | 1.0                 | 9.0           |        |                    |                |                              |                               |                                  |                  |                               |         |
| 79.6  | 7.9   | 1.0                 | 9.0           |        |                    |                |                              |                               |                                  | H.W.             | G. 4th Tri., 1905, p. 295     | 67      |
| 82.0  | 1.6   | 2.2                 |               |        |                    |                |                              |                               |                                  |                  |                               |         |
| 60.7  | 1.6   | 1.6                 |               |        |                    |                |                              |                               |                                  |                  |                               |         |
| 83.2  | 8.3   | 1.0                 | †9.0          |        |                    |                |                              | †37.6                         |                                  | H.W.             | G. 1st Tri., 1904             | 68      |
| 84.9  | 26.9  | 2.0                 | 3.3           |        |                    |                |                              | †32.0                         |                                  | H.W.             | N. Dec. 9, 1905, p. 645       | 69      |
| 30.2  | 13.1  |                     |               |        |                    |                |                              |                               |                                  |                  |                               |         |
| 85.3  | 5.9   | 1.5                 | 4.8           |        |                    |                |                              | †18.0                         |                                  | H.W.             | G. 1st Tri., 1904             | 70      |
| 85.3  | 8.5   | †3.5                | †10.0         |        |                    | 1½" J          | †.76                         | 53.1                          |                                  | H.W.             | B. Dec. 31, 1903              | 71      |
| 86.3  | 20.6  | 1.1                 | †1.5          |        |                    |                |                              |                               |                                  | H.W.             | G. 1st Tri., 1904             | 72      |
| 87.0  |       |                     |               |        |                    | 1½" T          |                              | 64.3                          | 9.0                              | H.W.             | B. Dec. 1, 1904, p. 489       | 73      |
| 87.0  | 8.0   | 1.6                 | 2.8           |        |                    |                | 1.16                         |                               |                                  |                  |                               |         |
| *88.0 | 11.0  | 1.5                 | 3.0           |        |                    |                | 1.22                         |                               |                                  |                  |                               |         |
| 89.0  | 9.5   | 1.3                 | 5.5           |        |                    |                | 1.37                         | 54.0                          | 8.8                              | H.W.             | B. May 19, 1904               | 74      |
| 88.5  | 9.5   | 1.3                 | 5.5           |        |                    |                | 1.37                         | 50.8                          | 8.5                              | H.W.             | X.                            | 75      |
| 95.0  | 11.1  |                     |               |        |                    | ½" O           |                              | †18.0                         |                                  |                  | B. March 16, 1899             | 76      |
| 95.2  | 2.3   | 3.3                 |               |        |                    |                |                              |                               |                                  |                  | B. May 11, 1905               | 77      |
| 100.0 | 27.7  | 1.8                 | 4.0           |        | 68.8               |                |                              |                               |                                  |                  | G. 4th Tri., 1905             | 78      |
|       | 23.9  | 1.9                 | 4.2           |        | 79.2               |                |                              |                               |                                  |                  | Cement, July, 1902            |         |
|       | 20.0  | 1.9                 | 4.3           |        | 93.6               |                |                              | 42.7                          | 10.0                             | H.W.             |                               |         |

† About. † T = Thacher bars. O = Round bars. J = Johnson bars.

bars. 61. Seventeen 9" 21-lb. I beams; 36" centre to centre. 62. Seven 9" 18-lb. I beams.  
 63. Spandrel wall tied to ring. 64. Hennebique type. 65. Four 3"×3"×6-lb. angles; 33" centre to centre. 66. Hennebique type. 67. Large arch has three "hinges." 68. Wülnch type.  
 69. Total length = 354'. 70. Melan type. 71. Skew. 72. Moniertype. 74. Four angles, 2½"×2½"×½"; 36" centre to centre. 75. Seventeen 10" 25-lb. I beams. 35½" centre to centre.  
 77. Five ribs; four angles latticed. 78. Melan type, four angles latticed.

TABLE II.—DATA FOR ABOUT 500 ARCH BRIDGES

REINFORCED CON

| Number. | Name.                | Place.                                    | Over.                    | Date.        | Engineer.                     | No. of Spans. |
|---------|----------------------|-------------------------------------------|--------------------------|--------------|-------------------------------|---------------|
| 70      | Icy Glen             | Stockbridge, Mass., U.S.A.                | Housatonic R.            | 1895         | M. A. C. Co.                  | 1             |
| 80      | François-Joseph      | Buda-Pesth (?), Austria-Hungary           | Danube R. ?              | 1900         |                               | 1             |
| 81      |                      | Laibach, Austria                          | Laibach R.               | 1900-1       | Melan                         | 1             |
| 82      | Green Island         | Niagara F'ls, N.Y., U.S.A.                | Niagara R.               | 1900-1       | Buck (Con.)<br>Waldo (State)  | 1             |
| 83      | Third Street         | Dayton, Ohio, U.S.A.                      | Great Miami R.           | 1904-        | Turner                        | 2             |
| 84      | Wayne St.            | Peru, Ind., U.S.A.                        | Wabash R.                | 1905         | Luten                         | 2             |
| 85      |                      | Portugal                                  |                          |              |                               | 2             |
| 86      | Lake Park            | Milwaukee, Wis., U.S.A.                   | Pena R.                  | 1901         |                               | 2             |
| 87      |                      | Yellowstone Nat. Park, U.S.A.             | Ravine<br>Yellowstone R. | 1905<br>1903 | Newton Eng. Co.<br>Chittenden | 5             |
| 88      | Jacaquas R.          | Military Road, San Juan-Ponce, Porto Rico | Jacaquas R.              | 1900-1       | Jackson                       | 1             |
| 89      | Washington Ave., So. | Lansing, Mich., U.S.A.                    | Grand R.                 | 1902         | Collar                        | 2             |
| 90      | Y-Bridge             | Zanesville, Ohio, U.S.A.                  | Muskingum R.             | 1900-2       | Landor                        | 1             |
|         |                      |                                           |                          |              |                               | 4             |
|         |                      |                                           |                          |              |                               | 3             |
| 91      |                      | Route Wildegg, Switz                      |                          | 1890         |                               | 1             |
| 92      | Kansas Ave.          | Topeka, Kan., U.S.A.                      | Kansas R.                | 1896-98      | M. A. C. Co.                  | 1             |
|         |                      |                                           |                          |              |                               | 1             |
| 93      | Park Ave.            | Newark, N. J., U.S.A.                     | Park                     | 1905         | Reynolds                      | 2             |
| 94      | Schwimmschulbrücke   | Steyr                                     | Stream                   | 1897†        |                               | 2             |
| 95      |                      | Playa-del-Rey, Cal., U.S.A.               |                          | 1906         | De Palo                       | 1             |
| 96      |                      | Route Waidhofen, Austria                  |                          |              |                               | 1             |
| 97      | St. Pierre Hollow    | Schenley Park, Pittsburgh, Penn., U.S.A.  | St. Pierre Hollow        | Proposed     | Keepers and Thacher           | 1             |
| 98      |                      | Chatellerault, France                     | Vienne R.                | 1900         |                               | 1             |
| 99      |                      | Route Bormida, Italy                      |                          | 1902         |                               | 2             |
| 100     |                      | Decize, France                            | Loire R.                 |              |                               | 1             |
| 101     | Gruenwald            | Munich, Bavaria                           | Isar R.                  | 1904         | Mörsch                        | 2             |

\* Maximum.

REMARKS.—70. Four 7" 15-lb. I beams; 28" centre to centre. 80. Three "hinges." Lattice ribs. 81. Three "hinges." Fourteen lattice ribs. 83. Four angles, 2½" × 2½" × ⅜". 34" centre to centre. 85. Hennebique type. 87. Four 2½" × 3" × ½" angles; 30" centre to centre. 88. Thacher type. 89. Melan type. 90. Thacher type. In plan, Y-shaped.



## ARRANGED ACCORDING TO SPAN—(Continued).

## CRETE ARCHES.

| Span. | Rise. | Thickness at Crown. | At Springing. | Curve. | Radius R at Crown. | Kind of Steel. | Per Cent Steel at the Crown. | Width, Face to Face at Crown. | Thickness of Piers at Springing. | Class of Bridge. | Reference.                              | Number. |
|-------|-------|---------------------|---------------|--------|--------------------|----------------|------------------------------|-------------------------------|----------------------------------|------------------|-----------------------------------------|---------|
| 100.0 | 10.0  | 0.8                 | 2.5           |        |                    |                | 2.30                         | 7.5                           |                                  | Foot-bridge      | X. B. Nov. 7, '95                       | 79      |
| 108.2 | 14.4  | 1.6                 | †2.2          |        |                    |                |                              | †45.9                         |                                  | H.W.             | G. 1st Tri., 1904                       | 80      |
| 108.3 | 14.6  | 1.7                 |               |        | 123.0              | 6"×4"          |                              | †50.0                         |                                  | H.W.             | B. July 16, 1903                        | 81      |
| 110.0 | 11.5  | 3.2                 | 5.9           |        |                    |                | .63                          | 40.0                          | 13.5                             | H.W.             | B. Dec. 6, 1900                         | 82      |
|       |       |                     |               |        |                    |                |                              |                               |                                  |                  | N. Feb. 16, 1901, p. 146                |         |
| 103.5 | 10.0  | 3.3                 | 6.3           |        |                    |                | .66                          | †62.0                         |                                  |                  |                                         |         |
| 110.0 | 14.3  |                     |               |        |                    |                | .69                          |                               |                                  | H.W.             | T. Mar. 4, 1904, p. 154                 | 83      |
| 100.0 | 13.3  | 2.1                 |               |        |                    |                | .69                          |                               | 11.0                             |                  |                                         |         |
| 90.0  | 11.3  | 2.1                 |               |        |                    |                | .91                          |                               | 10.0                             |                  |                                         |         |
| 80.0  | 9.7   | 1.6                 |               |        |                    |                |                              |                               | 9.0                              | H.W.             | B. Mar. 29, 1906, p. 347                | 84      |
| 100   |       |                     |               |        |                    | ‡ O ‡          |                              | 32?                           |                                  |                  |                                         |         |
| 95    |       |                     |               |        |                    |                |                              |                               |                                  |                  |                                         |         |
| 85    |       |                     |               |        |                    |                |                              |                               |                                  |                  |                                         |         |
| 75    |       |                     |               |        |                    |                |                              |                               |                                  |                  |                                         |         |
| 114.8 | 14.4  |                     |               |        |                    | "×3"K          |                              | 11.8                          |                                  | Tramway          | G. 1st Tri., 1904                       | 85      |
| 118.0 | 18.0  | 5.0                 | 5.0           |        |                    |                |                              | 14.0                          |                                  | H.W.             | N. Nov. 25, 1905                        | 86      |
| 120.0 | 15.0  | 2.0                 | 4.0           |        |                    |                | †1.39                        | 17.5                          |                                  | H.W.             | B. Jan. 14, 1904, p. 25                 | 87      |
| 120.0 | 12.0  | 2.3                 |               | 3C     | 226.0              | 4"×4"          | .63                          | 20.0                          | 12.0                             | H.W.             | N. Aug. 3, 1901                         | 88      |
|       |       |                     |               |        |                    |                | .80                          |                               |                                  |                  | B. Aug. 1, 1901, p. 66                  |         |
| 100.0 | 11.3  | 1.9                 |               |        | 167.2              | 3" & 5" × 4"   |                              | 54.0                          |                                  | H.W.             | Cement, Mar. 1902                       | 89      |
| 120.0 | 14.5  | 2.5                 |               |        |                    |                |                              | 43.0                          |                                  | H.W.             | N. Mar. 1, 1902, p. 194                 | 90      |
| 122.0 | 11.5  |                     |               |        |                    |                |                              |                               |                                  | El. Ry.          |                                         |         |
| 81.0  | 14.5  | 1.5                 |               |        |                    |                |                              |                               |                                  |                  |                                         |         |
|       | 10.9  |                     |               |        |                    |                |                              |                               |                                  |                  |                                         |         |
| 99.0  | 6.3   |                     |               |        |                    |                |                              | †12.8                         |                                  | H.W.             | G. 1st Tri., 1904                       | 91      |
| 122.0 | 11.4  | 0.6                 | †0.8          |        |                    |                | 1.58                         | 36.0                          |                                  | H.W.             | M. A. C. Co. B. April 2, 1896           | 92      |
| 125.0 | 18.9  | 1.8                 | 6.0           |        |                    |                |                              |                               |                                  |                  |                                         |         |
|       |       |                     |               |        |                    |                | 1.58                         |                               |                                  |                  |                                         |         |
| 110.0 | 16.3  | 1.8                 | 6.0           |        |                    |                | 1.73                         | †72.0                         |                                  | H.W.             | N. Aug. 12, 1905                        | 93      |
| 97.5  | 14.6  | 1.7                 | 5.0           |        |                    |                |                              | 19.7                          |                                  | H.W.             | Z. Oe. Ing. u. Arch. Ver., Dec. 23, '98 | 94      |
| 132.0 | 16.2  |                     |               |        |                    |                |                              |                               |                                  |                  | N. Mar. 31, 1906                        | 95      |
| 138.4 | 9.4   | 2.0                 | 2.3           |        |                    | 4 angles       |                              |                               |                                  | H.W.             | G. 1st Tri., 1904                       | 96      |
|       |       |                     |               |        |                    |                | .94                          | 84.0                          |                                  | H.W.             | K. & T. Blues                           | 97      |
| 146.0 | 18.0  | 2.0                 |               |        |                    |                |                              |                               |                                  |                  |                                         |         |
| 144.3 |       |                     |               |        |                    |                |                              |                               |                                  | H.W.             |                                         | 98      |
| 150.0 | 29.8  | 2.0                 | 8.0           |        |                    |                |                              |                               |                                  |                  |                                         |         |
|       |       |                     |               |        |                    |                |                              | †19.0                         |                                  |                  |                                         |         |
| 164.0 | 15.8  | 1.8                 | †3.0          |        |                    |                |                              |                               |                                  | H.W.             | B. April 10, 1902                       | 99      |
| 131.2 | 13.2  | 1.4                 | †2.6          |        |                    |                |                              |                               |                                  | H.W.             | G. 4th Tri., 1905                       | 100     |
| 167.3 | 16.7  | 1.9                 |               |        |                    |                |                              | 34.4                          |                                  | H.W.             | B. Feb. 20, 1905                        | 101     |
| 183.7 | 15.3  | 1.6                 |               |        |                    | 1.1" O         | .19                          | 26.3                          |                                  |                  |                                         |         |
| 230.0 | 42.0  | 2.5                 | 2.5           |        |                    |                |                              |                               |                                  |                  |                                         |         |

† About.

‡ O = Round bars.

K = Khan bars.

91. Monier type. 92. Four angles, 3"×3"×7.2 lbs. Twelve ribs. 93. Four angles 3"×3"×4"; 36" centre to centre. Thacher bars. 94. Melan type. 95. Foot-bridge type. 96. Monier type. 97. Twenty-seven ribs. Four angles, 3"×3"×7.2 lbs. 98. Hennebique type. 101. Three steel "hinges."



## SUPPLEMENTARY TABLE OF REINFORCED CONCRETE ARCHES.

| No. | Name and Location.                             | Approx.<br>Date. | Max.<br>Span.      | Rise. | <i>t</i> <sub>0</sub> | Reference.        |
|-----|------------------------------------------------|------------------|--------------------|-------|-----------------------|-------------------|
| 1   | Kinman Road, Cleveland, Ohio....H              | 1910-            | 100                | ..... | 5.0                   | N Sept. 30, 1911  |
| 2   | Husbands Forge, Hartford, Conn....H            | 1911-            | 100                | 11.0  | ....                  | N Dec. 30, 1911   |
| 3   | Broad St., Bethlehem, Pa.....H                 | 1909-            | 100                | ..... | ....                  | N Apr. 23, 1910   |
| 4   | Near Nashville, Tenn.....H                     | 1910             | 100                | ..... | ....                  | N Nov. 5, 1910    |
| 5   | Mulberry St., Harrisburg, Pa....H & E          | 1907-9           | 100                | 14.0  | 3.0                   | { N Aug. 15, 1908 |
| 6   | Cameron St., Harrisburg, Pa....H & E           | 1907-            | 100                | 14.0  | 3.0                   | { Apr. 3, 1909    |
| 7   | Ross Drive, Washington, D.C.....H              | 1908             | 100 <sup>3</sup>   | 15.0  | 2.5                   | B May 21, 1908    |
| 8   | Bridge No. 113, near Marshall, Ill....R        | 1906             | 100                | 40.0  | 4.0                   | Blue Prints       |
| 9   | Highland Boulevard, Milwaukee Wis.H            | 1909             | 100                | 21.7  | 1.8                   | N June 12, 1909   |
| 10  | Gilbert Ave., Cincinnati, Ohio....H & E        | 1912             | 100                | ..... | ....                  | N March 1, 1913   |
| 11  | Ansonia, Conn....H & E                         | 1913             | 101                | 19.0  | 2.3                   | N Feb. 22, 1913   |
| 12  | Cumberland River, Ky. & Tenn. Ry..R            | 1906-7           | 102 1              | 18.0  | 3.6                   | T Mch. 22, 1907   |
| 13  | Red Bridge, S. W. Huntington, Ind..H           | 1907             | 105                | 14.0  | 1.8                   | Nat. Br. Co.      |
| 14  | Pelham, Pelham Bay Park, N. Y....H             | 1908             | 105                | 16.5  | 2.0                   | N Oct. 31, 1908   |
| 15  | Factory St., Canal Dover, Ohio.H & E           | 1905-6           | 106. 7             | 11.8  | 2.0                   | N Feb. 9, 1907    |
| 16  | San Luis Rey, Oceanside, Cal.....H             | 1910             | 107 <sup>3</sup>   | 19.0  | 2.0                   | B Apr. 13, 1911   |
| 17  | Wagaraw Br., Paterson, N.J.....H               | 1907             | 108                | 12.0  | 2.3                   | N Mch. 7, 1908    |
| 18  | Cedar St., Mishawaka, Ind.....H                | 1903             | 110                | 14.0  | ....                  | N July 7, 1906    |
| 19  | Indianapolis, Ind.....H                        | 1904-5           | 110                | 13.3  | 2.0                   | Blue Prints       |
| 20  | Jefferson St., So. Bend, Ind.....H             | 1905             | 110                | 14.3  | 2.3                   | N July 28, 1906   |
| 21  | Penn St., Reading, Pa....H & E                 | 1912-            | 110                | 24.0  | 3.5                   | N Nov. 30, 1912   |
| 22  | Avranches, France.....R                        | 1908-            | 110. 2             | 23.5  | 1.2                   | B Apr. 22, 1909   |
| 23  | Austin, Texas.....H                            | 1910             | 115                | 18.0  | ....                  | B June 23, 1910   |
| 24  | Paulins Kill, near Hainsburg, N. J....R        | 1908-11          | 120                | 60.0  | 6.0                   | { N Aug. 15, 1908 |
|     |                                                |                  |                    |       |                       | { Apr. 24, 1909   |
|     |                                                |                  |                    |       |                       | { July 16, 1910   |
| 25  | Yellowstone Nat. Park.....H                    | 1903             | 120                | 15.0  | 2.0                   | B Jan. 14, 1904   |
| 26  | Bridge St., Peoria Ill.....H                   | 1907-            | 125                | ..... | 2.5                   | B May 13, 1909    |
| 27  | St. Paul, Minn.....H                           | 1908-9           | 125                | ..... | 3.0                   | N Apr. 3, 1909    |
| 28  | Tempe, Arizona.....H                           | 1911-12          | 125 <sup>3</sup>   | ..... | 3.0                   | B March 28, 1912  |
| 29  | Charles River, Boston, Mass....H & E           | 1907-11          | 128 <sup>2</sup>   | 19.3  | 4.5                   | B July 13, 1911   |
| 30  | Columbus, Georgia, H                           | 1911-12          | 128                | 16.5  | 2.5                   | N July 6, 1912    |
| 31  | Rye Outlet, 3 m. N. White Plains<br>N. Y.....H | 1911             | 128.6              | 24.4  | 2.8                   | N Oct. 14, 1911   |
| 32  | Oakland Ave., Piedmont, Cal....H & E           | 1911             | 130                | 18.8  | 2.3                   | B Sept. 14, 1911  |
| 33  | Amélie-les-Bains, France....H                  | .....            | 134.5 <sup>3</sup> | 15.4  | ....                  | V Aug. 27, 1910   |
| 34  | Bannock St., Denver, Col.....H                 | 1907             | 138 <sup>2</sup>   | 13.0  | 2.0                   | N Mch. 21, 1908   |
| 35  | Edmonson Ave., Baltimore, Md.H & E             | 1909             | 139                | 43.8  | 3.8                   | { N June 19, 1909 |
|     |                                                |                  |                    |       |                       | { N Feb. 9, 1907  |
| 36  | Grand Ave., Milwaukee, Wis....H & E            | 1910             | 145                | 28.0  | 2.2                   | { Mch. 11, 1911   |
| 37  | French Broad River, Asheville, N. C.<br>H & E  | 1909-10          | 145                | 42.0  | 4.0                   | N May 6, 1911     |
| 38  | Playa del Rey, Cal.....H                       | 1906             | 146                | 18.0  | 2.0                   | N Mar. 31, 1906   |
| 39  | St. Jean La Rivière Viaduct.....E              | 1908             | 149.2              | 40.6  | 1.8                   | N Dec. 3, 1910    |
| 40  | Grand River, Painsville, Ohio....R             | 1908             | 160                | 58.3  | 7.3                   | N May 1, 1909     |
| 41  | Catawba River, nr. Charlotte, N. C.H           | .....            | 161 C-C            | ..... | ....                  | N Mch. 8, 1913    |
| 42  | Tavanasa, Switz.....H                          | 1908             | 167.3 <sup>3</sup> | 18.0  | 0.6                   | B Feb. 18, 1909   |
| 43  | Trégnier, France.....R                         | .....            | 167.3 <sup>3</sup> | 21.3  | 3.5                   | G Pt. IV, 1907    |
| 44  | Pyrimont, France.....H                         | 1907             | 169                | 25.0  | 2.0                   | G Pt. V, 1907     |
| 45  | Kings Highway Viad., St. Louis, Mo.H           | 1912             | 170                | 17.0  | ....                  | N Nov. 23, 1912   |
| 46  | Guindy River, N. E. Coast, France.R            | 1907             | 177 <sup>3</sup>   | 21.3  | 4.9                   | B Mch. 26, 1908   |
| 47  | Humbolt Co., Cal.....H                         | 1911-            | 180                | 26.3  | ....                  | B Mch. 23, 1911   |
| 48  | Wiesen, Switz.....R                            | 1908-9           | 180.4              | 109.4 | 5.9                   | B Sept. 16, 1909  |
| 49  | Berlin, Germany....H & R                       | 1910             | 183.7              | ..... | ....                  | N Aug. 13, 1910   |
| 50  | Meadow St., Pittsburgh, Pa.....H               | 1910             | 209                | 46.1  | 5.0                   | B Dec. 1, 1910    |
| 51  | Colorado St., Los Angeles, Cal....H            | 1912             | 223                | ..... | ....                  | N May 24, 1913    |
| 52  | Sitter Arch, Switzerland.....H                 | 1908             | 259                | 87.0  | 3.9                   | V Mch. 13, 1909   |
| 53  | Valserine River, Montanges, France.R           | 1909-            | 262                | 66.0  | 5.0                   | N Oct. 21, 1911   |
| 54  | 42d St., Philadelphia, Pa.....H                | 1909             | 262                | 53.0  | 3.0                   | B May 20, 1909    |
| 55  | Bellegarde-Chizery, France.....R               | 1910             | 262.8              | ..... | 4.9                   | N June 4, 1911    |
| 56  | Larimer Ave., Pittsburgh, Pa....H & E          | 1911-            | 300                | 67.0  | 6.5                   | N Dec. 9, 1910    |
| 57  | Grafton Br. Aukland, N. Z.....H                | 1907-            | 320 <sup>3</sup>   | 84.   | 5.5                   | B Aug. 4, 1910    |
| 58  | Risorgimento, Rome, Italy.....H                | 1910-            | 328                | 32.8  | 0.7                   | B Nov. 30, 1911   |
| 59  | Hudson Memorial Br., 220th St. N. Y.           | Proposed         | 703                | 177.0 | 15.0                  | N Nov. 16, 1907   |

\* = Three hinges. 2 = Two hinges. H = Highway. E = Electric railway. R = Steam railway.

## SUPPLEMENTARY TABLE OF PLAIN CONCRETE ARCHES.

| No. | Name and Location.                     | Approx. Date. | Max. Span.         | Rise. | $t_0$ | Reference.      |
|-----|----------------------------------------|---------------|--------------------|-------|-------|-----------------|
| 1   | Nashua to Hudson, N. H. . . . .        | H 1910        | 100                | 32.0  | 5.0   | N Feb. 25, 1911 |
| 2   | Scotland, Pa. . . . .                  | R 1909        | 100                | 32.0  | 5.0   | B Apr. 8, 1909  |
| 3   | Latah Creek, Spokane, Wash. . . H & E  | 1911-         | 150                | 40.0  | 6.0   | N Mch. 22, 1913 |
| 4   | Delaware River, Portland, Pa. . . . R  | 1910          | 150                | 40.0  | 6.0   | B Dec. 30, 1909 |
| 5   | Near Auggersbach, Switz. . . . .       | H 1906        | 169                | 28.0  | 3.7   | B Apr. 30, 1908 |
| 6   | Tunkhannock Viad., Nicholson, Pa. R    | 1912          | 180                | 8.0   | 8.0   | N May 13, 1913  |
| 7   | Wiesen, Switz. . . . .                 | R 1908-9      | 180.4              | 32.2  | 5.0   | N Nov. 21, 1908 |
| 8   | Lautrach, Bavaria . . . . .            | R 1906        | 187.5 <sup>3</sup> | 18.1  | 4.4   | B May 2, 1907   |
| 9   | Mannheim, Germany. . . . .             | H 1906-8      | 192.3 <sup>3</sup> | 70.3  | 9.5   | B June 18, 1908 |
| 10  | Near Kempten, S. Germany. . . . .      | R 1906        | 211.5 <sup>3</sup> | 86.7  | 4.4   | N Feb. 23, 1907 |
| 11  | Walnut Lane, Philadelphia, Pa. . . . H | 1906-8        | 233.0              | 70.3  | 9.5   | B Aug. 15, 1907 |
| 12  | Teufen to Stein, Switz. . . . .        | H 1908        | 259.3              | 80.8  | 6.0   | B Dec. 3, 1908  |
| 13  | Rocky River, Cleveland, Ohio . H & E   | 1909-         | 280                | 115.0 | 6.8   | N Jan. 23, 1909 |
| 14  | Monroe St., Spokane, Wash. . . H & E   | 1909-         | 281                | 115.0 | 6.8   | B May 4, 1911   |

<sup>3</sup> = Three hinges; H = Highway; E = Electric railway; R = Steam railway

## KEY TO REFERENCES.

| Symbol. | Name of Periodical or Book, etc.                             | Address.                              |
|---------|--------------------------------------------------------------|---------------------------------------|
| A       | Civil Engineers' and Architects' Journal . . .               | New York City, U. S. A.               |
| B       | Engineering News and Am. Ry. Journal. . .                    |                                       |
| C       | Weale's Bridges. . . . .                                     |                                       |
| D       | Penn. Ry. Co.'s Blues. . . . .                               | Philadelphia, Pa., U. S. A.           |
| E       | Wm. H. Brown, Ch. Eng'r Penn. Ry. Co. . .                    | Philadelphia, Pa., U. S. A.           |
| F       | Construction des Viaducs, Tony Fontenay. .                   | Paris, France.                        |
| G       | Annales des Ponts et Chausses. . . . .                       | Paris, France.                        |
| H       | Mahan's Civil Engineering. . . . .                           | John Wiley & Sons, New York, U. S. A. |
| I       | Masonry Construction by Baker. . . . .                       | John Wiley & Sons, New York, U. S. A. |
| J       | Spon's Dictionary of Engineering. . . . .                    | London, England.                      |
| K       | Engineering. . . . .                                         |                                       |
| L       | Edinburgh Encyclopædia, 9th ed. . . . .                      |                                       |
| M       | Scientific American Supplement. . . . .                      | New York City, U. S. A.               |
| N       | Engineering Record. . . . .                                  | New York City, U. S. A.               |
| O       | Engineering Magazine. . . . .                                | New York City, U. S. A.               |
| P       | Journal of the Association of Engineering Societies. . . . . | New York City, U. S. A.               |
| Q       | Encyclopædia Britannica, 9th ed. . . . .                     |                                       |
| R       | Railway and Engineering Journal. . . . .                     |                                       |
| S       | Cresy' Bridges. . . . .                                      | New York City, U. S. A.               |
| T       | Railroad Gazette. . . . .                                    |                                       |
| U       | Murray's Handbook of Northern Italy. . . .                   |                                       |
| V       | Le Genie Civil. . . . .                                      | Paris, France.                        |
| W       | Messrs. Keepers & Thacher . . . . .                          | Paterson, N. J., U. S. A.             |
| X       | The Melan Arch Construction Co. . . . .                      | New York City, U. S. A.               |
| Y       | Transactions of Am. Soc. C. E. . . . .                       | New York City, U. S. A.               |
| Z       | Concrete-steel Engineering Co. . . . .                       | New York City, U. S. A.               |



## APPENDIX B.

ARCH COEFFICIENTS. TRANSFORMATION OF DIMENSIONS IN SYMMETRICAL RIB ARCHES; WITH TABLES OF COEFFICIENTS FOR TWENTY ARCHES OF DIFFERENT PROPORTIONS.\*

Assume that for a certain arch ring, of span  $l$  and rise  $f$  and of shape symmetrical about the center line of span, the values of  $H$ ,  $M_1$ ,  $M_2$ ,  $V_1$ , and  $V_2$  have been computed for unit loads, as explained in Chapter II. The loads are ver-

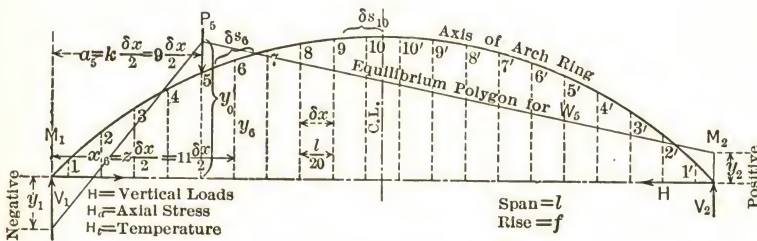


FIG. 1.

tical and, for convenience, are applied at the numbered points, Fig. 1.

If it is found necessary to change the thickness of the ring, the rise or the span, the new values of the horizontal thrust and bending moments can be readily found without making a new set of computations as explained below.

\*See Engineering News. June 2, 1910.



## HORIZONTAL THRUST.

The horizontal thrust is given by the following formula;  
Art. 9:

$$H = \frac{\Sigma m_x \Delta \left( y - \frac{\Sigma y \Delta}{\Sigma \Delta} \right)}{\Sigma y \Delta \left( y - \frac{\Sigma y \Delta}{\Sigma \Delta} \right)} = \frac{\Sigma m_x \Delta (y - K)}{\Sigma y \Delta (y - K)}, \quad \cdot \cdot \quad (1)$$

where  $\Sigma$  signifies a sum taken over the entire span; and

$m_x$  = the moment produced by the loading considering the arch rib as a simple beam resting on two supports.

$$\Delta_1, \Delta_2, \text{ etc.}, = \frac{\delta s_1}{I_1}, \frac{\delta s_2}{I_2}, \text{ etc.}$$

$\delta s_1, \delta s_2, \text{ etc.},$  = portions of arch axis as shown in Fig. 1.

$I_1, I_2, \text{ etc.},$  = the moments of inertia of the radial section of the arch ring at points 1, 2, etc.

$x, y$  = coordinates of any point on the arch,

and  $K = \frac{\Sigma y \Delta}{\Sigma \Delta}$  = constant for a given arch.

For the present purpose let the span always be divided into twenty equal divisions, i.e.,  $\delta x = l \div 20$ .

If both numerator and denominator of a fraction are divided by the same quantity, the value of the fraction remains unchanged. In eq. (1), let both terms of  $K$  be divided by  $\Delta_{10}$ , the value of  $\Delta$  in the section adjacent to the crown; and let both terms of the entire expression for  $H$  be divided by  $f \Delta_{10}$ , where  $f$  is the rise of the arch. Then,

$$\begin{aligned}
 H &= \frac{\Sigma m_x \frac{\Delta}{\Delta_{10}} \left( \frac{y}{f} - \frac{\Sigma \frac{y}{f} \frac{\Delta}{\Delta_{10}}}{\Sigma \frac{\Delta}{\Delta_{10}}} \right)}{f \Sigma \frac{y}{f} \frac{\Delta}{\Delta_{10}} \left( \frac{y}{f} - \frac{\Sigma \frac{y}{f} \frac{\Delta}{\Delta_{10}}}{\Sigma \frac{\Delta}{\Delta_{10}}} \right)} = \frac{1}{f} \frac{\Sigma m_x \Delta' \left( y' - \frac{\Sigma y' \Delta'}{\Sigma \Delta'} \right)}{\Sigma y' \Delta' \left( y' - \frac{\Sigma y' \Delta'}{\Sigma \Delta'} \right)} \\
 &= \frac{1}{f} \frac{\Sigma m_x \Delta' \left( y' - \frac{K}{f} \right)}{\Sigma y' \Delta' \left( y' - \frac{K}{f} \right)} = \frac{1}{f} \frac{\Sigma m_x \Delta' (y' - K')}{\Sigma y' \Delta' (y' - K')} \quad \dots \quad (2)
 \end{aligned}$$

Suppose the arch to be loaded with a single load  $P$  distant  $a = k \frac{\delta x}{2}$  from the left support; then

$$m_x = P \left[ z \frac{40 - k}{40} - (z - k, \text{ when } z \text{ exceeds } k) \right] \frac{\delta x}{2} = m_x' \frac{\delta x}{2},$$

in which  $m_x'$  is independent of the span and rise of the arch axis. Substituting this value in eq. (2) we get

$$H = \frac{\Sigma m_x' \Delta' (y' - K')}{\Sigma y' \Delta' (y' - K')} \frac{1}{f} \frac{\delta x}{2} = H' \frac{\delta x}{2f} \quad \dots \quad (3)$$

The coefficient  $H'$  is the horizontal thrust for an arch ring having a *span of 40*, a *rise of 1*, and values of  $y'$  and  $\Delta'$  which vary in the same manner as the values of  $y$  and  $\Delta$  for the original or base arch ring. Knowing the value of  $H$ , the value of the coefficient  $H'$  becomes

$$H' = \frac{2f}{\delta x} H = \frac{40f}{l} H \text{ [for twenty divisions]} \quad \dots \quad (4)$$

By the use of  $H'$  the value of the thrust  $H''$  for an arch ring with a span of  $l''$  and a rise of  $f''$  is readily found, for

$$H'' = \frac{\delta x''}{2f''} H' = \frac{l''}{4of''} H' \quad . \quad . \quad . \quad (5)$$

The dimensions of the new arch ring and the shape of its axis are readily found. The values of  $y'$  were obtained by dividing each value of  $y$  of the original or base arch axis by its rise  $f$ ; and the values of  $\Delta'$  are the quotients of each original value of  $\Delta$  divided by  $\Delta_{10}$ , the original value of  $\Delta$  at point No. 10. Therefore in the new arch, for a span of  $l''$  and a rise of  $f''$ ,

$$y_1'' = f'' y_1', \quad y_2'' = f'' y_2', \quad y_3'' = f'' y_3', \text{ etc.} \quad . \quad . \quad (6)$$

$$\Delta_1'' = \Delta_{10}'' \Delta_1', \quad \Delta_2'' = \Delta_{10}'' \Delta_2', \quad \Delta_3'' = \Delta_{10}'' \Delta_3', \text{ etc.} \quad (7)$$

Knowing the span  $l''$  and the ordinates  $y''$  the arch axis may be drawn to scale and the values of  $\delta s_1''$ ,  $\delta s_2''$ ,  $\delta s_3''$  to  $\delta s_{10}''$  determined. The next step is to assume the depth of the ring and the reinforcement at point No. 10 and compute the corresponding moment of inertia  $I_{10}''$ , then  $\Delta_{10}'' = \delta s_{10}'' \div I_{10}''$  and the values of  $\Delta''$  at other points can be found by simple multiplication. Since  $\Delta'' = \delta s'' \div I''$ ;  $I_1'' = \delta s_1'' \div \Delta_1''$ ;  $I_2'' = \delta s_2'' \div \Delta_2''$ ;  $I_3'' = \delta s_3'' \div \Delta_3''$ , etc., thus determining the moments of inertia at each point. From the values of the moments of inertia the corresponding depths of ring are found.

The curves forming the upper and lower limits of the ring will not be regular curves as a rule, but usually they can be very closely approximated by circular arcs.

If the axis of the original arch axis was an arc of a circle the new axis will be a portion of an ellipse.

## MOMENTS AT THE SUPPORTS.

The formula used in routine calculations when the span is divided into twenty equal parts is, for unit loads, Art. 22.

$$\left. \begin{matrix} M_1 \\ M_2 \end{matrix} \right\} = H \frac{\Sigma y \Delta}{\Sigma \Delta} - \left[ \sum_{z=0}^{z=k} z \Delta + k \sum_{z=k}^{z=19} \Delta \right] \frac{\delta x}{2 \Sigma \Delta}$$

$$\mp \left[ (20-k) \sum_{z=0}^{z=k} 20(z-20) \Delta - k \sum_{z=k}^{z=19} (z-20)^2 \Delta \right]$$

$$\left[ \frac{\delta x}{-4 \sum_0^{l/2} (z-20)^2 \Delta} \right] \dots \dots \dots (8)$$

If  $H \frac{\Sigma y \Delta}{\Sigma \Delta}$  is replaced by

$$H' \frac{\delta x}{2f} \cdot \frac{\Sigma y' \Delta'}{\Sigma \Delta'} f = H' \frac{\Sigma y' \Delta'}{\Sigma \Delta'} \frac{\delta x'}{2},$$

and the remainder of the expression divided (numerators and denominators) by  $\Delta_{10}$ , the value of the expression will not be changed, but it will become apparent that the whole expression is independent of span, rise, and thickness of ring. It may be written

$$\left. \begin{matrix} M_1 \\ M_2 \end{matrix} \right\} = \left\{ \begin{matrix} M_1' \\ M_2' \end{matrix} \right\} \frac{\delta x}{2}, \dots \dots \dots (9)$$

where  $M_1'$  and  $M_2'$  are the moments at the supports for a span of 40 and a rise of unity and a ring having values of  $y'$  and  $\Delta'$  as explained above. The values of  $M_1'$  and  $M_2'$  are quickly found by dividing the values of  $M_1$  and  $M_2$  found for the original or base arch ring by  $\frac{\delta x}{2} = \frac{l}{40}$ .



For an arch ring having a span of  $l''$  and a rise of  $f''$

$$M_1'' = M_1' \frac{\delta x''}{2} = M_1' \frac{l''}{40},$$

and

$$M_2'' = M_2' \frac{\delta x''}{2} = M_2' \frac{l''}{40}.$$

Note that the change in rise has no influence upon the values of the moments when the coefficients are employed. In general then

$$M_2 : M_2'' :: l : l'' \text{ and } M_1 : M_1'' :: l : l''. \quad . \quad . \quad (10)$$

#### VERTICAL REACTIONS.

The vertical reaction at the left support is

$$V_1 = \frac{M_2 - M_1}{l} + R_1, \quad . \quad . \quad . \quad (11)$$

where  $R_1$  is the reaction assuming the arch to be a beam simply supported at the ends. This latter is independent of the length of span; hence for a span  $l''$

$$V_1'' = \frac{M_2'' - M_1''}{l''} + R_1, \quad . \quad . \quad . \quad (12)$$

but

$$M_2'' = \frac{M_2 l''}{l} \quad \text{and} \quad M_1'' = \frac{M_1 l''}{l},$$

therefore

$$V_1'' = \frac{M_2 - M_1}{l} + R_1, \quad . \quad . \quad . \quad (13)$$

or, the value of the vertical reaction at the left support remains unchanged for changes in length of span. The rise of the arch axis does not appear and hence has no effect.

## ORDINATES LOCATING THE EQUILIBRIUM POLYGON.\*

The ordinates  $y_1$ ,  $y_0$ , and  $y_2$ , Fig. 1, which locate the equilibrium polygon, change with the rise only:

$$y_1 = \frac{M_1}{H} = \frac{M_1' \frac{\delta x}{2}}{H' \frac{\delta x}{2f}} = \frac{M_1'}{H'} f. \quad \dots \quad (14)$$

$$y_1'' = \frac{M_1''}{H''} = \frac{M_1' \frac{\delta x''}{2}}{H' \frac{\delta x''}{2f''}} = \frac{M_1'}{H'} f'' \quad \dots \quad (15)$$

$$\therefore y_1 : y_1'' :: f : f'' \quad \text{or} \quad y_1'' = y_1 \frac{f''}{f}. \quad \dots \quad (16)$$

In like manner

$$y_0'' = y_0 \frac{f''}{f} \quad \text{and} \quad y_2'' = y_2 \frac{f''}{f}. \quad \dots \quad (17)$$

## INTERMEDIATE MOMENTS.

The moment at any intermediate point of the arch, for a single load  $P$ , can be found from the general formula:

$$M_x = M_1 + V_1 x - H y - \left[ \begin{array}{l} \text{when } x > a \\ P(x-a) \end{array} \right] \quad \dots \quad (18)$$

or

$$M_x = \left\{ M_1' \frac{\delta x}{2} + V_1 z \frac{\delta x}{2} - H' y' \frac{\delta x}{2} - \left[ \begin{array}{l} \text{when } z > k \\ P(z-k) \end{array} \right] \frac{\delta x}{2} \right\}, \quad (19)$$

and for a unit load

$$M_x = \left\{ M_1' + V_1 z - H' y' - \left[ \begin{array}{l} \text{when } z > k \\ (z-k) \end{array} \right] \right\} \frac{\delta x}{2}. \quad \dots \quad (20)$$

---

$y_1$  and  $y_2$  in this paragraph are not ordinates at points 1 and 2, but ordinates to the equilibrium polygon at the left and right supports respectively.

Then for any other span  $l'' = 20\delta x''$ , the corresponding moment is

$$M_x'' = \left\{ M_1' + V_1 z - H' y' - \left[ \begin{array}{c} \text{when } z > k \\ (z - k) \end{array} \right] \right\} \frac{\delta x''}{2}. \quad (21)$$

Therefore

$$M_x : M_x'' :: \frac{\delta x}{2} : \frac{\delta x''}{2} :: l : l'',$$

and

$$M_x'' = M_x \frac{l''}{l}. \quad (22)$$

#### TEMPERATURE.

The horizontal thrust produced by a change in temperature is, Art. 9,

$$H_t = \frac{et^\circ El}{\Sigma y \Delta \left( y - \frac{\Sigma y \Delta}{\Sigma \Delta} \right)}, \quad (23)$$

where  $e$  = coefficient of expansion;

$t^\circ$  = number of degrees change;

$E$  = Young's modulus.

If the denominator of equation (23) be multiplied and divided by  $f^2 \Delta_{10}$ , the whole expression is not changed in value. It becomes

$$H_t = \frac{et^\circ El}{\Sigma y' \Delta' \left( y' - \frac{\Sigma y' \Delta'}{\Sigma \Delta'} \right) f^2 \Delta_{10}} = \frac{et^\circ El}{D' \Delta_{10} f^2}. \quad (24)$$

For a change of span to  $l''$  and change of rise to  $f''$ :

$$H_t'' = \frac{et^\circ El''}{D' \Delta_{10}'' f''^2}. \quad (25)$$

Then

$$H_t : H_t'' :: \frac{l}{D' \Delta_{10} f^2} : \frac{l''}{D' \Delta_{10}'' f''^2} \quad \cdot \cdot \cdot \quad (26)$$

or

$$H_t'' = H_t \frac{\Delta_{10}}{\Delta_{10}''} \cdot \frac{l''}{l} \left( \frac{f}{f''} \right)^2 \quad \cdot \cdot \cdot \quad (27)$$

The value of  $H_t''$  can be found either from equation (25), using the coefficient  $D'$ , already found in computing  $H$ , or from the proportion given in equation (27).

The moment at either support is, for the original arch:

$$M_{1t} = M_{2t} = H_t \frac{\Sigma y \Delta}{\Sigma \Delta} = H_t \frac{\Sigma y' \Delta'}{\Sigma \Delta'} f = H_t K' f,$$

and for the arch with span  $l''$  and rise  $f''$ :

$$M_{1t}'' = M_{2t}'' = H_t'' K' f'', \quad \cdot \cdot \cdot \quad (28)$$

or

$$M_{1t} : M_{1t}'' :: H_t K' f : H_t'' K' f'',$$

and

$$M_{1t}'' = \frac{H_t'' f''}{H_t f} M_{1t} \quad \cdot \cdot \cdot \quad (29)$$

The value of  $M_{1t}''$  can be found from (28) by using the coefficient  $K'$ , already found, or from equation (29).

#### THE AXIAL OR DIRECT STRESS.

The effect of the axial stress may be considered as the effect of a drop in temperature, and consequently it is necessary to consider its effect upon the horizontal thrust alone. (Art. 9.)

$$H_a = -H \left\{ 1 - \frac{D}{D + (\delta x)^2 \Sigma \frac{1}{F \delta s}} \right\} \quad \cdot \cdot \cdot \quad (30)$$



$$H_a = -H \frac{(\delta x)^2 \Sigma \frac{1}{F \delta s}}{D' \Delta_{10} f^2 + (\delta x)^2 \Sigma \frac{1}{F \delta s}}. \quad \dots \quad (31)$$

For a ring with a span  $l''$  and a rise  $f''$ ,

$$H_a'' = -H'' \frac{(\delta x'')^2 \Sigma \frac{1}{F'' \delta s''}}{D' \Delta_{10}'' f''^2 + (\delta x'')^2 \Sigma \frac{1}{F'' \delta s''}}. \quad \dots \quad (32)$$

In the above equations  $F$  is the effective area of the radial section of the ring at the point being considered.

#### SUMMARY.

Given an arch ring of a certain shape and certain rate of variation of the ring-thickness function  $\Delta (= \delta s \div I)$ , it is possible to change the span  $l$  and the rise  $f$  of the axis of this ring to a span  $l''$  and a rise  $f''$ , provided that the ordinates  $y''$  and the function  $\Delta''$  for the new ring vary according to the same laws as  $y$  and  $\Delta$  of the original ring. Then the values of  $H''$ ,  $M_1''$ ,  $M_2''$ , etc., may be found directly from the values of  $H$ ,  $M_1$ ,  $M_2$ , etc., of the first arch ring, by the help of the following expressions, in which the square brackets denote derived coefficients which are independent of span and rise.

Ordinates:

$$y'' = \frac{y}{f} f'' = [y'] f''. \quad \dots \quad (6)$$

Ring — Thickness function:

$$\Delta'' = \frac{\Delta}{\Delta_{10}} \Delta_{10}'' = [\Delta'] \Delta_{10}''. \quad \dots \quad (7)$$

Unit-load thrust:

$$H'' = \frac{\delta x''}{2f''} \cdot \frac{2f}{\delta x} H = \frac{\delta x''}{2f''} [H']. \quad (5)$$

Temperature thrust:

$$H_t'' = \frac{\Delta_{10}}{\Delta_{10}''} \cdot \frac{l''}{l} \left( \frac{f}{f''} \right)^2 H_t. \quad (27)$$

Temperature moment at left support:

$$M_{1t}'' = M_{2t}'' = \frac{H_t''}{H_t} \frac{f''}{f} M_{1t} = H_t'' f'' [K']. \quad (29)$$

Unit-load moment at left:

$$M_1'' = \frac{l''}{l} M_1 = \frac{\delta x''}{2} [M_1']. \quad . \quad . \quad . \quad . \quad . \quad (9)$$

At right support:

$$M_2'' = \frac{l''}{l} M_2 = \frac{\delta x''}{2} [M_2']. \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Reactions:

$$V_1''=[V_1] \text{ and } V_2''=[V_2]. \quad . \quad . \quad . \quad . \quad . \quad (I_3)$$

Ordinates of equilibrium polygon for single load:

$$y_1'' = y_1 \frac{f''}{f} \quad y_0'' = y_0 \frac{f''}{f} \quad y_2'' = y_2 \frac{f''}{f}. \quad (I6)-(I7)$$

Unit-load moment at any point  $x$ :

$$M_x'' = \frac{l''}{l} M_x. \quad (22)$$

Thrust due to axial stress:

$$H_a'' = -H'' \frac{(\delta x'')^2 \Sigma \frac{I}{F'' \delta S''}}{[D'] \Delta_{10}'' f''^2 + (\delta x'')^2 \Sigma \frac{I}{F'' \delta S''}}. \quad (32)$$

The formulas for the coefficients are:

$$[y'] = \frac{y}{f},$$

$$[\Delta'] = \frac{\Delta}{\Delta_{10}},$$

$$[K'] = \frac{\Sigma y' \Delta'}{\Sigma \Delta'} = \frac{1}{f} \frac{\Sigma y \Delta}{\Sigma \Delta} = \frac{K}{f},$$

$$[D'] = \Sigma y' \Delta' (y' - K') = \frac{D}{f^2 \Delta_{10}},$$

$$[H'] = H \frac{40f}{l} = \frac{\Sigma m_x' \Delta' (y' - K')}{D'},$$

in which  $m_x'$  is the simple-beam moment of a single load of unity for a span of 40, or

$$m_x' = m_x \frac{40}{l},$$

$$\left. \begin{matrix} [M_1'] \\ [M_2'] \end{matrix} \right\} = \left\{ \begin{matrix} M_1 \frac{40}{l} \\ M_2 \frac{40}{l} \end{matrix} \right\} = H' K' - \frac{1}{\Sigma \Delta'} \left( \sum_{z=0}^{z=k} z \Delta' + k \sum_{z=k}^{z=19} \Delta' \right) \\ \mp \frac{\left( (20-k) \sum_{z=0}^{z=k} 20(z-20) \Delta' - k \sum_{z=k}^{z=19} (z-20)^2 \Delta' \right)}{-2 \sum_{z=0}^{z=19} (z-20)^2 \Delta'}.$$

$$[V_1] = R_1 + \frac{M_2 - M_1}{l},$$

$$[V_2] = R_2 - \frac{M_2 - M_1}{l}.$$

## TABLES OF COEFFICIENTS OF TWENTY ARCHES

The appended tables contain the above coefficients for twenty different arch rings. The original or base arches had either segmental or false elliptic intrados. By the methods above given, these tables may be used for analyzing any similar arch. Of course the greatest utility of the coefficients is for comparatively small changes in span or rise or both.

The fields of loading which produce maximum moments at sections 0, 6 and the crown are designated below.

## POINTS TO BE LOADED FOR MAXIMUM MOMENTS

| Arch No. | Point 0.  |           | Point 6.  |           | Crown.      |           |
|----------|-----------|-----------|-----------|-----------|-------------|-----------|
|          | Negative. | Positive. | Negative. | Positive. | Negative.   | Positive. |
| 1        | 1-8       | 9-1'      | 8-1'      | 1-7       | 1-8 & 8'-1' | 9-9'      |
| 2        | 1-8       | 9-1'      | 8-1'      | 1-7       | 1-8 & 8'-1' | 9-9'      |
| 3        | 1-8       | 9-1'      | 10-1'     | 1-9       | 1-8 & 8'-1' | 9-9'      |
| 4        | 1-8       | 9-1'      | 8-1'      | 1-7       | 1-8 & 8'-1' | 9-9'      |
| 5        | 1-8       | 9-1'      | 9-1'      | 1-8       | 1-8 & 8'-1' | 9-9'      |
| 6        | 1-7       | 8-1'      | 8-1'      | 1-7       | 1-8 & 8'-1' | 9-9'      |
| 7        | 1-7       | 8-1'      | 8-1'      | 1-7       | 1-8 & 8'-1' | 9-9'      |
| 8        | 1-7       | 8-1'      | 8-1'      | 1-7       | 1-8 & 8'-1' | 9-9'      |
| 9        | 1-8       | 9-1'      | 9-1'      | 1-8       | 1-8 & 8'-1' | 9-9'      |
| 10       | 1-8       | 9-1'      | 8-1'      | 1-7       | 1-8 & 8'-1' | 9-9'      |
| 11       | 1-8       | 9-1'      | 9-1'      | 1-8       | 1-8 & 8'-1' | 9-9'      |
| 12       | 1-7       | 8-1'      | 8-1'      | 1-7       | 1-8 & 8'-1' | 9-9'      |
| 13       | 1-6       | 7-1'      | .....     | .....     | .....       | 9-9'      |
| 14       | 1-7       | 8-1'      | 9-1'      | 1-8       | 1-8 & 8'-1' | 9-9'      |
| 15       | 1-8       | 9-1'      | 8-1'      | 1-7       | 1-8 & 8'-1' | 9-9'      |
| 16       | 1-8       | 9-1'      | 9-1'      | 1-8       | 1-8 & 8'-1' | 9-9'      |
| 17       | 1-7       | 8-1'      | 8-1'      | 1-7       | 1-8 & 8'-1' | 9-9'      |
| 18       | 1-8       | 9-1'      | 9-1'      | 1-8       | 1-8 & 8'-1' | 9-9'      |
| 19       | 1-8       | 9-1'      | 9-1'      | 1-8       | 1-8 & 8'-1' | 9-9'      |
| 20       | 1-7       | 8-1'      | 9-1'      | 1-8       | 1-8 & 8'-1' | 9-9'      |



## TABLES OF ARCH COEFFICIENTS

## ARCH No. 1.

Base:  $l=100$ ;  $f=8.333$ ;  $f \div l=0.0833$ ;  $D'=0.2498$ ;  $K'=0.878$ ;  
 $\Sigma M_1' = -21.550 + 21.850$ .

| Point No. | $y'$   | $\Delta'$ | $H'$   | $M_1'$ | $M_2'$ | $V_1$ | $V_2$ |
|-----------|--------|-----------|--------|--------|--------|-------|-------|
| 1         | 0.0996 | 0.0579    | 0.000  | -1.000 | 0.000  | 1.000 | 0.000 |
| 2         | .2820  | .0680     | .401   | -2.474 | .218   | .992  | .008  |
| 3         | .4392  | .06       | 1.163  | -3.444 | .614   | .976  | .024  |
| 4         | .5820  | .1108     | 2.260  | -3.909 | 1.150  | .951  | .049  |
| 5         | .7008  | .1461     | 3.649  | -3.870 | 1.779  | .916  | .084  |
| 6         | .8004  | .1990     | 5.269  | -3.365 | 2.406  | .870  | .130  |
| 7         | .8796  | .2821     | 7.026  | -2.411 | 3.000  | .811  | .189  |
| 8         | .9348  | .3905     | 8.779  | -1.071 | 3.420  | .737  | .263  |
| 9         | .9768  | .5718     | 10.338 | .466   | 3.542  | .651  | .348  |
| 10        | .9972  | 1.0000    | 11.405 | 2.040  | 3.155  | .552  | .447  |
|           |        | 2.9118    | 50.290 | $M_2'$ | $M_1'$ | $V_2$ | $V_1$ |

## ARCH No. 2.

Base:  $l=100$ ;  $f=10$ ;  $f \div l=0.10$ ;  $D'=0.2539$ ;  $K'=0.876$ ;  
 $\Sigma M_1' = -23.776 + 25.528$ .

| Point No. | $y'$  | $\Delta'$ | $H'$   | $M_1'$ | $M_2'$ | $V_1$ | $V_2$ |
|-----------|-------|-----------|--------|--------|--------|-------|-------|
| 1         | 0.102 | 0.0518    | 0.000  | -1.000 | 0.000  | 1.000 | 0.000 |
| 2         | .287  | .0769     | .316   | -2.612 | .193   | .995  | .005  |
| 3         | .450  | .1134     | .990   | -3.766 | .591   | .984  | .016  |
| 4         | .590  | .1696     | 2.043  | -4.394 | 1.186  | .964  | .036  |
| 5         | .706  | .2486     | 3.479  | -4.430 | 1.942  | .934  | .066  |
| 6         | .805  | .3644     | 5.248  | -3.846 | 2.784  | .891  | .109  |
| 7         | .882  | .5041     | 7.221  | -2.674 | 3.576  | .831  | .169  |
| 8         | .940  | .6967     | 9.170  | -1.054 | 4.129  | .755  | .245  |
| 9         | .976  | .8200     | 10.769 | .766   | 4.210  | .661  | .339  |
| 10        | .997  | 1.0000    | 11.720 | 2.471  | 3.680  | .555  | .445  |
|           |       | 4.0455    | 50.956 | $M_2'$ | $M_1'$ | $V_2$ | $V_1$ |

## ARCH No. 3.

Base:  $l=40$ ;  $f=4$ ;  $f \div l=0.10$ ;  $D'=0.3288$ ;  $K'=0.860$ ;  
 $\Sigma M_1' = -21.367 + 29.369$ .

| Point No. | $y'$   | $\Delta'$ | $H'$   | $M_1'$ | $M_2'$ | $V_1$ | $V_2$ |
|-----------|--------|-----------|--------|--------|--------|-------|-------|
| 1         | 0.1200 | 0.0641    | 0.000  | -1.000 | 0.000  | 1.000 | 0.000 |
| 2         | .3175  | .1181     | .288   | -2.665 | .185   | .996  | .004  |
| 3         | .4875  | .2126     | .968   | -3.848 | .606   | .986  | .014  |
| 4         | .6250  | .3260     | 2.128  | -4.380 | 1.286  | .97   | .033  |
| 5         | .7375  | .4724     | 3.752  | -4.190 | 2.168  | .934  | .066  |
| 6         | .8250  | .5638     | 5.728  | -3.284 | 3.126  | .88   | .115  |
| 7         | .9000  | .6830     | 7.820  | -1.856 | 3.984  | .821  | .179  |
| 8         | .9500  | .8032     | 9.748  | -0.144 | 4.538  | .742  | .258  |
| 9         | .9800  | .8866     | 11.236 | 1.591  | 4.621  | .651  | .349  |
| 10        | .9975  | 1.0000    | 12.072 | 3.107  | 4.157  | .551  | .449  |
|           |        | 5.1298    | 53.740 | $M_2'$ | $M_1'$ | $V_2$ | $V_1$ |

## ARCH No. 4.

Base:  $l=80$ ;  $f=8$ ;  $f \div l=0.10$ ;  $D'=0.3323$ ;  $K'=0.862$ ;  
 $\Sigma M_1' = -22.868 + 24.088$ .

| Point No. | $y'$   | $\Delta'$ | $H'$   | $M_1'$ | $M_2'$ | $V_1$ | $V_2$ |
|-----------|--------|-----------|--------|--------|--------|-------|-------|
| 1         | 0.1013 | 0.0726    | 0.000  | -1.000 | 0.000  | 1.000 | 0.000 |
| 2         | .2850  | .1076     | .332   | -2.592 | .197   | .995  | .005  |
| 3         | .4475  | .1549     | 1.038  | -3.698 | .597   | .982  | .018  |
| 4         | .5872  | .2213     | 2.128  | -4.261 | 1.186  | .961  | .039  |
| 5         | .7062  | .3167     | 3.586  | -4.240 | 1.917  | .929  | .071  |
| 6         | .8037  | .4383     | 5.338  | -3.628 | 2.703  | .883  | .117  |
| 7         | .8812  | .5888     | 7.242  | -2.487 | 3.414  | .823  | .177  |
| 8         | .9400  | .7646     | 9.076  | -0.962 | 3.876  | .746  | .254  |
| 9         | .9788  | .9230     | 10.548 | 0.711  | 3.900  | .655  | .345  |
| 10        | .9975  | 1.0000    | 11.366 | 2.236  | 3.351  | .553  | .447  |
|           |        | 4.5877    | 50.654 | $M_2'$ | $M_1'$ | $V_2$ | $V_1$ |

## ARCH No. 5.

Base:  $l=100$ ;  $f=10$ ;  $f \div l=0.10$ ;  $D'=0.5729$ ;  $K'=0.811$ ;  
 $\Sigma M_1' = -19.730 + 20.638$ .

| Point No. | $y'$  | $\Delta'$ | $H'$   | $M_1'$ | $M_2'$ | $V_1$ | $V_2$ |
|-----------|-------|-----------|--------|--------|--------|-------|-------|
| 1         | 0.101 | 0.1660    | 0.000  | -1.000 | 0.000  | 1.000 | 0.000 |
| 2         | .285  | .2264     | .412   | -2.487 | .215   | .993  | .007  |
| 3         | .446  | .2935     | 1.239  | -3.413 | .626   | .976  | .024  |
| 4         | .585  | .3862     | 2.441  | -3.773 | 1.185  | .949  | .051  |
| 5         | .704  | .4608     | 3.948  | -3.579 | 1.824  | .910  | .090  |
| 6         | .802  | .5682     | 5.627  | -2.920 | 2.446  | .859  | .141  |
| 7         | .879  | .6979     | 7.325  | -1.901 | 2.944  | .796  | .204  |
| 8         | .939  | .8182     | 8.858  | -0.658 | 3.203  | .722  | .278  |
| 9         | .977  | .8999     | 10.027 | 0.632  | 3.121  | .637  | .363  |
| 10        | .997  | 1.0000    | 10.675 | 1.790  | 2.652  | .547  | .453  |
|           |       | 5.5171    | 50.552 | $M_2'$ | $M_1'$ | $V_2$ | $V_1$ |

## ARCH No. 6.

Base:  $l=79$ ;  $f=8$ ;  $f \div l=0.1012$ ;  $D'=0.3827$ ;  $K'=0.858$ ;  
 $\Sigma M_1' = -18.157 + 33.851$ .

| Point No. | $y'$   | $\Delta'$ | $H'$   | $M_1'$ | $M_2'$ | $V_1$ | $V_2$ |
|-----------|--------|-----------|--------|--------|--------|-------|-------|
| 1         | 0.1438 | 0.0893    | 0.000  | -1.000 | 0.000  | 1.000 | 0.000 |
| 2         | .3600  | .1846     | .333   | -2.618 | .222   | .996  | .004  |
| 3         | .5350  | .3233     | 1.147  | -3.645 | .737   | .985  | .015  |
| 4         | .6638  | .4258     | 2.505  | -3.921 | 1.545  | .962  | .038  |
| 5         | .7688  | .5513     | 4.295  | -3.492 | 2.532  | .926  | .074  |
| 6         | .8563  | .6852     | 6.340  | -2.462 | 3.545  | .875  | .125  |
| 7         | .9125  | .7780     | 8.390  | -1.019 | 4.386  | .810  | .190  |
| 8         | .9563  | .8757     | 10.215 | 0.622  | 4.910  | .732  | .268  |
| 9         | .9850  | .9638     | 11.590 | 2.231  | 4.998  | .644  | .356  |
| 10        | .9975  | 1.0000    | 12.320 | 3.592  | 4.541  | .549  | .451  |
|           |        | 5.8770    | 57.135 | $M_2'$ | $M_1'$ | $V_2$ | $V_1$ |

## ARCH No. 7.

Base:  $l=79$ ;  $f=8.84$ ;  $f \div l=0.1119$ ;  $D'=0.3415$ ;  $K'=0.864$ ;  
 $\Sigma M_1' = -18.980 + 35.831$ .

| Point No. | $y'$   | $\Delta'$ | $H'$   | $M_1'$ | $M_2'$ | $V_1$ | $V_2$ |
|-----------|--------|-----------|--------|--------|--------|-------|-------|
| 1         | 0.1425 | 0.0766    | 0.000  | -1.000 | 0.000  | 1.000 | 0.000 |
| 2         | .3598  | .1522     | .322   | -2.636 | .219   | .996  | .004  |
| 3         | .5317  | .2782     | 1.097  | -3.714 | .714   | .986  | .014  |
| 4         | .667   | .4103     | 2.412  | -4.096 | 1.542  | .966  | .034  |
| 5         | .7715  | .5726     | 4.198  | -3.725 | 2.569  | .931  | .069  |
| 6         | .8507  | .7117     | 6.298  | -2.674 | 3.651  | .883  | .117  |
| 7         | .9095  | .8095     | 8.455  | -1.135 | 4.586  | .818  | .182  |
| 8         | .9547  | .9325     | 10.397 | 0.637  | 5.191  | .739  | .261  |
| 9         | .9842  | .9667     | 11.844 | 2.368  | 5.292  | .648  | .352  |
| 10        | .9952  | 1.0000    | 12.614 | 3.824  | 4.831  | .550  | .450  |
|           |        | 5.9103    | 57.637 | $M_2'$ | $M_1'$ | $V_2$ | $V_1$ |

## ARCH No. 8.

Base:  $l=76$ ;  $f=8.9$ ;  $f \div l=0.1171$ ;  $D'=0.4023$ ;  $K'=0.848$ ;  
 $\Sigma M_1' = -19.656 + 30.437$ .

| Point No. | $y'$   | $\Delta'$ | $H'$   | $M_1'$ | $M_2'$ | $V_1$ | $V_2$ |
|-----------|--------|-----------|--------|--------|--------|-------|-------|
| 1         | 0.1236 | 0.0846    | 0.000  | -1.000 | 0.000  | 1.000 | 0.000 |
| 2         | .3326  | .1700     | .304   | -2.651 | .195   | .996  | .004  |
| 3         | .5056  | .3072     | 1.045  | -3.766 | .651   | .986  | .014  |
| 4         | .6461  | .4573     | 2.305  | -4.167 | 1.379  | .964  | .036  |
| 5         | .7472  | .5599     | 4.024  | -3.816 | 2.290  | .928  | .072  |
| 6         | .8337  | .6721     | 6.024  | -2.837 | 3.241  | .877  | .123  |
| 7         | .9011  | .7868     | 8.066  | 1.419  | 4.051  | .812  | .188  |
| 8         | .9506  | .8766     | 9.902  | 0.222  | 4.559  | .734  | .266  |
| 9         | .9832  | .9448     | 11.284 | 1.838  | 4.621  | .645  | .355  |
| 10        | .9978  | 1.0000    | 12.034 | 3.215  | 4.175  | .549  | .451  |
|           |        | 5.8593    | 54.988 | $M_2'$ | $M_1'$ | $V_2$ | $V_1$ |



## ARCH No. 9.

Base:  $l=83.04$ ;  $f=9.99$ ;  $f \div l=0.1203$ ;  $D'=0.9263$ ;  $K'=0.767$ ;

$$\Sigma M_1' = -17.398 + 17.981.$$

| Point No. | $y'$   | $\Delta'$ | $H'$   | $M_1'$ | $M_2'$ | $V_1$ | $V_2$ |
|-----------|--------|-----------|--------|--------|--------|-------|-------|
| 1         | 0.1031 | 0.3478    | 0.000  | -1.000 | 0.000  | 1.000 | 0.000 |
| 2         | .2893  | .4223     | .500   | -2.366 | .231   | .992  | .008  |
| 3         | .4515  | .5031     | 1.434  | -3.133 | .645   | .976  | .024  |
| 4         | .5916  | .5839     | 2.704  | -3.344 | 1.172  | .949  | .051  |
| 5         | .7097  | .6833     | 4.205  | -3.082 | 1.728  | .910  | .090  |
| 6         | .8068  | .7640     | 5.780  | -2.438 | 2.224  | .859  | .141  |
| 7         | .8838  | .8447     | 7.295  | -1.534 | 2.577  | .796  | .204  |
| 8         | .9409  | .9317     | 8.594  | -0.501 | 2.712  | .721  | .279  |
| 9         | .9790  | .9752     | 9.542  | 0.529  | 2.575  | .637  | .363  |
| 10        | .9980  | 1.0000    | 10.042 | 1.344  | 2.144  | .546  | .454  |
|           |        | 7.0550    | 50.096 | $M_2'$ | $M_1'$ | $V_2$ | $V_1$ |

## ARCH No. 10.

Base:  $l=60$ ;  $f=7.5$ ;  $f \div l=0.1250$ ;  $D'=0.2761$ ;  $K'=.873$ ;

$$\Sigma M_1' = -23.410 + 25.503.$$

| Point No. | $y'$   | $\Delta'$ | $H'$   | $M_1'$ | $M_2'$ | $V_1$ | $V_2$ |
|-----------|--------|-----------|--------|--------|--------|-------|-------|
| 1         | 0.1027 | 0.0572    | 0.000  | -1.000 | 0.000  | 1.000 | 0.000 |
| 2         | .2894  | .0854     | .320   | -2.612 | .195   | .995  | .005  |
| 3         | .4533  | .1280     | 1.000  | -3.759 | .597   | .984  | .016  |
| 4         | .5920  | .1892     | 2.069  | -4.364 | 1.197  | .964  | .036  |
| 5         | .7107  | .2780     | 3.523  | -4.371 | 1.957  | .933  | .067  |
| 6         | .8066  | .3895     | 5.303  | -3.756 | 2.796  | .889  | .111  |
| 7         | .8840  | .5343     | 7.269  | -2.575 | 3.577  | .829  | .171  |
| 8         | .9414  | .7408     | 9.192  | -0.973 | 4.113  | .752  | .248  |
| 9         | .9786  | .8496     | 10.746 | 0.812  | 4.171  | .659  | .341  |
| 10        | .9973  | 1.0000    | 11.649 | 2.454  | 3.634  | .555  | .445  |
|           |        | 4.2520    | 51.071 | $M_2'$ | $M_1'$ | $V_2$ | $V_1$ |

## ARCH No. 11.

Base:  $l=100$ ;  $f=12.5$ ;  $f \div l=0.1250$ ;  $D'=0.5698$ ;  $K'=0.818$ ;  
 $\Sigma M_1' = -19.612 + 21.730$ .

| Point No. | $y'$   | $\Delta'$ | $H'$   | $M_1'$ | $M_2'$ | $V_1$ | $V_2$ |
|-----------|--------|-----------|--------|--------|--------|-------|-------|
| 1         | 0.1048 | 0.1685    | 0.000  | -1.000 | 0.000  | 1.000 | 0.000 |
| 2         | .2928  | .2280     | .417   | -2.485 | .224   | .993  | .007  |
| 3         | .4552  | .2938     | 1.254  | -3.408 | .654   | .977  | .023  |
| 4         | .5944  | .3865     | 2.465  | -3.772 | 1.239  | .950  | .050  |
| 5         | .7120  | .5055     | 3.979  | -3.590 | 1.907  | .912  | .088  |
| 6         | .8072  | .6026     | 5.681  | -2.911 | 2.562  | .862  | .138  |
| 7         | .8864  | .7422     | 7.405  | -1.860 | 3.092  | .799  | .201  |
| 8         | .9424  | .8870     | 8.952  | -0.586 | 3.366  | .724  | .276  |
| 9         | .9784  | .9398     | 10.112 | 0.728  | 3.273  | .639  | .361  |
| 10        | .9976  | 1.0000    | 10.742 | 1.903  | 2.782  | .547  | .453  |
|           |        | 5.7513    | 51.007 | $M_2'$ | $M_1'$ | $V_2$ | $V_1$ |

## ARCH No. 12.

Base:  $l=76$ ;  $f=9.68$ ;  $f \div l=0.1273$ ;  $D'=0.3484$ ;  $K'=0.861$ ;  
 $\Sigma M_1' = -19.964 + 31.127$ .

| Point No. | $y'$   | $\Delta'$ | $H'$   | $M_1'$ | $M_2'$ | $V_1$ | $V_2$ |
|-----------|--------|-----------|--------|--------|--------|-------|-------|
| 1         | 0.1291 | 0.0742    | 0.000  | -1.000 | 0.000  | 1.000 | 0.000 |
| 2         | .3358  | .1407     | .311   | -2.640 | .203   | .996  | .004  |
| 3         | .5062  | .2433     | 1.050  | -3.753 | .666   | .985  | .015  |
| 4         | .6457  | .3796     | 2.277  | -4.196 | 1.391  | .964  | .036  |
| 5         | .7521  | .4728     | 3.979  | -3.887 | 2.319  | .930  | .070  |
| 6         | .8368  | .5858     | 5.976  | -2.947 | 3.298  | .881  | .119  |
| 7         | .8998  | .7446     | 8.051  | -1.541 | 4.155  | .817  | .183  |
| 8         | .9504  | .8602     | 9.966  | -0.158 | 4.723  | .739  | .261  |
| 9         | .9835  | .9612     | 11.434 | 1.867  | 4.822  | .649  | .351  |
| 10        | .9990  | 1.0000    | 12.223 | 3.330  | 4.353  | .551  | .449  |
|           |        | 5.4624    | 55.267 | $M_2'$ | $M_1'$ | $V_2$ | $V_1$ |

## ARCH NO. 13.

Base:  $l=76$ ;  $f=9.68$ ;  $f \div l=0.1273$ ;  $D'=1.3506$ ;  $K'=.704$ ;  
 $\Sigma M_1' = -9.226 + 29.676$ .

| Point No. | $y'$   | $\Delta'$ | $H'$   | $M_1'$ | $M_2'$ | $V_1$ | $V_2$ |
|-----------|--------|-----------|--------|--------|--------|-------|-------|
| 1         | 0.1291 | 1.000     | 0.000  | -1.000 | 0.000  | 1.000 | 0.000 |
| 2         | .3358  | 1.000     | .857   | -2.008 | .420   | .986  | .014  |
| 3         | .5062  | 1.000     | 2.265  | -2.279 | 1.067  | .959  | .041  |
| 4         | .6457  | 1.000     | 3.966  | -1.992 | 1.792  | .910  | .090  |
| 5         | .7521  | 1.000     | 5.755  | -1.397 | 2.491  | .872  | .128  |
| 6         | .8368  | 1.000     | 7.472  | -0.550 | 3.070  | .715  | .185  |
| 7         | .8998  | 1.000     | 8.990  | 0.379  | 3.479  | .753  | .247  |
| 8         | .9504  | 1.000     | 10.218 | 1.304  | 3.672  | .684  | .316  |
| 9         | .9835  | 1.000     | 11.073 | 2.155  | 3.633  | .612  | .388  |
| 10        | .9990  | 1.000     | 11.514 | 2.855  | 3.359  | .537  | .463  |
|           |        | 10.000    | 62.110 | $M_2'$ | $M_1'$ | $V_2$ | $V_1$ |

## ARCH NO. 14.

Base:  $l=140$ ;  $f=18$ ;  $f \div l=0.1285$ ;  $D'=0.3356$ ;  $K'=0.849$ ;  
 $\Sigma M_1' = -17.350 + 35.663$ .

| Point No. | $y'$   | $\Delta'$ | $H'$   | $M_1'$ | $M_2'$ | $V_1$ | $V_2$ |
|-----------|--------|-----------|--------|--------|--------|-------|-------|
| 1         | 0.1500 | 0.1012    | 0.000  | -1.000 | 0.000  | 1.000 | 0.000 |
| 2         | .3667  | .1682     | .421   | -2.520 | .273   | .995  | .005  |
| 3         | .5222  | .2426     | 1.326  | -3.448 | .834   | .982  | .018  |
| 4         | .6556  | .3646     | 2.703  | -3.735 | 1.645  | .960  | .040  |
| 5         | .7500  | .4688     | 4.500  | -3.346 | 2.626  | .924  | .076  |
| 6         | .8333  | .5863     | 6.572  | -2.361 | 3.651  | .875  | .125  |
| 7         | .8944  | .7620     | 8.700  | -0.940 | 4.546  | .812  | .188  |
| 8         | .9389  | .8512     | 10.618 | 0.729  | 5.118  | .735  | .265  |
| 9         | .9667  | .9420     | 12.083 | 2.393  | 5.230  | .646  | .354  |
| 10        | .9833  | 1.0000    | 12.884 | 3.818  | 4.800  | .550  | .450  |
|           |        | 5.4869    | 59.807 | $M_2'$ | $M_1'$ | $V_2$ | $V_1$ |

## ARCH No. 15.

Base:  $l=60$ ;  $f=10$ ;  $f \div l=0.1667$ ;  $D'=0.3279$ ;  $K'=0.863$ ;  
 $\Sigma M_1' = -21.676 + 25.658$ .

| Point No. | $y'$   | $\Delta'$ | $H'$   | $M_1'$ | $M_2'$ | $V_1$ | $V_2$ |
|-----------|--------|-----------|--------|--------|--------|-------|-------|
| 1         | 0.1082 | 0.0780    | 0.000  | -1.000 | 0.000  | 1.000 | 0.000 |
| 2         | .3009  | .1127     | .359   | -2.561 | .215   | .994  | .006  |
| 3         | .4651  | .1600     | 1.103  | -3.624 | .647   | .982  | .018  |
| 4         | .6041  | .2353     | 2.237  | -4.138 | 1.268  | .960  | .040  |
| 5         | .7199  | .3167     | 3.741  | -4.052 | 2.030  | .927  | .073  |
| 6         | .8143  | .4520     | 5.533  | -3.387 | 2.852  | .881  | .119  |
| 7         | .8885  | .5866     | 7.444  | -2.217 | 3.572  | .820  | .180  |
| 8         | .9434  | .7507     | 9.263  | -0.697 | 4.035  | .743  | .257  |
| 9         | .9797  | .8754     | 10.713 | 0.963  | 4.063  | .652  | .348  |
| 10        | .9977  | 1.0000    | 11.207 | 2.465  | 3.548  | .552  | .448  |
|           |        | 4.5674    | 51.600 | $M_2'$ | $M_1'$ | $V_2$ | $V_1$ |

## ARCH No. 16.

Base:  $l=100$ ;  $f=16.67$ ;  $f \div l=0.1667$ ;  $D'=0.5890$ ;  $K'=0.817$ ;  
 $\Sigma M_1' = -18.819 + 22.333$ .

| Point No. | $y'$   | $\Delta'$ | $H'$   | $M_1'$ | $M_2'$ | $V_1$ | $V_2$ |
|-----------|--------|-----------|--------|--------|--------|-------|-------|
| 1         | 0.1086 | 0.1840    | 0.000  | -1.000 | 0.000  | 1.000 | 0.000 |
| 2         | .3012  | .2395     | .443   | -2.455 | .241   | .992  | .008  |
| 3         | .4656  | .3180     | 1.305  | -3.347 | .687   | .976  | .024  |
| 4         | .6048  | .4087     | 2.547  | -3.667 | 1.289  | .949  | .051  |
| 5         | .7224  | .5115     | 4.085  | -3.443 | 1.970  | .910  | .090  |
| 6         | .8160  | .6266     | 5.787  | -2.752 | 2.625  | .859  | .141  |
| 7         | .8892  | .7564     | 7.491  | -1.706 | 3.142  | .796  | .204  |
| 8         | .9444  | .8728     | 9.010  | -0.450 | 3.406  | .721  | .279  |
| 9         | .9804  | .9580     | 10.152 | 0.838  | 3.320  | .637  | .363  |
| 10        | .9972  | 1.0000    | 10.763 | 1.979  | 2.836  | .546  | .454  |
|           |        | 5.8755    | 51.583 | $M_2'$ | $M_1'$ | $V_2$ | $V_1$ |



## ARCH No. 17.

Base:  $l=53.6$ ;  $f=9.12$ ;  $f \div l=0.1701$ ;  $D'=0.19404$ ;  $K'=0.893$ ;  
 $\Sigma M_1' = -22.735 + 37.903$ .

| Point No. | $y'$   | $\Delta'$ | $H'$   | $M_1'$ | $M_2'$ | $V_1$ | $V_2$ |
|-----------|--------|-----------|--------|--------|--------|-------|-------|
| 1         | 0.1206 | 0.0246    | 0.000  | -1.000 | 0.000  | 1.000 | 0.000 |
| 2         | .3345  | .0563     | .196   | -2.782 | .142   | .998  | .002  |
| 3         | .5110  | .1276     | .718   | -4.179 | .509   | .992  | .008  |
| 4         | .6525  | .2944     | 1.743  | -4.939 | 1.196  | .978  | .022  |
| 5         | .7654  | .3885     | 3.504  | -4.661 | 2.290  | .948  | .052  |
| 6         | .8520  | .4463     | 5.782  | -3.480 | 3.578  | .901  | .099  |
| 7         | .9102  | .5679     | 8.254  | -1.694 | 4.806  | .837  | .163  |
| 8         | .9551  | .7194     | 10.636 | 0.465  | 5.751  | .757  | .243  |
| 9         | .9847  | .9090     | 12.568 | 2.678  | 6.153  | .662  | .338  |
| 10        | .9999  | 1.0000    | 13.656 | 4.561  | 5.774  | .555  | .445  |
|           |        | 4.5340    | 57.057 | $M_2'$ | $M_1'$ | $V_2$ | $V_1$ |

## ARCH No. 18.

Base:  $l=40$ ;  $f=8$ ;  $f \div l=0.2000$ ;  $D'=0.6252$ ;  $K'=0.812$ ;  
 $\Sigma M_1' = -17.857 + 22.787$ .

| Point No. | $y'$   | $\Delta'$ | $H'$   | $M_1'$ | $M_2'$ | $V_1$ | $V_2$ |
|-----------|--------|-----------|--------|--------|--------|-------|-------|
| 1         | 0.1138 | 0.2084    | 0.000  | -1.000 | 0.000  | 1.000 | 0.000 |
| 2         | .3113  | .2735     | .454   | -2.429 | .251   | .992  | .008  |
| 3         | .4755  | .3527     | 1.368  | -3.269 | .717   | .975  | .025  |
| 4         | .6150  | .4404     | 2.648  | -3.529 | 1.337  | .947  | .053  |
| 5         | .7288  | .5423     | 4.208  | -3.265 | 2.019  | .907  | .043  |
| 6         | .8212  | .6588     | 5.904  | -2.559 | 2.663  | .856  | .144  |
| 7         | .8925  | .7722     | 7.584  | -1.517 | 3.165  | .792  | .208  |
| 8         | .9462  | .8736     | 9.072  | -.289  | 3.417  | .718  | .282  |
| 9         | .9800  | .9507     | 10.184 | .903   | 3.387  | .637  | .363  |
| 10        | .9975  | 1.0000    | 10.776 | 2.054  | 2.874  | .546  | .454  |
|           |        | 6.0726    | 52.208 | $M_2'$ | $M_1'$ | $V_2$ | $V_1$ |

## ARCH NO. 19.

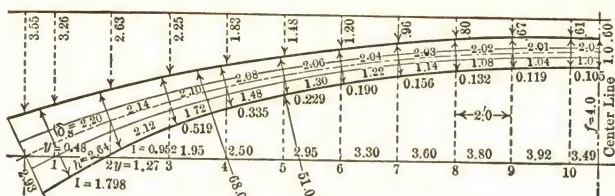
Base:  $l=72$ ;  $f=17.37$ ;  $f \div l=0.2411$ ;  $D'=0.6569$ ;  $K'=0.807$ ;  
 $\Sigma M_1' = -16.378 + 23.531$ .

| Point No. | $y'$   | $\Delta'$ | $H'$   | $M_1'$ | $M_2'$ | $V_1$ | $V_2$ |
|-----------|--------|-----------|--------|--------|--------|-------|-------|
| 1         | 0.1226 | 0.2534    | 0.000  | -1.000 | 0.000  | 1.000 | 0.000 |
| 2         | .3293  | .3213     | .515   | -2.368 | .280   | .991  | .009  |
| 3         | .4956  | .3891     | 1.485  | -3.188 | .777   | .972  | .028  |
| 4         | .6315  | .4797     | 2.816  | -3.301 | 1.415  | .943  | .057  |
| 5         | .7421  | .5702     | 4.395  | -2.980 | 2.100  | .902  | .098  |
| 6         | .8302  | .6652     | 6.084  | -2.249 | 2.735  | .850  | .150  |
| 7         | .8987  | .8010     | 7.728  | -1.234 | 3.220  | .786  | .214  |
| 8         | .9488  | .8824     | 9.152  | -0.058 | 3.451  | .713  | .287  |
| 9         | .9816  | .9322     | 10.205 | 1.117  | 3.360  | .631  | .369  |
| 10        | .9983  | 1.0000    | 10.773 | 2.152  | 2.924  | .544  | .456  |
|           |        | 6.2945    | 53.153 | $M_2'$ | $M_1'$ | $V_2$ | $V_1$ |

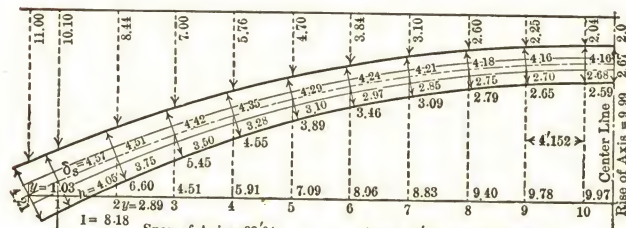
## ARCH NO. 20.

Base:  $l=40$ ;  $f=10.6$ ;  $f \div l=0.265$ ;  $D'=0.5858$ ;  $K'=0.821$ ;  
 $\Sigma M_1' = -16.311 + 25.443$ .

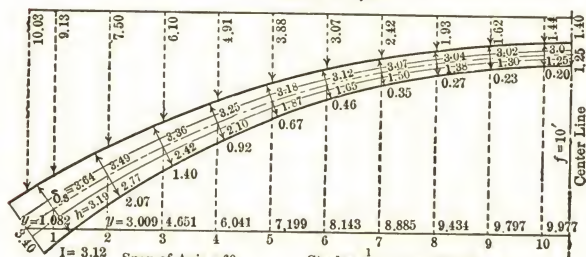
| Point No. | $y'$   | $\Delta'$ | $H'$   | $M_1'$ | $M_2'$ | $V_1$ | $V_2$ |
|-----------|--------|-----------|--------|--------|--------|-------|-------|
| 1         | 0.1274 | 0.2180    | 0.000  | -1.000 | 0.000  | 1.000 | 0.000 |
| 2         | .3396  | .2784     | .519   | -2.372 | .296   | .992  | .008  |
| 3         | .5056  | .3519     | 1.473  | -3.151 | .805   | .974  | .026  |
| 4         | .6415  | .4379     | 2.809  | -3.348 | 1.472  | .946  | .054  |
| 5         | .7500  | .5364     | 4.431  | -3.009 | 2.219  | .906  | .094  |
| 6         | .8367  | .6448     | 6.169  | -2.252 | 2.914  | .854  | .146  |
| 7         | .9038  | .7794     | 7.876  | -1.179 | 3.455  | .791  | .209  |
| 8         | .9509  | .8595     | 9.360  | .058   | 3.724  | .717  | .283  |
| 9         | .9727  | .9613     | 10.473 | 1.304  | 3.658  | .634  | .366  |
| 10        | .9981  | 1.0000    | 11.024 | 2.362  | 3.176  | .545  | .455  |
|           |        | 6.0676    | 54.133 | $M_2'$ | $M_1'$ | $V_2$ | $V_1$ |



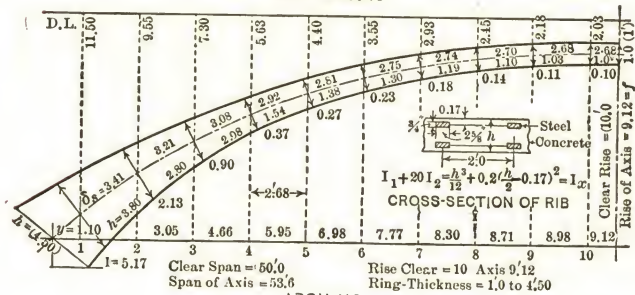
ARCH NO. 3



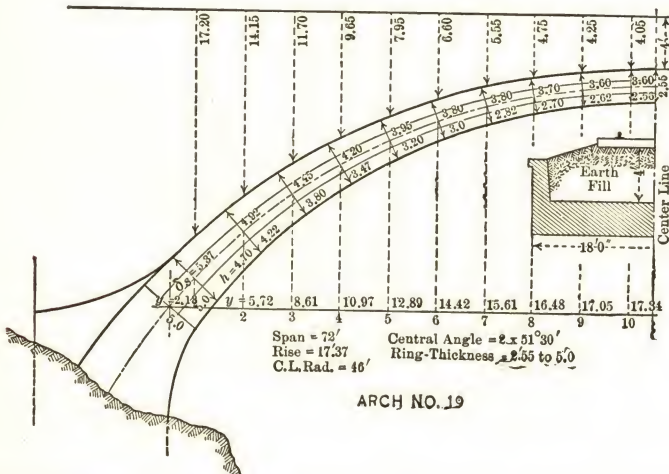
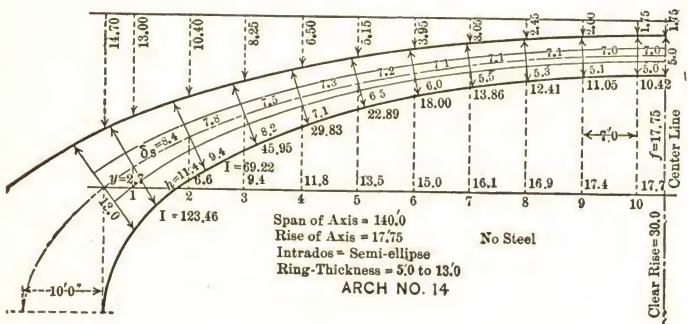
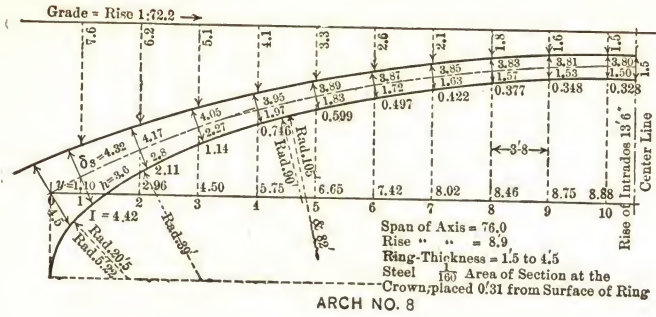
ARCH NO. 9



ARCH NO. 15



DIAGRAMS OF TYPICAL ARCHES INCLUDED IN THE TABLES OF COEFFICIENTS.



DIAGRAMS OF TYPICAL ARCHES INCLUDED IN THE TABLES OF COEFFICIENTS.



*Example.\**

Table A, p. 223, contains a complete set of coefficients which are based upon an arch having a span of 100 ft. and a rise of 16.67 ft. The depth of the ring at the crown and the springing being in the ratio of about 1 to 2.

Columns 1 and 2 Table A (this is arch No. 16) contain the coefficients which must be satisfied by each arch ring to which the coefficients in the remaining columns apply. That is, each  $y$  for the axis must bear the ratio to the rise  $f$  of the axis that the coefficient in column 1 bears to unity, and each value of  $\Delta = \delta s \div I$  must bear the relation to  $\Delta_{10}$  that the coefficient in column 2 bears to unity.

Assume that for some reason the rise must be changed from 16.67 ft. to 12.5 ft. while the span remains unchanged or 100 ft.

In Table B, p. 224, column 1, gives the new values of  $y = 12.5 y'$ . In Fig. 1b divide the span into 20 equal parts and, at the centers of these divisions, lay off the ordinates  $y$  and draw a curve through the extremities. This will be the new arch axis. Then scale the values of  $\delta s$  and place the results in column 2 of Table B.

The next step will be the assumption of the depth of the arch ring at the crown, or for all practical purposes at the section numbered 10. This depth will depend upon the loading and the material composing the ring as well as the radius of curvature at the crown. Let it be assumed that the ring is to be composed of concrete and that the fill will be composed of gravel and about 3 ft. deep over the

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\* *Railroad Gazette*, Aug. 20, 1907, March 13, 1908.

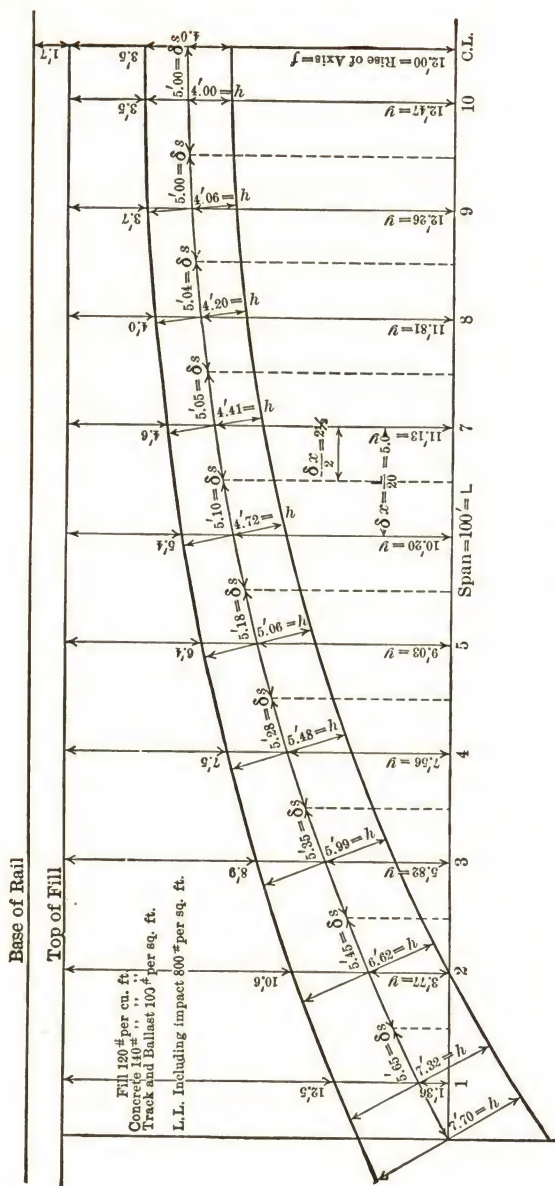


FIG. 1b.

crown. The moving load will be assumed at 5000 lbs. per foot of span with 100 per cent impact. In other words, a single-track railroad bridge is the structure to be designed. The depth at the crown by various empirical formulas will be somewhere in the vicinity of 4 ft., and, as a little error now can be easily corrected later, the depth will be assumed as 4 ft. at the section numbered 10. A longitudinal slice of the bridge 1 ft. thick is assumed. Then  $I_{10} = 5.333$  and  $\Delta_{10} = \delta s_{10} \div I_{10} = 5.00 \div 5.33 = 0.938$ , and hence  $\Delta_n = 0.938 \Delta_n'$ . These values are given in column 3 of Table B.

$$\text{Since } \Delta = \frac{\delta s}{I}, I = \frac{\delta s}{\Delta} = \frac{h^3}{12} \therefore h = \sqrt[3]{\frac{12 \delta s}{\Delta}} \text{ for each point.}$$

The values of  $h$  are given in column 4 of Table B.

Assuming that the fill weighs 120 lbs. per cubic foot, the rails, ties and ballast to be equivalent to 100 lbs. per square foot, and that the concrete in the ring weighs 140 lbs. per cubic foot, the dead load concentrated at each point becomes as shown in column 5 of Table B. The dimensions employed are shown in Fig. 1b. In column 6 of Table B are given the values of  $H$  for unit loads at points 1, 2, 3, etc., respectively. These coefficients multiplied by the actual loads at these points will give the values of  $H$  as shown in column 7 of Table B. The values of  $H$  for the loads 1 to 10 is 62,230 lbs. and for the entire dead load this becomes  $2 \times 62,230 = 124,500$  lbs. The vertical reactions  $V_1$  and  $V_2$  each equal one-half the entire dead load, or 79,500 lbs.

The computation of  $M_1$  now follows: Since the coefficients  $M_1'$  are each multiplied by  $\frac{\delta x}{2} = 2.5$  to obtain the values of  $M_1$  for unit loads, this may be considered as a





constant factor and introduced at the end of the calculation. In column 8 of Table B are given the sums of each pair of symmetrical values of  $M_1'$  as given in columns 5 and 6 of Table A. These multiplied by the dead load (column 9, Table B), and the sum multiplied by  $\frac{\delta x}{2} = 2.5$  gives the value of  $M_1$  and  $M_2$  for the total dead load, or  $M_1 = M_2 = -27,359 \times 2.5 = -68,400$  lbs.-ft.

The equilibrium polygon is now located since  $\frac{M_1}{H} = y_1 = -0.55$  ft., or the polygon starts 0.55 ft. below the axis at the support. For convenience in drawing the equilibrium polygon, the moment at crown should be computed by the formula,

$$M_x = M_1 + V_1x - Hy - \sum P(x-a),$$

$$\text{or} \quad M_c = M_1 + \frac{1}{2}lV_1 - Hf - \sum P(\frac{1}{2}l - a).$$

In this case  $M_c = -13,900$  and  $y_c = -\frac{-13,900}{124,500} = -0.11$  ft.,

or the polygon passes the crown 0.11 ft. below the axis. This polygon is shown in Fig. 2*b*, one-half only being drawn, as it is symmetrical. For convenience and accuracy in drawing the polygon, start at the support and at the crown and close at the quarter point.

Inspection shows that this polygon lies well within the middle third of the ring and that the maximum compression is very much below the safe strength of the concrete.

The live load will now be considered. Let this be 800 lbs. per square foot or  $5 \times 800 = 4000$  lbs. per division.

Table A, column 11, shows the fields of loading which produce maximum moments. (Approx.)

*Live Load 1 to 8.*

$$H = \frac{\delta x}{2f} H' = 0.2 H' \text{ or } H = 0.2 \times 30.668 \times 4000 = 24,500 \text{ lbs.};$$

$$M_1 = \frac{\delta x}{2} M_1' = 2.5 M_1' \text{ or } M_1 = -18.819 \times 2.5 \times 4000 =$$

$$-188,200 \text{ lbs.-ft.};$$

$$V_1 = 7.203 \times 4000 = 28,800 \text{ lbs.};$$

$$V_2 = 32,000 - 28,800 = 3,200 \text{ lbs.};$$

$$M_2 = +13.360 \times 2.5 \times 4000 = +133,600 \text{ lbs.-ft.};$$

$$M_c = -14,200 \text{ lbs.-ft.}$$

Combining these values with those obtained for the dead load the following results obtain:

*Dead Load and Live Load 1 to 8.*

$$M_1 \dots \dots \text{D.L.} = -68,400 \text{ lbs.-ft.}$$

$$\text{L.L.} = -188,200 \text{ "}$$

$$\text{D.L.} + \text{L.L.} = -256,600 \text{ lbs.-ft.}$$

$$H \dots \dots \text{D.L.} = 124,500 \text{ lbs.}$$

$$\text{L.L.} = 24,500 \text{ "}$$

$$\text{D.L.} + \text{L.L.} = 149,000 \text{ lbs.}$$

$$V_1 \dots \dots \text{D.L.} = 79,500 \text{ lbs.}$$

$$\text{L.L.} = 28,800 \text{ "}$$

$$\text{D.L.} + \text{L.L.} = 108,300 \text{ lbs.}$$

$$y_1 = \frac{-256,600}{149,000} = -1.72 \text{ ft.}$$

$$M_2 \dots \dots \text{D.L.} = -68,400 \text{ lbs.-ft.}$$

$$\text{L.L.} = +133,600 \text{ "}$$

$$\text{D.L.} + \text{L.L.} = +65,200 \text{ lbs.-ft.}$$

$$M_c \dots \dots \text{D.L.} = -13,900 \text{ lbs.-ft.}$$

$$\text{L.L.} = -14,200 \text{ "}$$

$$\text{D.L.} + \text{L.L.} = -28,200 \text{ lbs.-ft.}$$

$$V_2 \dots \dots \text{D.L.} = 79,500 \text{ lbs.}$$

$$\text{L.L.} = 3,200 \text{ "}$$

$$\text{D.L.} + \text{L.L.} = 82,700 \text{ lbs.}$$

$$y_c = \frac{-28,200}{149,000} = -0.19 \text{ ft.}$$

$$y_2 = \frac{+65,000}{149,000} = +0.43 \text{ ft.}$$

In like manner each set of loadings is considered. Any other field can be taken by simply adding the values of  $H'$ ,  $M_1'$ ,  $V_1$ , etc., in Table A corresponding to the loading used.

In Table C, p. 224, are given the values of  $H$ ,  $M_1$ , etc., for the various loadings considered. Since the first four sets are symmetrical, only one-half of each equilibrium polygon is drawn, as shown in Fig. 1*b*. In the last two sets the loading is not symmetrical and hence the polygons must be constructed for the entire span. Proceed as follows: Lay off  $y_1$  and  $y_2$  at the left support and  $y_c$  at the crown and construct both halves of the polygon on the left of the crown as shown in Fig. 3*b*.

All of the above results are obtained with comparatively little labor and in a short time. As usually considered the arch rib, as shown in Fig. 1*b*, would be passed as quite safe for the given loadings. But the effect of the axial thrust and temperature changes remains to be considered.

The effect of the axial thrust will be first taken up. The horizontal thrust is

$$H_a = H \left\{ 1 - \frac{D}{D + \sum \frac{(\delta x)^2}{\delta s F}} \right\}$$

where

$$D = D' f^2 \Delta_{10} = 0.589 f^2 \Delta_{10} \text{ or } D = 86.32 \text{ (see Table A);}$$

$$\delta x = 5.0 \text{ ft.};$$

$$F = \text{area of rib at section considered};$$

$$\sum \frac{(\delta x)^2}{F \delta s} = 19.42 \quad \therefore H_a = 18\% H \text{ (nearly).}$$

The moment at the support is

$$M_1 = H_a \frac{\sum y \Delta}{\sum \Delta} = H_a K' f = 0.817(12.5) H_a = (10.21) H_a$$

(see Table A).

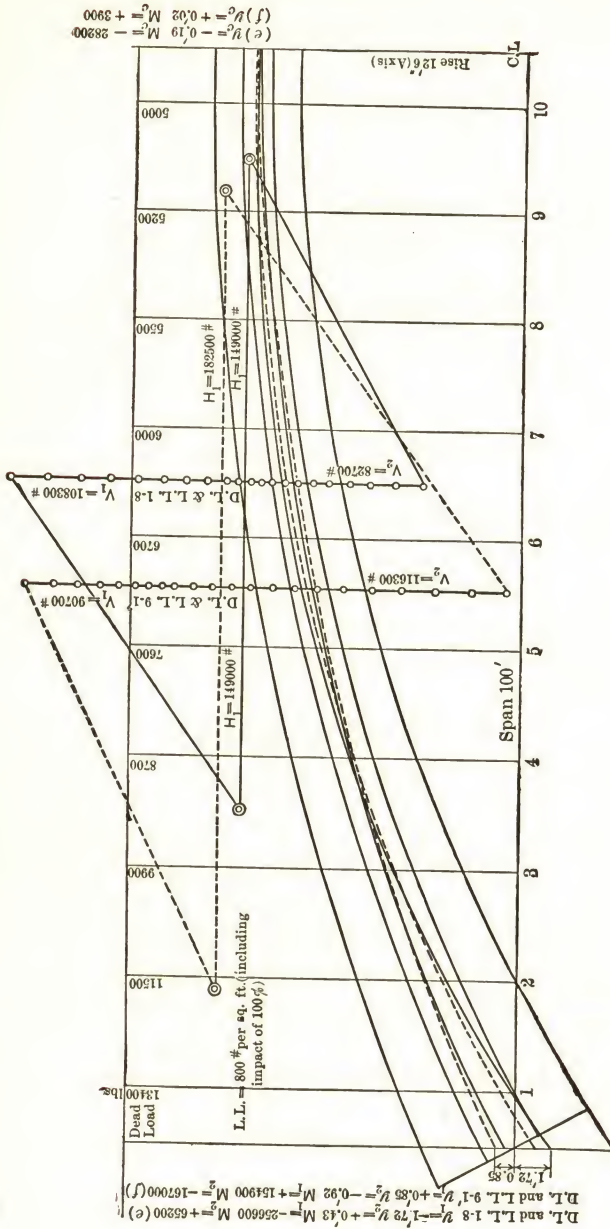


FIG. 3b.



By a brief calculation it becomes evident that the concrete will be subjected to tensile stresses greater than its safe strength when the axial thrust is considered without taking into account the effect of a drop in temperature; hence it becomes necessary to either reinforce the rib with steel or increase its depth. Assuming that the latter is not feasible to any great extent the rib will be reinforced with steel.

Let  $E_c = 1,500,000$  lbs. per square inch, and  $E_s = 30,000,000$  lbs. per square inch, then  $\frac{E_s}{E_c} = 20$  and each square inch of steel is equivalent to 20 square inches of concrete.

Let 1 per cent of the area at the crown be assumed as steel, or 0.8 of a square foot of equivalent concrete, and place the center of the steel 0.25 ft. from the outside of the concrete. The effective area is now  $4.0 + 0.8 = 4.8$  sq.ft. =  $F$  at the crown, and practically at section number 10.

$I$  = the moment of inertia = 7.78 and the section modulus  $S = 3.89$ .

Since the values of  $\Delta$  must always remain in the relative magnitudes shown in column 2, Table A, if the moment of inertia is increased at any section, the moment of inertia at each other section must be increased in like proportion.

At the crown the original  $I$  was 5.33 and  $7.78 \div 5.33 = 1.46$ , or each section must have its  $I$  increased 46 per cent.

For example take section number 1. The original  $I$  was 32.68, and adding 46 per cent to this,  $I$  becomes 47.72, then

$$\frac{h^3}{12} + 0.8 \left( \frac{h-0.5}{2} \right)^2 = 47.72 \quad \text{or} \quad h = 7.65 \text{ ft. (nearly).}$$

The effective area is  $7.65 + 0.80 = 8.45$  and the section modulus is 12.42.

In like manner the proper depths of the arch ring at other sections can be determined.

It is not feasible or necessary here to compute the fiber stresses for all loads at all points. The method is identical in each case, so one example only will be given. Point number 1 will be selected and the fiber stress for the dead load, dead load axial thrust and a change of temperature of  $\pm 20$  deg. be determined.

#### POINT NUMBER 1.

$$\begin{array}{lll} F = 8.45 \text{ sq.ft.} & S = 12.42. & \sin \phi = 0.454. \\ \cos \phi = 0.885. & y = 1.36 \text{ ft.} & x = 2.5 \text{ ft.} \end{array}$$

#### DEAD LOAD.

The direct stress  $= N_x = H \cos \phi + V_1 \sin \phi = 146,300$  lbs.

The moment  $M_x = M_1 + V_1 x - H y = -39,200$  lbs.-ft.

$$\text{The fiber stress } p = \frac{N_x}{F} \pm \frac{M_x}{S} = \frac{146,300}{8.45} \pm \frac{-39,200}{12.42}.$$

Or, the stress in the upper fiber  $= +14,100$  lbs.,

and the stress in the lower fiber  $= +20,500$  lbs.

#### DEAD LOAD AXIAL THRUST.

Assume  $H_a = 1000$  lbs. and determine the fiber stresses.

$N_x = 1000 (0.885) = 885$  lbs.  $M_x = 1000 (10.21 - 1.36) = 8860$  lbs.-ft.

$$p = \frac{-885}{8.45} \mp \frac{8860}{12.42}.$$

Or, the stress in the upper fiber  $= -818$  lbs.,

and the stress in the lower fiber  $= +609$  lbs.

Then for  $H_a = 1$  lb., these quantities become  $-0.818$  lb. and  $+0.609$  lb. respectively.

For the dead load  $H_a = 18$  per cent of  $H = 18$  per cent of  $124,500 = 22,400$  lbs.

Therefore, the stress in the upper fiber  $= 22,400 \times -0.818 = -18,300$  lbs., and the stress in the lower fiber  $= 22,400 \times 0.609 = +13,600$  lbs.

Temperature change of  $\pm 20^\circ$ ;

$$H_t = \frac{et^\circ El}{0.589f^2 \Delta_{10}} = \frac{et^\circ El}{86.32};$$

$$e = 0.000006;$$

$$E = 1,500,000 \times 144 \text{ per sq.ft.};$$

$$l = 100 \text{ ft};$$

$$t^\circ = 20^\circ;$$

and  $H_t = 30,000$  lbs.

Reducing this for the effect of the axial thrust

$$H_t = 82 \text{ per cent of } 30,000 = 24,600 \text{ lbs.}$$

The stress in the upper fiber  $= 24,600 \times 0.818 = \pm 20,100$  lbs., and in the lower fiber  $24,600 \times 0.609 = \mp 15,000$  lbs.

In precisely the same manner any other loading can be considered or any other section of the arch ring examined.

Table D, p. 225, contains the fiber stresses for section number 1 and at the crown for various loadings. Exclusive of the temperature effect the maximum tension in the concrete is 12,700 lbs. per square foot, or a little less than 100 lbs. per square inch. The maximum compression is at point 1 and equals 52,800 lbs. per square foot, or about 370 lbs. per square inch. The stress in the steel is less than  $20 \times 370 = 7400$  lbs. per square inch.



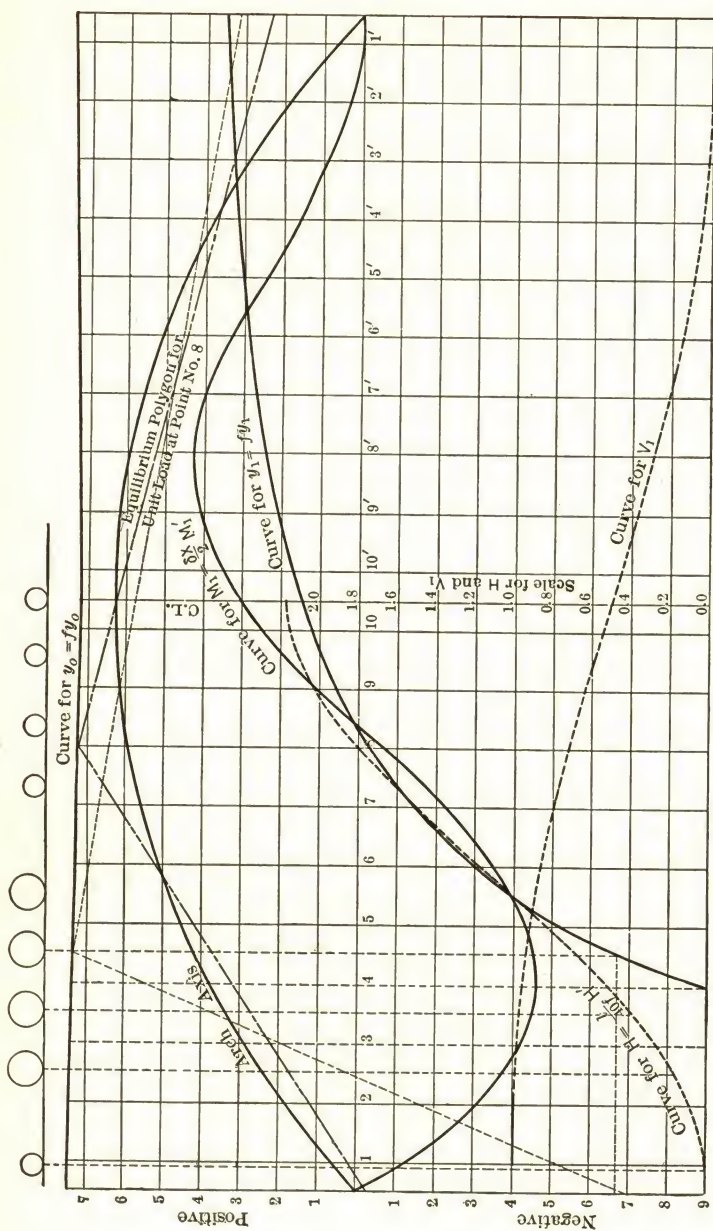


FIG. 4b.



If the temperature effect is considered, the maximum tension in the concrete is 38,500 lbs. per square foot, or about 270 lbs. per square inch, and in the steel less than  $20 \times 270 = 5400$  lbs. per square inch. The maximum compression, including the effect of temperature, is 67,800 lbs. per square foot, or about 470 lbs. per square inch, and in the steel, less than  $20 \times 470 = 9400$  lbs. per square inch.

In case the tension values are considered too large the arch ring can be made a little deeper.

The dead load was based upon a plain concrete ring which now has been increased in depth a maximum amount at the supports and not at all at the crown. At section number 1 the additional concrete increases the dead load about 150 lbs., which is of no practical importance when it is considered that the dead load assumed for this section was 13,400 lbs. Increasing the depth of the ring actually decreases the effect of the axial thrust and hence the values given in Table D are a little large. Since the stresses due to temperature changes include the axial thrust effect, the values given in Table D are a little small.

The above solution shows that the use of unit loads in determining the values of  $H$ ,  $M_1$ ,  $V_1$ , etc., for an arch ring, enables the designer to change at will the loads, the span and the rise of the arch as well as the depth of the ring with very little labor.

Although the loads have been assumed as concentrated at the points of division as being most convenient for determining  $H_1$ ,  $M_1$ ,  $V_1$ , etc., for unit loads, it is not at all necessary that the actual loading should be so arranged. The actual loading may be placed anywhere and the unit values of  $H$ ,  $M_1$ ,  $V_1$ , etc., are determined from a curve

drawn with the computed values of these quantities based on unit loads placed at the points of division. (See Fig. 4b.)

TABLE A.

| Point No. | 1<br>$y = fy'$<br>$y'$ | 2<br>$\Delta = \Delta_{10}\Delta'$<br>$\Delta'$ | 3<br>$\frac{\delta x}{H} \frac{H'}{2f}$<br>$H'$ | 4<br>$\frac{\delta x}{M_1} \frac{M_1'}{2}$<br>$M_1'$ | 5<br>$\frac{\delta x}{M_2} \frac{M_2'}{2}$<br>$M_2'$ | 6<br>$V_1 = V_1$<br>$V_1$ | 7<br>$V_2 = V_2$<br>$V_2$ | 8<br>$y_1 = fy_1'$<br>$y_1'$ |
|-----------|------------------------|-------------------------------------------------|-------------------------------------------------|------------------------------------------------------|------------------------------------------------------|---------------------------|---------------------------|------------------------------|
| 1         | 0.1086                 | 0.1840                                          | 0.000                                           | -1.000                                               | 0.000                                                | 1.000                     | 0.000                     |                              |
| 2         | 0.3012                 | 0.2395                                          | 0.443                                           | -2.455                                               | 0.241                                                | 0.992                     | 0.008                     | -5.967                       |
| 3         | 0.4656                 | 0.3180                                          | 1.305                                           | -3.347                                               | 0.687                                                | 0.976                     | 0.024                     | -2.719                       |
| 4         | 0.6048                 | 0.4087                                          | 2.547                                           | -3.667                                               | 1.289                                                | 0.949                     | 0.051                     | -1.530                       |
| 5         | 0.7224                 | 0.5115                                          | 4.085                                           | -3.443                                               | 1.970                                                | 0.910                     | 0.090                     | -0.902                       |
| 6         | 0.8160                 | 0.6266                                          | 5.787                                           | -2.752                                               | 2.625                                                | 0.859                     | 0.141                     | -0.513                       |
| 7         | 0.8892                 | 0.7564                                          | 7.491                                           | -1.706                                               | 3.142                                                | 0.796                     | 0.204                     | -0.251                       |
| 8         | 0.9444                 | 0.8728                                          | 9.010                                           | -0.450                                               | 3.406                                                | 0.721                     | 0.279                     | -0.066                       |
| 9         | 0.9804                 | 0.9580                                          | 10.152                                          | 0.838                                                | 3.320                                                | 0.637                     | 0.363                     | 0.072                        |
| 10        | 0.9972                 | 1.0000                                          | 10.763                                          | 1.979                                                | 2.836                                                | 0.546                     | 0.454                     | 0.177                        |
|           | .....                  | .....                                           | 51.583                                          |                                                      |                                                      |                           | $D' = 0.5890$             |                              |

| Point No. | 9<br>$y_2 = fy_2'$<br>$y_2'$ | 10<br>$y_0 = fy_0'$<br>$y_0'$ | 11<br>Points loaded<br>(inclusive). | 12<br>$\frac{\delta x}{H} \frac{H'}{2f}$<br>$H'$ | 13<br>$\frac{\delta x}{M_1} \frac{M_1'}{2}$<br>$M_1'$ | 14<br>$\frac{\delta x}{M_2} \frac{M_2'}{2}$<br>$M_2'$ | 15<br>$V_1 = V_1$<br>$V_1$ |
|-----------|------------------------------|-------------------------------|-------------------------------------|--------------------------------------------------|-------------------------------------------------------|-------------------------------------------------------|----------------------------|
| 1         |                              |                               |                                     |                                                  |                                                       |                                                       |                            |
| 2         | 0.538                        | 1.186                         | 1-8                                 | 30.668                                           | -18.819                                               | +13.360                                               | 7.203                      |
| 3         | 0.522                        | 1.178                         | 9-1'                                | 72.498                                           | +22.333                                               | -9.846                                                | 2.797                      |
| 4         | 0.502                        | 1.168                         | 1-8 and 8'-1'                       | 61.336                                           | -5.459                                                | -5.459                                                | 8.000                      |
| 5         | 0.479                        | 1.161                         | 9-9'                                | 41.830                                           | +8.973                                                | +8.973                                                | 2.000                      |
| 6         | 0.451                        | 1.157                         | 1-1'                                | 103.166                                          | +3.514                                                | +3.514                                                | 10.000                     |
| 7         | 0.418                        | 1.151                         | 1-10                                | 51.583                                           | -16.002                                               | +19.516                                               | 8.386                      |
| 8         | 0.376                        | 1.147                         |                                     |                                                  |                                                       |                                                       |                            |
| 9         | 0.324                        | 1.146                         |                                     |                                                  |                                                       |                                                       |                            |
| 10        | 0.259                        | 1.144                         |                                     |                                                  |                                                       |                                                       |                            |
|           | $K' = 0.817$                 |                               |                                     |                                                  |                                                       |                                                       |                            |

TABLE B.

| Point No. | 1<br>$y = fy'$ or<br>12.5y'<br>feet. | 2<br>$\delta s$ Scaled<br>from Fig. 1b<br>feet. | 3<br>$\Delta = \Delta' \Delta_{10}$ or<br>0.938 $\Delta'$ | 4<br>$h = \sqrt[3]{\frac{12 \delta s}{\Delta}}$<br>feet. | 5<br>Dead Load<br>$P$<br>lbs. | 6<br>$\frac{L H'}{H}$ or $\frac{40 f H'}{0.2 H'}$<br>unit lds. | 7<br>$PH$ | 8<br>Symmetrical<br>values of $M_1'$<br>Combined = $M$ | 9<br>$PM$                     |
|-----------|--------------------------------------|-------------------------------------------------|-----------------------------------------------------------|----------------------------------------------------------|-------------------------------|----------------------------------------------------------------|-----------|--------------------------------------------------------|-------------------------------|
| 1         | 1.36                                 | 5.65                                            | 0.173                                                     | 7.32                                                     | 13,400                        | 0.0000                                                         | 0,000     | -1.000                                                 | -13,400                       |
| 2         | 3.77                                 | 5.45                                            | 0.225                                                     | 6.62                                                     | 11,500                        | 0.0886                                                         | 1,019     | -2.214                                                 | -25,461                       |
| 3         | 5.82                                 | 5.35                                            | 0.298                                                     | 5.99                                                     | 9,900                         | 0.2610                                                         | 2,584     | -2.660                                                 | -26,334                       |
| 4         | 7.56                                 | 5.28                                            | 0.384                                                     | 5.48                                                     | 8,700                         | 0.5094                                                         | 4,432     | -2.378                                                 | -20,689                       |
| 5         | 9.03                                 | 5.18                                            | 0.480                                                     | 5.06                                                     | 7,600                         | 0.8170                                                         | 6,209     | -1.473                                                 | -11,195                       |
| 6         | 10.20                                | 5.10                                            | 0.588                                                     | 4.72                                                     | 6,700                         | 1.1574                                                         | 7,755     | -0.127                                                 | -851                          |
| 7         | 11.13                                | 5.05                                            | 0.709                                                     | 4.41                                                     | 6,000                         | 1.4982                                                         | 8,989     | 1.436                                                  | 8,616                         |
| 8         | 11.81                                | 5.04                                            | 0.819                                                     | 4.20                                                     | 5,500                         | 1.8020                                                         | 9,911     | 2.956                                                  | 16,258                        |
| 9         | 12.26                                | 5.00                                            | 0.899                                                     | 4.06                                                     | 5,200                         | 2.0304                                                         | 10,558    | 4.158                                                  | 21,622                        |
| 10        | 12.47                                | 5.00                                            | 0.938                                                     | 4.00                                                     | 5,000                         | 2.1526                                                         | 10,763    | 4.815                                                  | 24,075                        |
|           |                                      |                                                 |                                                           |                                                          |                               |                                                                | 62,230    |                                                        | -97,930<br>+70,571<br>-27,359 |

TABLE C.

| r  | Loading.      | (a)<br>D.L. | (b)<br>D.L. and L.<br>L., 1-1'. | (c)<br>D.L. and<br>L.L., 1-8<br>and 8'-1'. | (d)<br>D.L. and<br>L.L., 9-9'. | (e)<br>D.L. and<br>L.L., 1-8. | (f)<br>D.L. and<br>L.L. 9-1'. |
|----|---------------|-------------|---------------------------------|--------------------------------------------|--------------------------------|-------------------------------|-------------------------------|
| 2  | $M_1$ . . .   | -68,400     | -33,300                         | -122,990                                   | +21,300                        | -256,600                      | +154,900                      |
| 3  | $M_2$ . . .   | -68,400     | -33,300                         | -122,990                                   | +21,300                        | +65,200                       | -167,000                      |
| 4  | $H$ . . . .   | 124,500     | 207,000                         | 173,600                                    | 158,000                        | 149,000                       | 182,500                       |
| 5  | $V_1$ . . .   | 79,500      | 119,500                         | 111,500                                    | 87,500                         | 108,300                       | 90,700                        |
| 6  | $V_2$ . . .   | 79,500      | 119,500                         | 111,500                                    | 87,500                         | 82,700                        | 116,300                       |
| 7  | $Mc$ . . .    | -13,900     | -10,000                         | -41,900                                    | +17,500                        | +28,200                       | +3,900                        |
| 8  | $y_1$ . . . . | -0.55       | -0.16                           | -0.71                                      | +0.14                          | -1.72                         | +0.85                         |
| 9  | $y_0$ . . . . | -0.11       | -0.05                           | -0.24                                      | +0.11                          | -0.19                         | +0.02                         |
| 10 | $y_2$ . . . . | -0.55       | -0.16                           | -0.71                                      | +0.14                          | +0.43                         | -0.92                         |

TABLE D.

| 1                          | 2            | 3             | 4                          | 5           | 6             |
|----------------------------|--------------|---------------|----------------------------|-------------|---------------|
| Loading.                   | Point No. 1. |               | Loading.                   | Crown.      |               |
|                            | Top Fiber.   | Bottom Fiber. |                            | Top Fiber.  | Bottom Fiber. |
| D.L.....                   | +14,100      | +29,400       | D.L.....                   | +22,400     | +29,600       |
| D.L. axial.....            | -18,300      | +22,700       | Axial.....                 | + 8,400     | -17,800       |
| Sum.....                   | - 4,200      | +34,100       | Sum.....                   | +30,800     | +11,800       |
| Temp. $\pm 20^\circ$ ..... | $\pm 20,100$ | $\mp 15,000$  | Temp. $\pm 20^\circ$ ..... | $\mp 9,300$ | $\pm 19,600$  |
| Total.....                 | -24,300      | +49,100       | Total.....                 | +40,100     | - 7,800       |
| D.L. and L.L. 1-8..        | + 6,200      | +36,600       | D.L. and 1-8 and 8-1'      | +25,300     | +47,100       |
| Do., axial.....            | -21,900      | +10,200       | Axial.....                 | +11,800     | -24,800       |
| Sum.....                   | -15,700      | +52,800       | Sum.....                   | +37,100     | +22,300       |
| Temp. $\pm 20^\circ$ ..... | $\pm 20,100$ | $\mp 15,000$  | Temp. $\pm 20^\circ$ ..... | $\mp 9,300$ | $\pm 19,600$  |
| Total.....                 | -38,500      | +67,800       | Total.....                 | +46,400     | +41,900       |
| D.L. and L.L. 1-1'..       | +26,800      | +29,400       | D.L. and L.L. 9-9'...      | +37,500     | +28,500       |
| Axial.....                 | -20,500      | +22,700       | Axial.....                 | +10,700     | -22,600       |
| Sum.....                   | - 3,700      | +52,100       | Sum.....                   | +48,200     | + 5,900       |
| Temp. $\pm 20^\circ$ ..... | $\pm 20,100$ | $\mp 15,000$  | Temp. $\pm 20^\circ$ ..... | $\mp 9,300$ | $\pm 19,600$  |
| Total.....                 | -23,800      | +67,100       | Total.....                 | +57,500     | -13,700       |

NOTE.—All stresses are expressed in pounds per square foot. The + sign signifies compression.





## APPENDIX C.

FORMULAS FOR THE SYMMETRICAL FIXED ARCH WHEN  
THE ORIGIN OF THE CO-ORDINATES  $x$  AND  $y$  IS AT  
THE CROWN.

Assume the arch cut at the crown as shown by Fig. 1. Each half is divided into ten equal parts as indicated by Fig. 5. Art. 15.

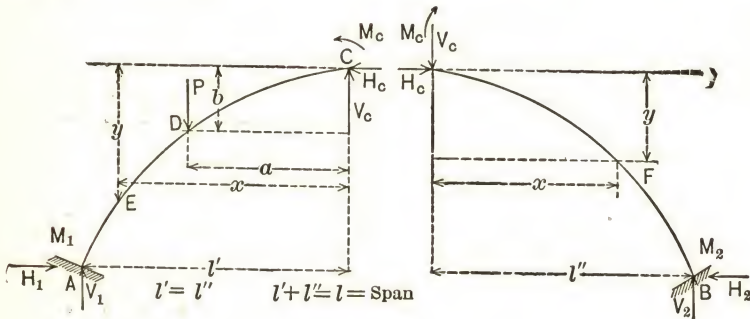


FIG. 1.

Considering a single vertical load  $P$  acting on the left of the crown the bending moment at  $E$  is

$$M_x = M' = M_c + V_c x + H_c y - m \quad (m = P(x - a)), \quad (1)$$

and that at  $F$  is

$$M_x = M'' = M_c - V_c x + H_c y. \quad (2)$$

From the general equations (29), (30), and (31), page 7,

$$+\sum_0^I M' \Delta = -\sum_0^{II} M'' \Delta. \quad . \quad . \quad . \quad . \quad (3)$$

$$\begin{aligned} -\sum_0^I M' y \Delta - H_c \sum_0^I \frac{(\delta x)^2}{F \delta s} + et^0 l' E = +\sum_0^{II} M'' y \Delta \\ + H_c \sum_0^{II} \frac{(\delta x)^2}{F \delta s} - et^0 l'' E. \quad . \quad (4) \end{aligned}$$

$$\begin{aligned} +\sum_0^I M' x \Delta - H_c \sum_0^I \frac{\delta x \delta y}{F \delta s} + et^0 E \sum_0^I \delta y = +\sum_0^{II} M'' x \Delta \\ - H_c \sum_0^{II} \frac{\delta x \delta y}{F \delta s} + et^0 E \sum_0^{II} \delta y. \quad . \quad . \quad . \quad (5) \end{aligned}$$

Since the arch is symmetrical about the crown these equations become

$$+\sum_0^I M' \Delta + \sum_0^{II} M'' \Delta = 0. \quad . \quad . \quad . \quad . \quad (6)$$

$$-\sum_0^I M' y \Delta - H_c \sum_0^I \frac{(\delta x)^2}{F \delta s} + et^0 l' E - \sum_0^{II} M'' y \Delta = 0. \quad . \quad (7)$$

$$+\sum_0^I M' x \Delta - \sum_0^{II} M'' x \Delta = 0. \quad . \quad . \quad . \quad . \quad (8)$$

Substituting (1) and (2), in (6), (7), and (8)

$$+M_c \Sigma \Delta + H_c \Sigma y \Delta - \sum_0^I m \Delta = 0. \quad . \quad . \quad . \quad (9)$$

$$-M_c \Sigma y \Delta - H_c \Sigma y^2 \Delta - H_c \Sigma \frac{(\delta x)^2}{F \delta s} + \sum_0^I m y \Delta + et^0 l' E = 0. \quad (10)$$

$$+V_c \Sigma x^2 \Delta - \sum_0^I m x \Delta = 0. \quad . \quad . \quad . \quad (11)$$

From (9) and (10), eliminating  $M_c$

$$H_c = \frac{\frac{\nu}{0} \Sigma m \Delta \left\{ \gamma - \frac{\Sigma y \Delta}{\Sigma \Delta} \right\} + e t^0 l E}{\Sigma y \Delta \left\{ \gamma - \frac{\Sigma y \Delta}{\Sigma \Delta} \right\} + \Sigma \frac{(\delta x)^2}{F \delta s}} \quad \dots \quad (12)$$

or

$$H_c = \frac{\frac{\nu}{0} \Sigma m \mathbf{B}' + e t^0 l E}{\Sigma y \mathbf{B}' + \Sigma \frac{(\delta x)^2}{F \delta s}} \quad \dots \quad (13)$$

For a single load

$$\frac{\nu}{0} \Sigma m \mathbf{B}' = \left\{ \frac{\nu}{a} \Sigma z \mathbf{B}' - k \frac{\nu}{a} \Sigma \mathbf{B}' \right\} \frac{\delta x}{2} P. \quad \dots \quad (14)$$

From (9)

$$M_c = -H_c \frac{\Sigma y \Delta}{\Sigma \Delta} + \frac{\frac{\nu}{0} \Sigma m \Delta}{\Sigma \Delta}. \quad \dots \quad (15)$$

For a single load

$$\frac{\nu}{0} \Sigma m \Delta = \left\{ \frac{\nu}{a} \Sigma z \Delta - k \frac{\nu}{a} \Sigma \Delta \right\} \frac{\delta x}{2} P. \quad \dots \quad (16)$$

From (11)

$$V_c = \frac{\frac{\nu}{0} \Sigma m x \Delta}{\Sigma x^2 \Delta}. \quad \dots \quad (17)$$

For a single load

$$\frac{\nu}{0} \Sigma m x \Delta = \left\{ \frac{\nu}{a} \Sigma z^2 \Delta - k \frac{\nu}{a} \Sigma z \Delta \right\} \left( \frac{\delta x}{2} \right)^2 P. \quad \dots \quad (18)$$

$$\Sigma x^2 \Delta = \Sigma z^2 \Delta \left( \frac{\delta x}{2} \right)^2. \quad \dots \quad (19)$$

The following tables show the application of the above formulas. The example selected is the second example given in Chapter IV. The values of  $H_c$  found in Table IIa should be the same as  $H$  found in Table II, Chapter IV.



TABLE Ia.  
DATA AND CONSTANTS. $n=20.$ 

| Point. | 1    | 2     | 3                | 4             | 5    | 6        | 7                               | 8                            | 9          | 10                     | 11   | 12               | 13                                       | 14                                                          | 15           |
|--------|------|-------|------------------|---------------|------|----------|---------------------------------|------------------------------|------------|------------------------|------|------------------|------------------------------------------|-------------------------------------------------------------|--------------|
|        | $h$  | $h^3$ | $\frac{h^3}{12}$ | $\frac{h}{2}$ | $h'$ | $(h')^2$ | $\frac{\frac{1}{6}(h')^3}{I_s}$ | $\frac{\bar{y}(3)+(7)}{I_x}$ | $\delta s$ | $\frac{\delta s}{I_x}$ | $y$  | $y\Delta$        | $y-\frac{\Sigma y\Delta}{\Sigma \Delta}$ | $\Delta\left(y-\frac{\Sigma y\Delta}{\Sigma \Delta}\right)$ | $yB'$        |
| 1      | 3.80 | 54.87 | 4.57             | 1.90          | 1.73 | 2.99     | 0.598                           | 5.17                         | 3.41       | 0.66                   | 8.02 | 5.2932           | +7.057                                   | +4.658                                                      | +37.357      |
| 2      | 2.80 | 21.95 | 1.83             | 1.40          | 1.23 | 1.51     | 0.302                           | 2.13                         | 3.21       | 1.51                   | 6.07 | 9.1657           | +5.107                                   | +7.711                                                      | +46.806      |
| 3      | 2.08 | 9.00  | 0.75             | 1.04          | 0.87 | 0.77     | 0.154                           | 0.90                         | 3.08       | 3.42                   | 4.46 | 15.2532          | +3.497                                   | +11.960                                                     | +53.342      |
| 4      | 1.54 | 3.65  | 0.30             | 0.77          | 0.60 | 0.36     | 0.072                           | 0.37                         | 2.92       | 7.89                   | 3.17 | 25.0113          | +2.207                                   | +17.413                                                     | +55.199      |
| 5      | 1.38 | 2.63  | 0.22             | 0.69          | 0.52 | 0.27     | 0.054                           | 0.27                         | 2.81       | 10.41                  | 2.14 | 22.2774          | +1.177                                   | +12.252                                                     | +26.219      |
| 6      | 1.30 | 2.20  | 0.18             | 0.65          | 0.48 | 0.23     | 0.046                           | 0.23                         | 2.75       | 11.96                  | 1.35 | 16.1460          | +0.387                                   | +4.629                                                      | +6.299       |
| 7      | 1.19 | 1.69  | 0.14             | 0.59          | 0.42 | 0.18     | 0.036                           | 0.18                         | 2.74       | 15.22                  | 0.82 | 12.4804          | - .142                                   | - 2.161                                                     | - 1.772      |
| 8      | 1.10 | 1.33  | 0.11             | 0.55          | 0.38 | 0.14     | 0.028                           | 0.14                         | 2.70       | 19.28                  | 0.41 | 7.9048           | - .552                                   | -10.643                                                     | - 4.364      |
| 9      | 1.03 | 1.09  | 0.09             | 0.51          | 0.34 | 0.12     | 0.024                           | 0.11                         | 2.68       | 24.36                  | 0.14 | 3.4104           | - .822                                   | -20.024                                                     | - 2.803      |
| 10     | 1.00 | 1.00  | 0.08             | 0.50          | 0.33 | 0.11     | 0.022                           | 0.10                         | 2.68       | 26.80                  | 0    | 0                | - .962                                   | -25.782                                                     | 0            |
|        |      |       |                  |               |      |          |                                 |                              |            |                        |      | 116.9424         |                                          | +58.623                                                     | +225.222     |
|        |      |       |                  |               |      |          |                                 |                              |            |                        |      | 2                |                                          | -58.610                                                     | - 8.939      |
|        |      |       |                  |               |      |          |                                 |                              |            |                        |      | 233.8848         |                                          |                                                             |              |
|        |      |       |                  |               |      |          |                                 |                              |            |                        |      | $\Sigma y\Delta$ |                                          |                                                             |              |
|        |      |       |                  |               |      |          |                                 |                              |            |                        |      |                  |                                          | $\frac{.013}{\frac{1}{2}\Sigma B'}$                         | +216.283     |
|        |      |       |                  |               |      |          |                                 |                              |            |                        |      |                  |                                          |                                                             | +432.566     |
|        |      |       |                  |               |      |          |                                 |                              |            |                        |      |                  |                                          |                                                             | $\Sigma yB'$ |

REMARKS. When the numerical work is correct  $\Sigma B'=0$ . $\Sigma yB'$  = the denominator of the expression for  $H_c$  when the effect of the axial stress is neglected.

All of the quantities in this table are independent of the loading and are constants for this arch.

TABLE IIa.  
HORIZONTAL THRUST PRODUCED BY UNIT VERTICAL LOADS.  
(Effect of Axial Stress Neglected.)

| Point. | 1        | 2         | 3       | 4                         | 5                        | 6                          | 7                 | 8                                                            |
|--------|----------|-----------|---------|---------------------------|--------------------------|----------------------------|-------------------|--------------------------------------------------------------|
| z or k |          | B'        | zB'     | $l' \frac{\Sigma zB'}{a}$ | $l' \frac{\Sigma B'}{a}$ | $l' \frac{k \Sigma B'}{a}$ | Col.<br>(4) - (6) | $\frac{\text{Col. (7) } \delta x}{\Sigma yB' \frac{2}{H_c}}$ |
| 1      | + 4.658  | + 88.502  | 0       | 0                         | 0                        | 0                          | 0                 | 0                                                            |
| 2      | + 7.711  | + 131.087 | 88.502  | 4.658                     | 79.186                   | 9.316                      | 0.029             |                                                              |
| 3      | + 11.960 | + 179.400 | 219.589 | 12.369                    | 185.535                  | 34.054                     | 0.106             |                                                              |
| 4      | + 17.413 | + 226.369 | 398.989 | 24.329                    | 316.277                  | 82.712                     | 0.256             |                                                              |
| 5      | + 12.252 | + 134.772 | 625.358 | 41.742                    | 459.162                  | 166.196                    | 0.515             |                                                              |
| 6      | + 4.629  | + 41.661  | 760.130 | 53.994                    | 485.946                  | 274.184                    | 0.850             |                                                              |
| 7      | - 2.161  | - 15.127  | 801.791 | 58.623                    | 410.361                  | 391.430                    | 1.213             |                                                              |
| 8      | - 10.643 | - 53.215  | 786.664 | 56.462                    | 282.310                  | 504.354                    | 1.563             |                                                              |
| 9      | - 20.024 | - 60.072  | 733.449 | 45.819                    | 137.457                  | 595.992                    | 1.847             |                                                              |
| 10     | - 25.782 | - 25.782  | 673.377 | 25.795                    | 25.795                   | 647.582                    | 2.007             |                                                              |
|        |          |           |         |                           |                          |                            |                   | 8.386                                                        |
|        |          |           |         |                           |                          |                            |                   | 2                                                            |
|        |          |           |         |                           |                          |                            |                   | 16.772                                                       |
|        |          |           |         |                           |                          |                            |                   | $\Sigma H_c$                                                 |

Use .0031

$\frac{\delta x}{2 \Sigma yB' \frac{2}{H_c}} = \frac{2.68}{2(432.566)} = .00309.$

$H_c = \frac{l' \frac{\Sigma zB' - k \Sigma B'}{a} \frac{\delta x}{a} \frac{P}{2}}{\Sigma yB' \frac{2}{H_c}}$

TABLE IIIa.

EFFECT OF THE AXIAL STRESS UPON THE HORIZONTAL THRUST.

 $n = 20.$ 

| Point.                                          | 1     | 2          | 3               | 4                             | 5                                                                                                                                                                                                                                                                                                                                                                |
|-------------------------------------------------|-------|------------|-----------------|-------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                                                 | $F_x$ | $\delta s$ | $F_x \delta s.$ | $\frac{1}{F_x \delta s}$      | Remarks.                                                                                                                                                                                                                                                                                                                                                         |
| 1                                               | 4.00  | 3.41       | 13.640          | 0.0733                        | $-H_a = \frac{\sum \frac{(\delta x)^2}{F_x \delta s} H_c}{\sum y B' + \sum \frac{(\delta x)^2}{F_x \delta s}} = \frac{4.2912(2.68)^2}{432.566 + 30.82} H_c.$ $\therefore H_a = -0.0665 H_c \text{ for vertical loads.}$ $F_x = h + 0.20. \text{ The area of the steel in equivalent concrete is } 0.20.$ $H_a = -0.0665 H_t \text{ for changes in temperature.}$ |
| 2                                               | 3.00  | 3.21       | 9.630           | .1038                         |                                                                                                                                                                                                                                                                                                                                                                  |
| 3                                               | 2.28  | 3.08       | 7.022           | .1424                         |                                                                                                                                                                                                                                                                                                                                                                  |
| 4                                               | 1.74  | 2.92       | 5.081           | .1968                         |                                                                                                                                                                                                                                                                                                                                                                  |
| 5                                               | 1.58  | 2.81       | 4.440           | .2252                         |                                                                                                                                                                                                                                                                                                                                                                  |
| 6                                               | 1.50  | 2.75       | 4.125           | .2424                         |                                                                                                                                                                                                                                                                                                                                                                  |
| 7                                               | 1.39  | 2.74       | 3.809           | .2625                         |                                                                                                                                                                                                                                                                                                                                                                  |
| 8                                               | 1.30  | 2.70       | 3.510           | .2849                         |                                                                                                                                                                                                                                                                                                                                                                  |
| 9                                               | 1.23  | 2.68       | 3.296           | .3034                         |                                                                                                                                                                                                                                                                                                                                                                  |
| 10                                              | 1.20  | 2.68       | 3.216           | .3109                         |                                                                                                                                                                                                                                                                                                                                                                  |
| See Table Ia for values of $h$ and $\delta s$ . |       |            |                 |                               |                                                                                                                                                                                                                                                                                                                                                                  |
|                                                 |       |            |                 | $2.1456$                      |                                                                                                                                                                                                                                                                                                                                                                  |
|                                                 |       |            |                 | $\frac{1}{2}$                 |                                                                                                                                                                                                                                                                                                                                                                  |
|                                                 |       |            |                 | $4.2912$                      |                                                                                                                                                                                                                                                                                                                                                                  |
|                                                 |       |            |                 | $\frac{1}{\sum F_x \delta s}$ |                                                                                                                                                                                                                                                                                                                                                                  |

TABLE IVa.  
BENDING MOMENT  $M_c$  PRODUCED BY UNIT VERTICAL LOADS.  
(Effect of Axial Stress Neglected.)

| Point. | 1      | 2        | 3         | 4                       | 5                      | 6                        | 7                 | 8                                                       | 9                                                   | 10                         |
|--------|--------|----------|-----------|-------------------------|------------------------|--------------------------|-------------------|---------------------------------------------------------|-----------------------------------------------------|----------------------------|
|        | z or k | $\Delta$ | $z\Delta$ | $\frac{\nu'}{2z\Delta}$ | $\frac{\nu'}{z\Delta}$ | $k\frac{\nu'}{2z\Delta}$ | Col.<br>(4) - (6) | Col. (7)<br>$\frac{\delta x}{2 \cdot z\Delta} \cdot D'$ | $-H_c \frac{\Sigma y \Delta}{\Sigma \Delta}$<br>-C' | Col.<br>(9) + (8)<br>$M_c$ |
| 1      | 19     | 0.66     | 12.54     | 0                       | 0                      | 0                        | 0                 | .0                                                      | 0                                                   | 0                          |
| 2      | 17     | 1.51     | 25.67     | 12.54                   | 0.66                   | 11.22                    | 1.32              | .0072                                                   | -.0279                                              | -.0207                     |
| 3      | 15     | 3.42     | 51.30     | 38.21                   | 2.17                   | 32.55                    | 5.66              | .0311                                                   | -.1020                                              | -.0709                     |
| 4      | 13     | 7.89     | 102.57    | 89.51                   | 5.59                   | 72.67                    | 16.84             | .0926                                                   | -.2464                                              | -.1538                     |
| 5      | 11     | 10.41    | 114.51    | 192.08                  | 13.48                  | 148.28                   | 43.80             | .2409                                                   | -.4956                                              | -.2547                     |
| 6      | 9      | 11.96    | 107.64    | 306.59                  | 23.89                  | 215.01                   | 91.58             | .5037                                                   | -.8180                                              | -.3143                     |
| 7      | 7      | 15.22    | 106.54    | 414.23                  | 35.85                  | 259.95                   | 163.28            | .8980                                                   | -.11674                                             | -.2694                     |
| 8      | 5      | 19.28    | 96.40     | 520.77                  | 51.07                  | 255.35                   | 265.42            | 1.4598                                                  | -.1.5042                                            | -.0443                     |
| 9      | 3      | 24.36    | 73.08     | 617.17                  | 70.35                  | 211.05                   | 406.12            | 2.2337                                                  | -.1.7775                                            | + .4560                    |
| 10     | 1      | 26.80    | 26.80     | 690.25                  | 94.71                  | 94.71                    | 595.54            | 3.2785                                                  | -.1.9315                                            | + 1.3470                   |

$$M_c = -H_c \frac{\Sigma y \Delta}{\Sigma \Delta} + \left\{ \frac{\nu'}{a} \frac{\nu'}{2z\Delta} - k \frac{\nu'}{2z\Delta} \right\} \frac{\delta x}{2 \Sigma \Delta} P.$$
  

$$\frac{\delta x}{2 \Sigma \Delta} = \frac{2.68}{2(243.02)} = 0.0055.$$

To include the effect of the Axial Stress use the values of  $H_c$  which include the effect of this stress.  
For a unit load at each of the twenty points the bending moment  $M_c = +1.3498$ .



TABLE Va.  
SHEAR  $V_c$  PRODUCED BY UNIT VERTICAL LOADS.

| Point. | 1            | 2     | 3        | 4           | 5                               | 6                             | 7                               | 8              | 9     |
|--------|--------------|-------|----------|-------------|---------------------------------|-------------------------------|---------------------------------|----------------|-------|
|        | $z$ or $k$ . | $z^2$ | $\Delta$ | $z^2\Delta$ | $\frac{V'}{a} \Sigma z^2\Delta$ | $\frac{V'}{a} \Sigma z\Delta$ | $\frac{V'}{a} k \Sigma z\Delta$ | Col. (5) - (7) | $V_c$ |
| 1      | 19           | 361   | 0.66     | 238.26      | 0                               | 0                             | 0                               | 0              | 0     |
| 2      | 17           | 289   | 1.51     | 436.39      | 238.26                          | 12.54                         | 213.18                          | 25.08          | .002  |
| 3      | 15           | 225   | 3.42     | 769.50      | 674.65                          | 38.21                         | 573.15                          | 101.50         | .008  |
| 4      | 13           | 169   | 7.89     | 1333.41     | 1444.15                         | 89.51                         | 1163.63                         | 280.52         | .022  |
| 5      | 11           | 121   | 10.41    | 1259.61     | 2777.56                         | 192.08                        | 2112.88                         | 664.68         | .051  |
| 6      | 9            | 81    | 11.96    | 968.76      | 4037.17                         | 306.59                        | 2759.31                         | 1277.86        | .099  |
| 7      | 7            | 49    | 15.22    | 745.78      | 5005.93                         | 414.23                        | 2899.61                         | 2106.32        | .163  |
| 8      | 5            | 25    | 19.28    | 482.00      | 5751.71                         | 520.77                        | 2603.85                         | 3147.86        | .243  |
| 9      | 3            | 9     | 24.36    | 219.24      | 6233.71                         | 617.17                        | 1851.51                         | 4382.20        | .338  |
| 10     | 1            | 1     | 26.80    | 26.80       | 6452.95                         | 690.25                        | 690.25                          | 5762.70        | .445  |

$$V_c = \frac{\frac{V'}{a} \Sigma mx\Delta}{\frac{V'}{a} \Sigma x^2\Delta} = \frac{\frac{V'}{a} \Sigma z^2\Delta - k \Sigma z\Delta}{\frac{V'}{a} \Sigma z^2\Delta}.$$

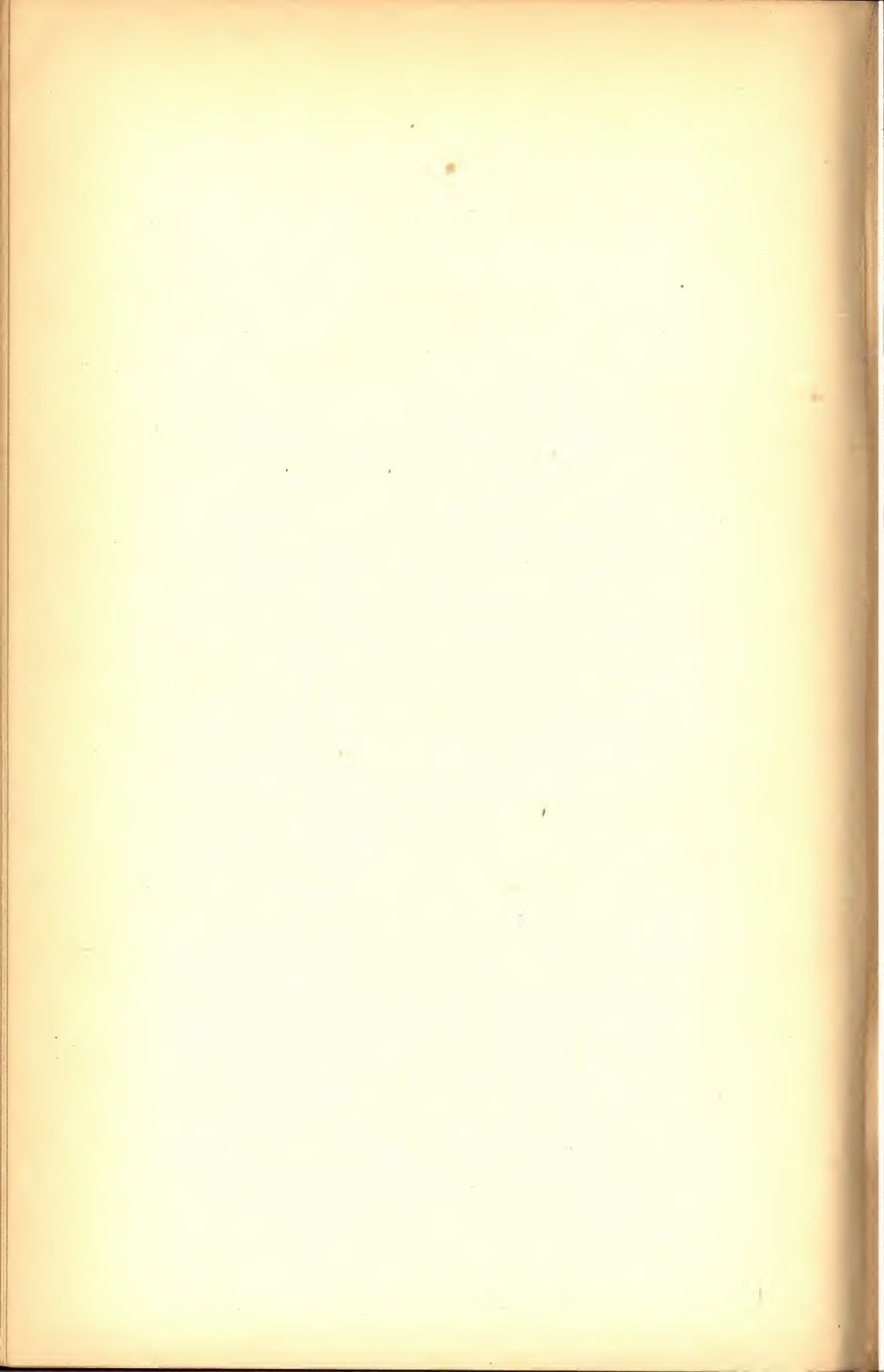
$\Sigma z^2\Delta = 12959.5$   
Use 12960.

Col. (6) is taken from Col. (4), Table IVa.

TABLE VIa.  
VALUES OF  $H_c$ ,  $M_c$ , AND  $V_c$  FOR UNIT VERTICAL LOADS.  
(Effect of Axial Stress Neglected.)

 $n=20.$ 

| Point. | 1            | 2            | 3            | Point. | 4                                                                                                       |
|--------|--------------|--------------|--------------|--------|---------------------------------------------------------------------------------------------------------|
|        | $H_c$        | $M_c$        | $V_c$        |        |                                                                                                         |
| 1      | 0            | 0            | 0            | 1'     | Symmetrical loads produce like thrusts and moments but unlike shears. For symmetrical loading $V_c=0$ . |
| 2      | .029         | — .0207      | .002         | 2'     |                                                                                                         |
| 3      | .106         | — .0709      | .008         | 3'     |                                                                                                         |
| 4      | .256         | — .1538      | .022         | 4'     |                                                                                                         |
| 5      | .515         | — .2547      | .051         | 5'     |                                                                                                         |
| 6      | .850         | — .3143      | .099         | 6'     |                                                                                                         |
| 7      | 1.213        | — .2694      | .163         | 7'     |                                                                                                         |
| 8      | 1.563        | — .0443      | .243         | 8'     |                                                                                                         |
| 9      | 1.847        | + .4560      | .338         | 9'     |                                                                                                         |
| 10     | 2.007        | +1.3470      | .445         | 10'    |                                                                                                         |
|        | 8.386        | —1.1281      | 1.371        |        |                                                                                                         |
|        | 2            | +1.8030      | $\Sigma V_c$ |        |                                                                                                         |
|        | 16.772       | + .6749      | $-V_c$       |        |                                                                                                         |
|        | $\Sigma H_c$ | 2            |              |        |                                                                                                         |
|        |              | +1.3498      |              |        |                                                                                                         |
|        |              | $\Sigma M_c$ |              |        |                                                                                                         |



## APPENDIX D.

### UNSYMMETRICAL FIXED ARCH.

The treatment of the unsymmetrical fixed arch for any loading is very similar to that for the symmetrical arch. The span is assumed divided into  $n$  equals parts. In addition to the general nomenclature let

- $c$  = the difference in elevation of the two supports;
- $y'$  = the ordinate to the arch axis measured vertically from a line joining the ends of the arch axis;
- $m_x$  = the bending moment produced by any loading considering the arch as hinged at the right support and on rollers at the left support.

$$H_1 = \frac{\Sigma m_x B'' \Sigma A'' - \Sigma m_x A'' \Sigma B'' - (\Sigma K' - \Sigma T)(W' \Sigma A'' - \Sigma B'') - (\Sigma K'' - \Sigma T') U' \Sigma A''}{\Sigma y' B'' \Sigma A'' - \Sigma y' A'' \Sigma B'' + (W' \Sigma A'' - \Sigma B'') U + W U' \Sigma A''}$$

$$A'' = \Delta \left\{ y - \frac{\Sigma z y \Delta}{\Sigma z \Delta} + {}^* \Sigma \frac{\delta y}{F_x \delta s} \cdot \frac{1}{\Sigma z \Delta} \right\}$$

$$B'' = \Delta \left\{ y - z \frac{\Sigma z y \Delta}{\Sigma z^2 \Delta} + y \Sigma \frac{(\delta y)^2}{F_x \delta s} \cdot \frac{1}{\Sigma z^2 \Delta} \left( \frac{2}{\delta x} \right)^2 + {}^* \Sigma \frac{\delta x \delta y}{F_x \delta s} \cdot \frac{1}{\Sigma z^2 \Delta} \cdot \frac{2}{\delta x} \right\}$$

$$U = \Sigma \frac{(\delta x)^2}{F_x \delta s} + \frac{c}{l} \Sigma \frac{\delta x \delta y}{F_x \delta s}$$

---

\* This term may be omitted without much error.



$$U' = \frac{\Sigma zy \Delta}{\Sigma z^2 \Delta} \frac{2}{\delta x} - \Sigma^* \frac{\delta x \delta y}{F_x \delta s} \cdot \frac{1}{\Sigma z^2 \Delta} \left( \frac{2}{\delta x} \right)^2$$

$$W = \Sigma \frac{\delta x \delta y}{F_x \delta s} + \frac{c}{l} \Sigma \frac{(\delta y)^2}{F_x \delta s}$$

$$W' = 1 + \Sigma^* \frac{(\delta y)^2}{F_x \delta s} \cdot \frac{1}{\Sigma z^2 \Delta} \left( \frac{2}{\delta x} \right)^2$$

$$\Sigma T = et^\circ El \quad \Sigma T' = et^\circ Ec$$

$$\Sigma K' = \left\{ P \frac{l-a}{l} - Q \frac{c-b}{l} \right\} \Sigma \frac{\delta x \delta y}{F_x \delta s} - P \Sigma_a^l \frac{\delta x \delta y}{F_x \delta s} - Q \Sigma_a^l \frac{(\delta x)^2}{F_x \delta s}$$

$$\Sigma K'' = \left\{ P \frac{l-a}{l} - Q \frac{c-b}{l} \right\} \Sigma \frac{(\delta y)^2}{F_x \delta s} - P \Sigma_a^l \frac{(\delta y)^2}{F_x \delta s} - Q \Sigma_a^l \frac{\delta x \delta y}{F_x \delta s}$$

$\Sigma K'$  and  $\Sigma K''$  are worked out for *each* value of  $P$  and  $Q$ .

$$M_1 = H_1 \frac{\Sigma y' G'' - W \frac{2}{\delta x}}{\Sigma G''} - \frac{\Sigma m_x G''}{\Sigma G''} - \frac{\Sigma K'' - \Sigma T'}{\Sigma G''} \frac{2}{\delta x}$$

$$M_2 = M_1 - \frac{H_1 \left( \Sigma y' G - W \frac{2}{\delta x} \right) - \Sigma m_x G - (\Sigma K'' - \Sigma T') \frac{2}{\delta x}}{\Sigma G''} \frac{2n \Sigma \Delta}{\Sigma z \Delta}$$

$$G = \Delta \left\{ z - \frac{\Sigma z \Delta}{\Sigma \Delta} \right\}$$

$$G'' = \Delta \left\{ z - \frac{\Sigma z^2 \Delta}{\Sigma z \Delta} - \Sigma^* \frac{(\delta y)^2}{F_x \delta s} \cdot \frac{1}{\Sigma z \Delta} \left( \frac{2}{\delta x} \right)^2 \right\}$$

---

\* This term may be omitted without much error.

## APPENDIX E.

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### INTERNAL TEMPERATURE RANGE IN CONCRETE BRIDGES.

In a recent\* publication of the Iowa State College of Agriculture and Mechanic Arts, is given an account of a series of experiments to determine the range of temperature change in Concrete Arch Bridges. The experiments were made upon two bridges (Boone Street Viaduct, Ames, Iowa, and Walnut Street Bridge, Des Moines, Iowa) and extended over a period of about two years. As a result of the experiments the following conclusions are drawn:

(a) The yearly range in temperature in a reinforced concrete arch bridge is not far from  $80^{\circ}$  Fahrenheit.

(b) The relation between the depth of concrete covering any point and the yearly temperature range for this point may be found from the following formula:

$$R = 90 - 0.53 D,$$

where  $R$  equals the yearly temperature range in degrees Fahrenheit and  $D$  equals the distance from the point to the nearest exposed surface in inches. The results obtained

---

\* Bulletin No. 30, Engineering Experiment Station, Iowa State College of Agriculture and Mechanic Arts, Ames, Iowa. The Determination of Internal Temperature Range in Concrete Bridges, by C. S. Nichols and C. B. McCullough.

from the experiments indicate that the range of temperature change in concrete is considerably larger than suggested in the body of this book.

Considering the length of time over which the experiments extended and the care with which they were made, the range of 80° Fahrenheit is probably about correct for ordinary concrete arch bridges located in places having the same climatic conditions as Iowa.

Experiments show that the heat generated in the setting of concrete is considerable. The average maximum temperature is about 100° Fahrenheit which is reached in one or two days after the concrete is placed. This apparently indicates that all subsequent variation in temperature is a drop. Probably this is not true and until more is known about the matter the variation in temperature may be taken as 40° Fahrenheit (see Bulletin referred to above) each way from an assumed temperature of no stress.

The effect of the large masses of masonry upon range in temperature is shown by the experiment made during the construction of the Boonton, N. J., dam.

#### RESULTS OF THERMOPHONE OBSERVATIONS, BOONTON, N. J., DAM.

(Trans. Am. Soc. Civ. Engrs., Vol. LXI, p. 420.)

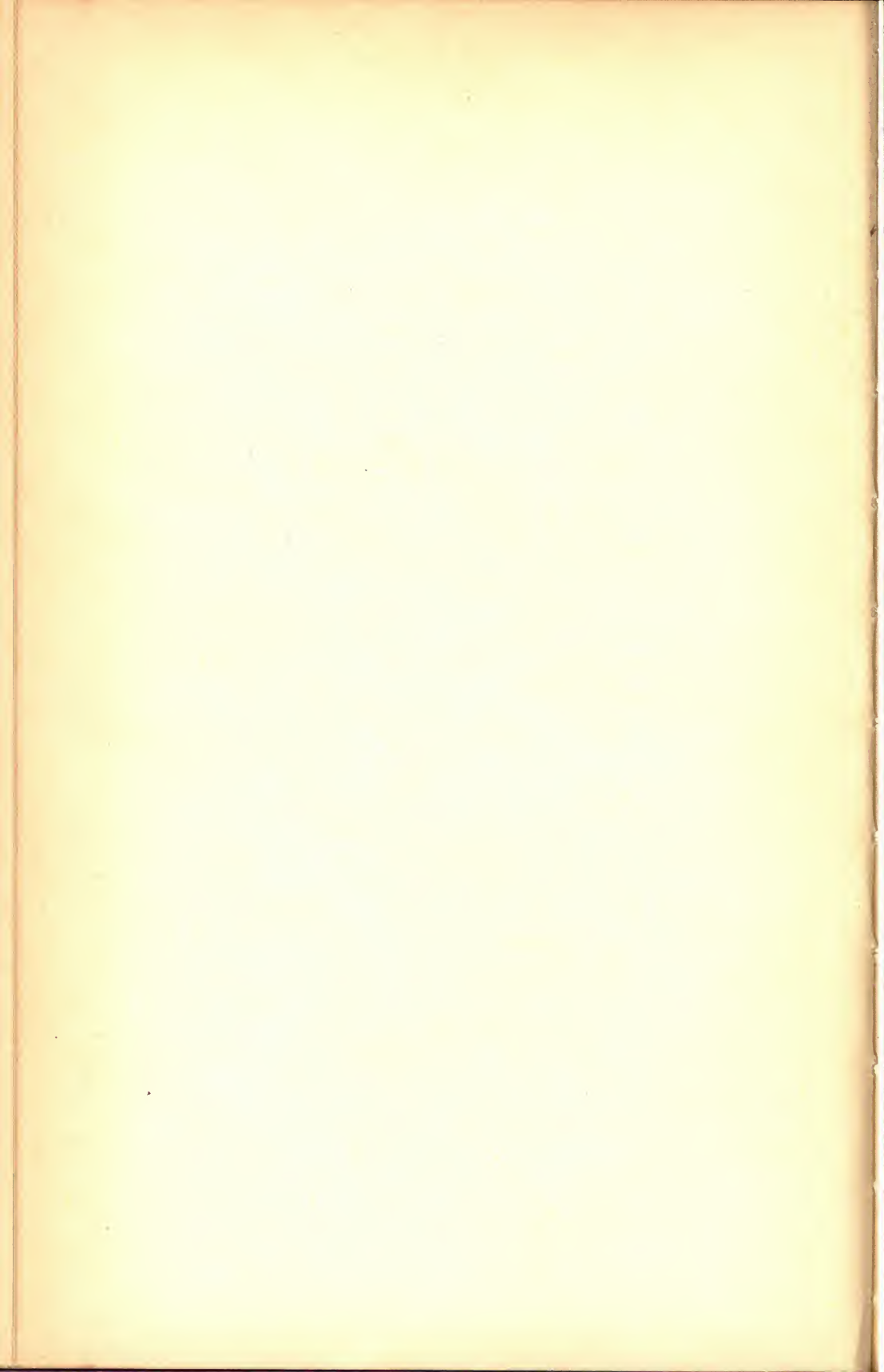
| Coil No. | Distance from Face in Feet. | Max. Temp., Degrees F. | Date of Max. Temp. | Minimum Temp. Degrees F. | Date of Minimum Temp. | Total Temp. Range, Degrees F. |
|----------|-----------------------------|------------------------|--------------------|--------------------------|-----------------------|-------------------------------|
| 5        | 2.0                         | 69                     | Aug. 1904          | 39                       | March, 1905           | 30                            |
| 6        | 11.3                        | 64                     | Sept. "            | 45                       | " "                   | 19                            |
| 8        | 11.0                        | 63                     | Sept. "            | 43                       | " "                   | 20                            |
| 9        | 2.5                         | 71                     | Aug. "             | 34                       | " "                   | 37                            |
| 10       | 16.0                        | 68                     | Oct. "             | 46                       | " "                   | 22                            |

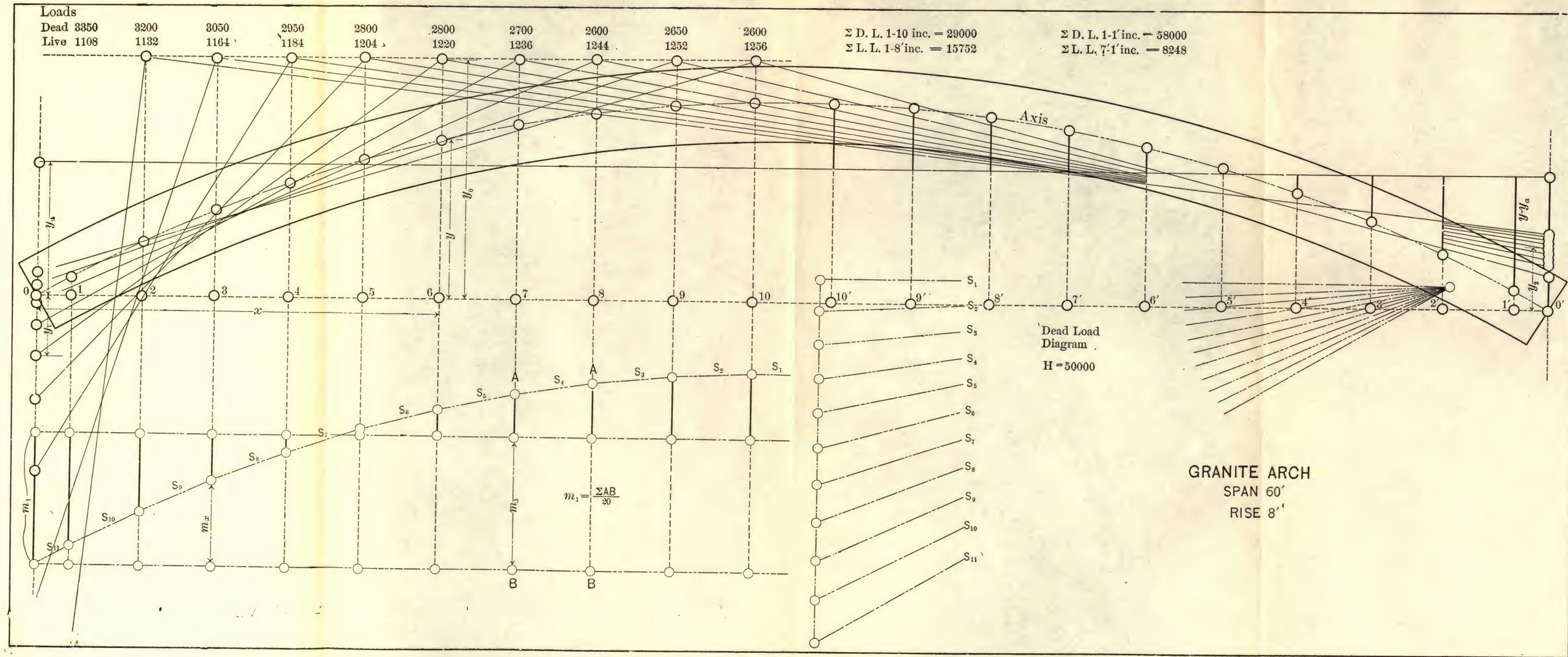
## COEFFICIENTS OF EXPANSION OF MORTAR, CONCRETE, ETC.

(Trans. Am. Soc. Civ. Engrs., Vol. LXI, p. 406.)

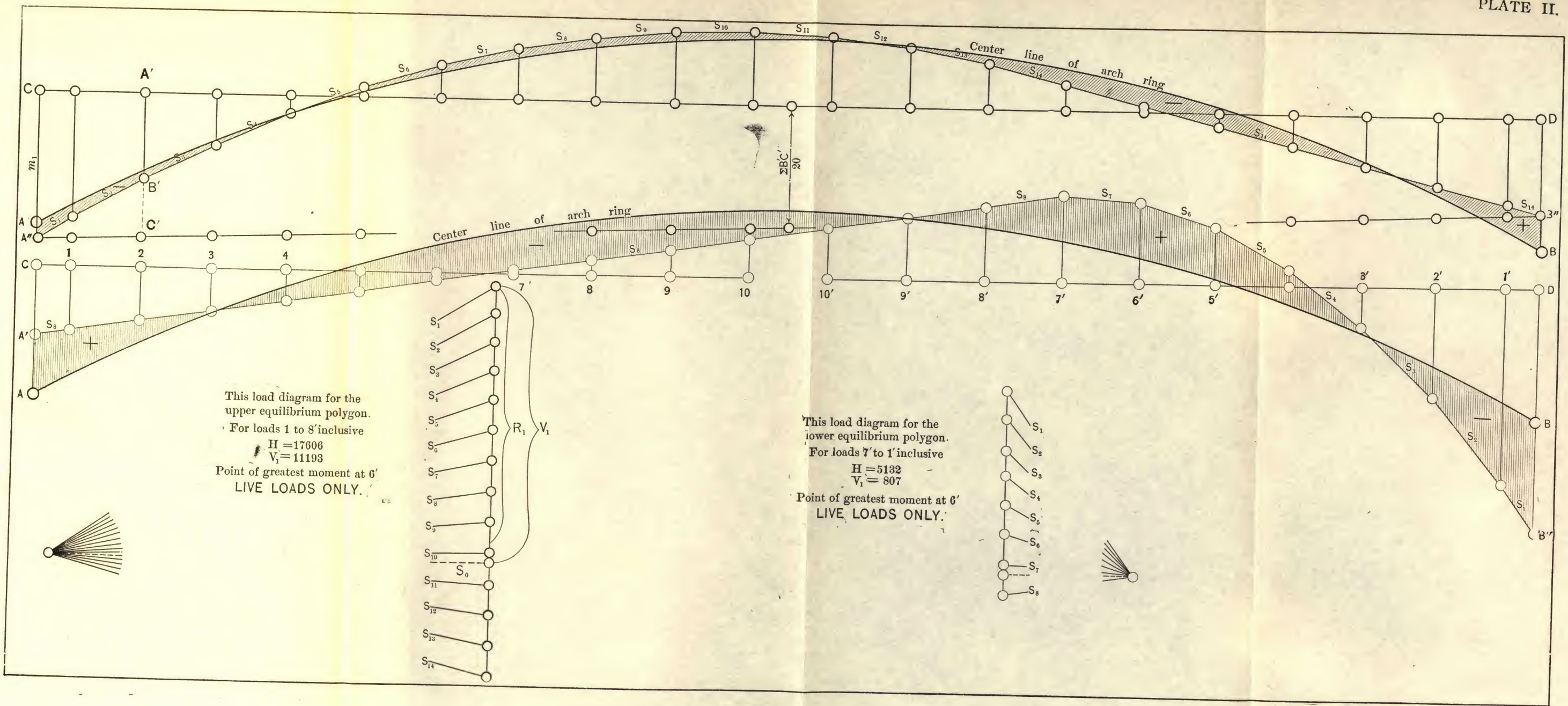
| Authority.                    | Material.                       | Coefficient<br>1° F.   |
|-------------------------------|---------------------------------|------------------------|
| Bonniceau.....                | Neat Portland cement            | .00000594              |
| Annales des Pontset Chaussées | Mortar 1 : 2                    | 655                    |
|                               | Concrete                        | 795                    |
| Christophe,.....              | Concrete                        | 667 to 805             |
| Le Béton Armé                 |                                 |                        |
| Sir A. R. Binnie.....         | Concrete 1 : 4 beam, 100 ft.    | } 436                  |
| Min. of Proc. Inst. C. E.     | long and 1 ft. square           |                        |
| Prof. Pence.....              | Broken stone concrete 1 : 2 : 4 | } 540                  |
| Purdue Univ.                  | 6×6×24 inches                   |                        |
|                               | Broken stone concrete 1 : 2 : 4 | } 560                  |
|                               | 4×4×36 inches                   |                        |
| Prof. Hallock.....            | Concrete 1 : 2 : 5 and 1 : 2    | 608                    |
| Columbia Univ.                |                                 |                        |
| Prof. Lyford.....             | Concrete bars                   | { 560 and<br>.00000640 |
| Worcester Polytechnic         |                                 |                        |



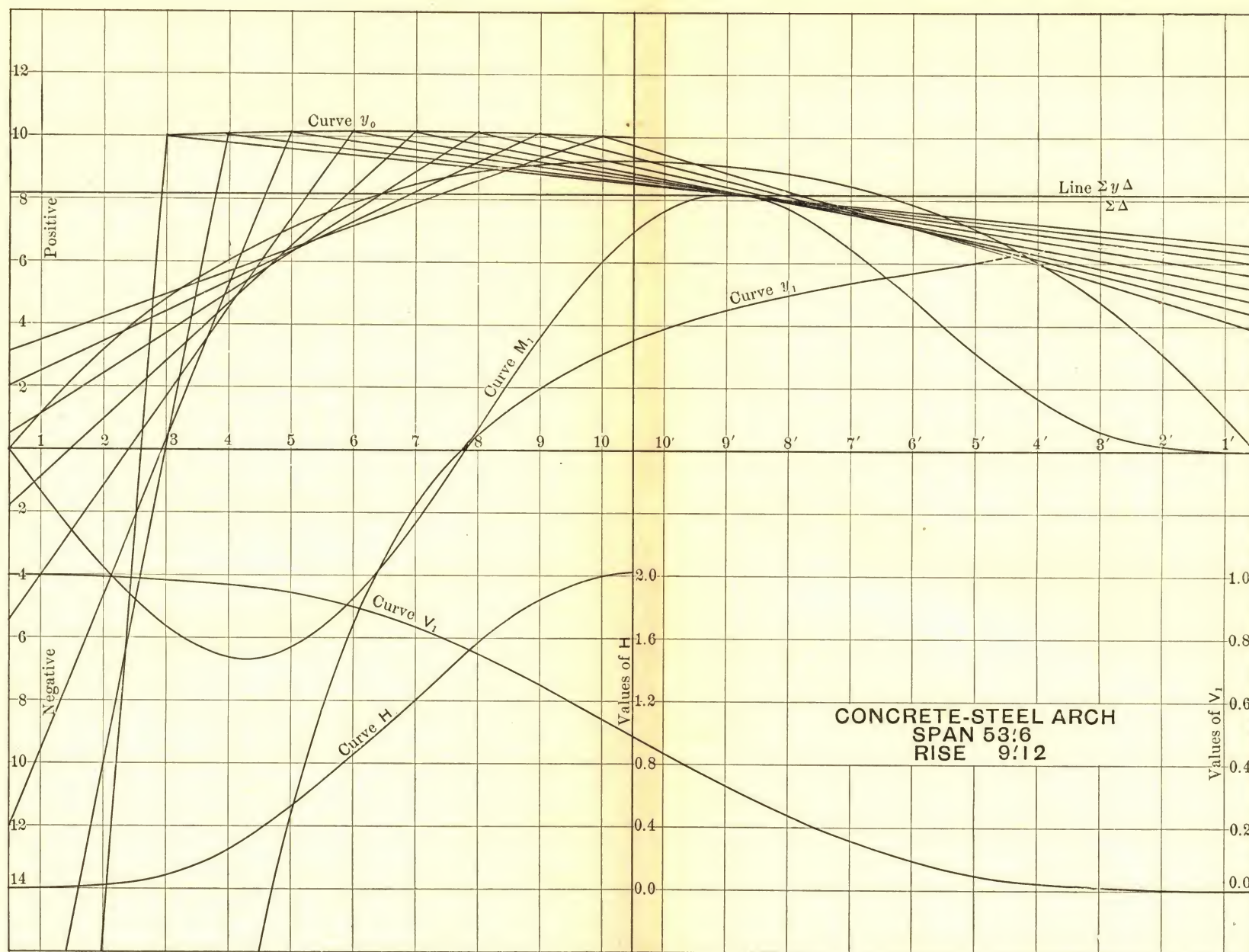










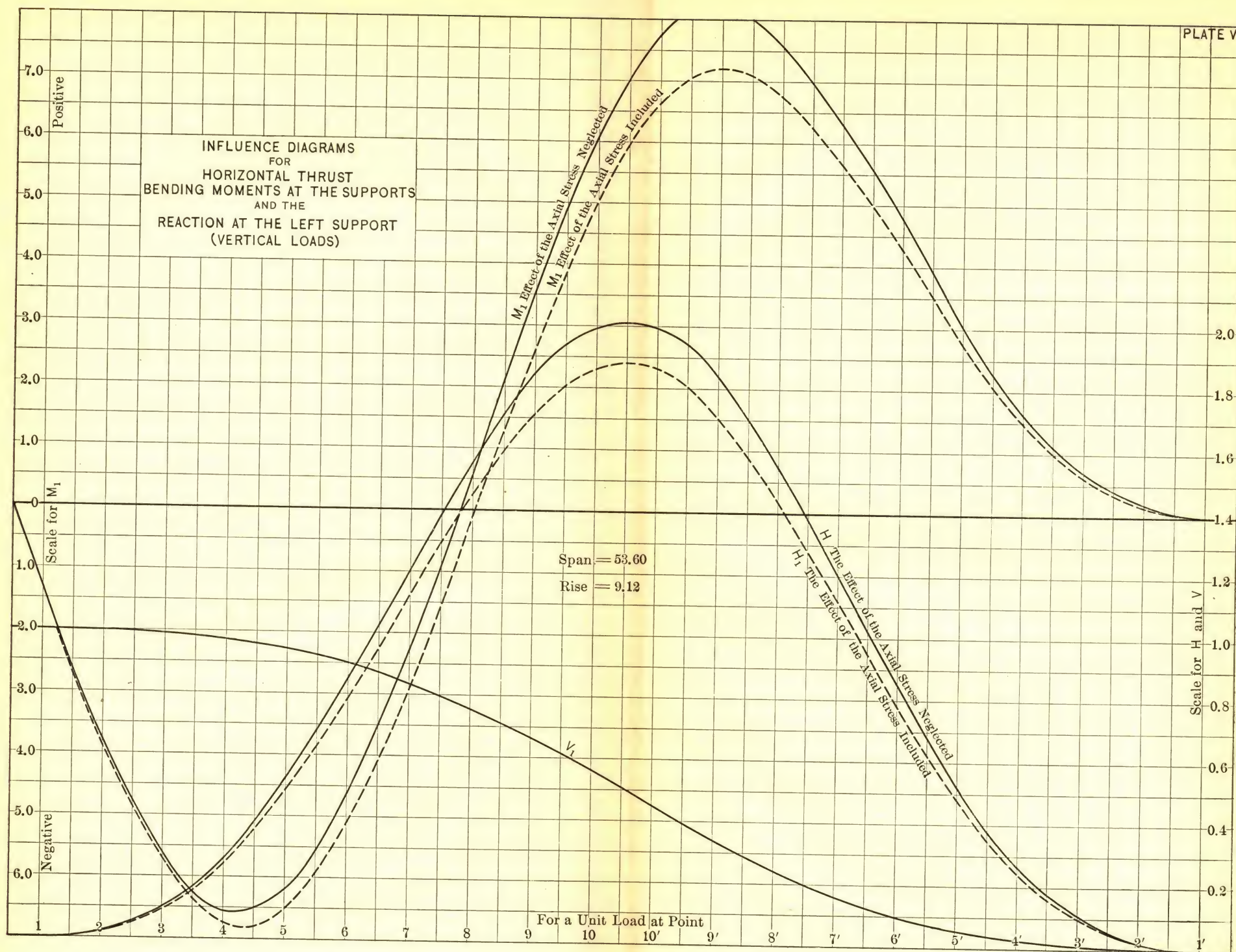




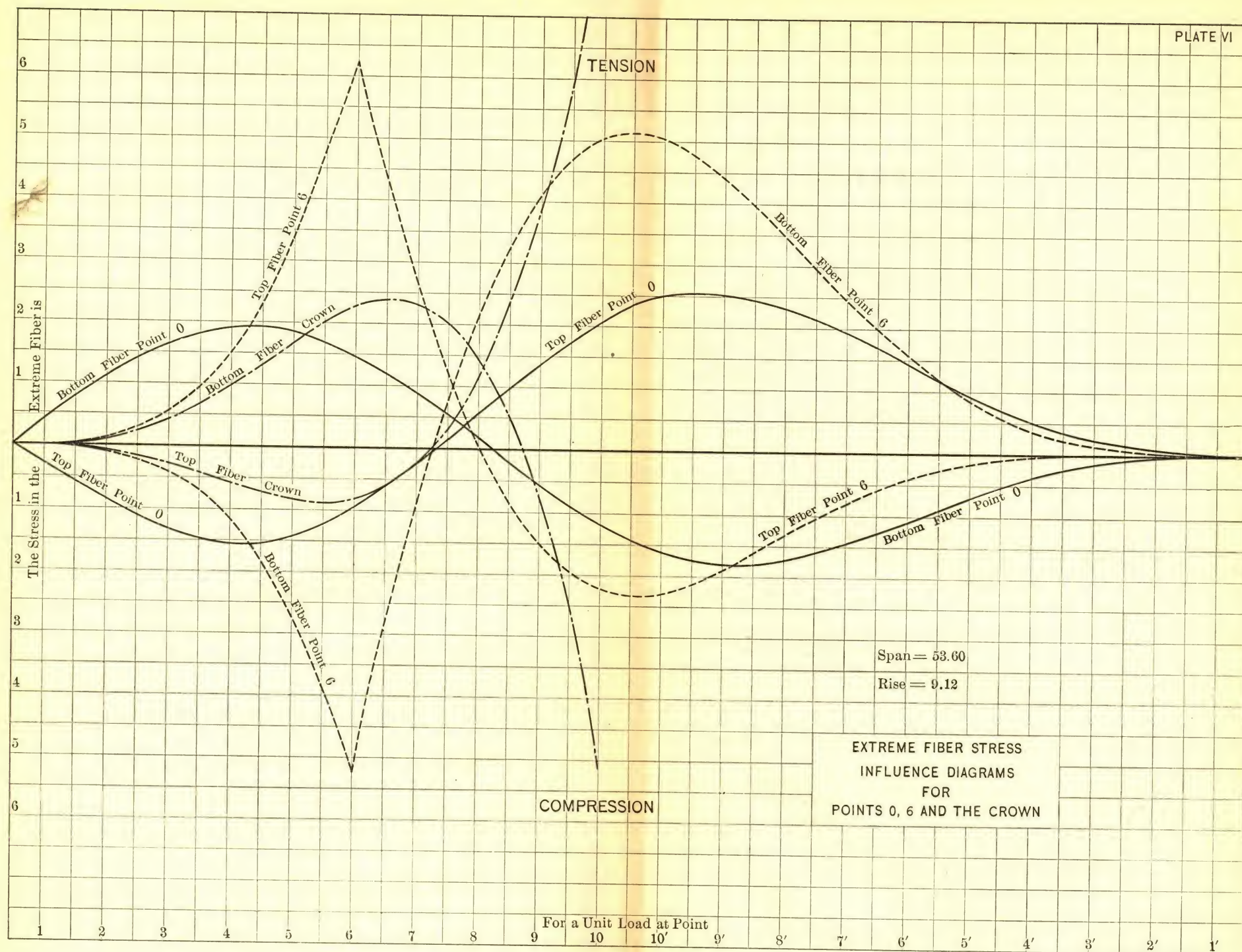




INFLUENCE DIAGRAMS  
FOR  
HORIZONTAL THRUST  
BENDING MOMENTS AT THE SUPPORTS  
AND THE  
REACTION AT THE LEFT SUPPORT  
(VERTICAL LOADS)







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